Family Gauge Bosons with an Inverted Mass Hierarchy

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@大阪大学
Quarks, leptons and Family Gauge Bosons

山下　敏史
（愛知医科大学）

based on
PLB**711**, (2012) 384 (arXiv:1203.2028)

w/ Y. Koide (Osaka U.)

Plan

- Introduction
- Sumino mechanism (in SUSY)
- VEV relations in SUSY
- Summary and discussions

Introduction

- charged lepton mass formula — Y. Koide (1982)

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\,)^2} = \frac{2}{3}$$

Koide-san's talk

exp.) $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$ — $O(10^{-5})$

⇒ $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ $@m_Z$ — $O(10^{-3})$

QED

- Sumino mechanism — Y. Sumino (2009)

family gauge symm. to cancel QED.

e.g. SU(3) w/ both l & e^c being **3** repr. — anomalous

Introduction

- "vector-like" version — Y. Koide & T.Y. (2012)
  - to be SUSY (to forbid unwanted terms)

    ⇨ Family Gauge Bosons
    with an Inverted Mass Hierarchy
    (for Sumino mech. to be worked)

    ⇨ characteristic phenomenology via $(A_\mu)_3^3$

- Sumino mechanism — Y. Sumino (2009)

  family gauge symm. to cancel QED.

  e.g. SU(3) w/ both l & e^c being **3** repr.

  anomalous

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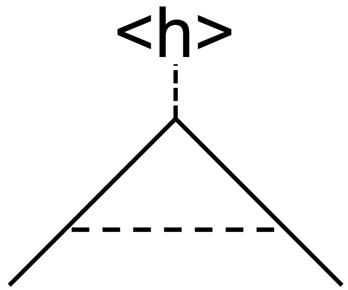
Sumino mechanism

Y. Sumino (2009)

- safe/dangerous corrections

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$



w/ $m_{ei} \to m_{ei}(1 + \varepsilon_0 + \varepsilon_i)$, $\delta K \propto \varepsilon_i$.

gauge etc. (ε_0) — Yukawa etc. (ε_i)

- naively: gauge etc. / Yukawa etc.

$$\frac{y^2}{16\pi^2} \le \frac{(0.01)^2}{100} = 10^{-6}$$

- RGE: decoupling

$$\times(\text{large log})$$

$$\Rightarrow \frac{\Lambda}{m_h} < e^{10}$$

Sumino mechanism

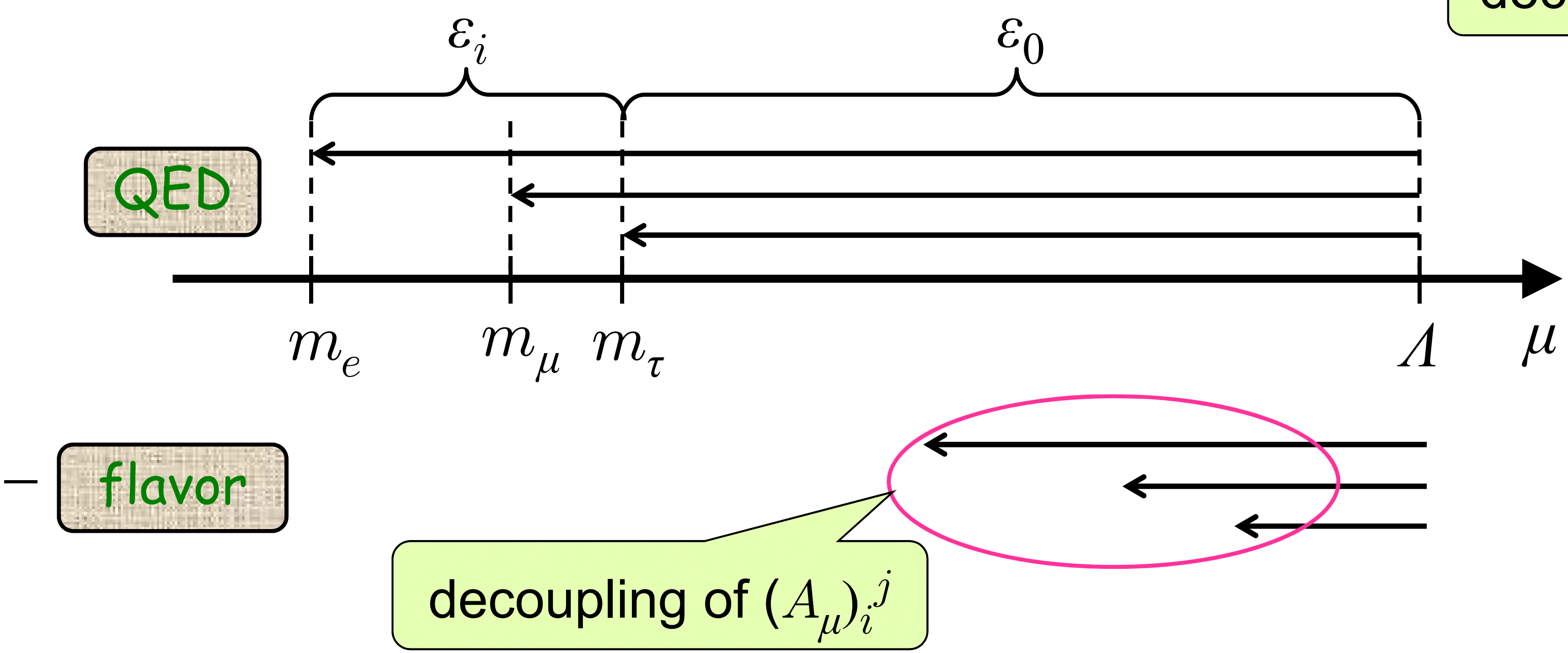
Y. Sumino (2009)

- QED correction (1-loop)

$$m_{ei}(\mu) = m_{ei}^{pole}\left[1 - \frac{\alpha_{em}(\mu)}{\pi}\left(1 + \frac{3}{4}\log\frac{\mu^2}{m_{ei}^2(\mu)}\right)\right]$$

log⇔RGE

decoupling



Sumino mechanism

Y. Sumino (2009)

- model

Q1) generality?

  - U(3)*O(3), w/ $l(\mathbf{3},\mathbf{1})$ & $e^c(\mathbf{3},\mathbf{1})$ (*i.e.* chiral)
  - U(3)-breaking by $<\Phi(\bar{\mathbf{3}},\mathbf{3})>_{\text{diag.}} = \text{diag.}(v_1, v_2, v_3)$

⇒

- ➢ charged lepton masses via $l_i \Phi_I^i \Phi_I^j e_j^c H$ as $le^c H$ is not allowed ⇒ $v_i \propto \sqrt{m_{ei}}$
- ➢ normal hierarchy: $m^2(A_i^j) \propto v_i^2 + v_j^2 \propto m_{ei} + m_{ej}$
- ➢ $A_i^{j\neq i}$ doesn't give vertex corr., which has opposite sign from QED (Q2) wf. renorm.?)
- ➢ $le^c H \times \Phi\Phi$ is not induced (Q3) chiral symm.?)
- ➢ anomalous (lepton/quark sector)

Sumino mechanism

Y. Sumino (2009)

- what happens if "vector-like"
  - U(3)*U(3), w/ $l(\mathbf{3},\mathbf{1})$ & $e^c(\bar{\mathbf{3}},\mathbf{1})$ (*i.e.* chiral)
  - U(3)-breaking by $<\Psi(\bar{\mathbf{3}},\mathbf{3})>_{\rm diag.} = {\rm diag.}(v_1, v_2, v_3)$

Q1) generality?

➢ charged lepton masses via $l_i \Phi_I^i \Phi_I^j e_j^c H$
as $l e^c H$ is ~~not~~ allowed ⇨ $v_i \propto \sqrt{m_{ei}}^{-1}$

➢ inverted hierarchy: $m^2(A_i^j) \propto v_i^2 + v_j^2 \propto m_{ei}^{-1} + m_{ej}^{-1}$

⇨ ➢ $A_i^{j\neq i}$ does~~n't~~ give vertex corr., which has same sign as QED

Q2) wf. renorm.?

SUSY $l e^c H \times \Phi\Phi$ is ~~not~~ induced

Q3) chiral symm.?

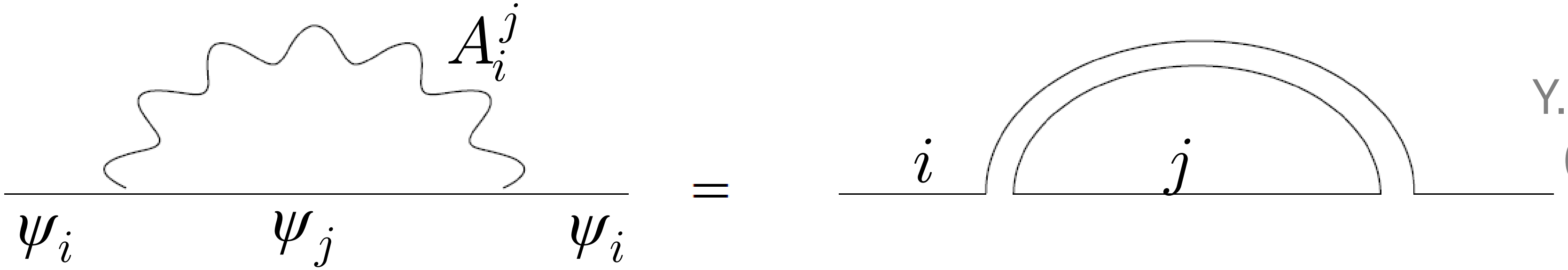
Y. Koide & T.Y. (2012)

less anomalous (lepton/quark sector)

Sumino mechanism in SUSY

Y. Koide & T.Y. (2012)

- wave function renormalization



Y. Sumino (2009)

$$\Rightarrow \delta m_{ei}^F = -m_{ei} \frac{\alpha_F}{2\pi} \sum_j \log \frac{\mu^2}{m(A_i^j)^2}$$

$$= m_{ei}(\varepsilon_0^F + \varepsilon_i^F)$$

$$m(A_i^j)^2 \propto \frac{1}{m_{ei}} + \frac{1}{m_{ej}}$$

$$\varepsilon_3^F = 0$$

y_τ?

$O(10^{-4})$

$$\varepsilon_1^F \propto \frac{1}{2}\log\frac{m_{e3}^2}{m_{e1}^2} + \frac{1}{2}\log\frac{m_{e2}^2}{m_{e1}^2} + \log\frac{1+m_{e1}/m_{e2}}{1+m_{e2}/m_{e3}}$$

$$\varepsilon_2^F \propto \frac{1}{2}\log\frac{m_{e3}^2}{m_{e2}^2} + \log\frac{1+m_{e1}/m_{e2}}{1+m_{e1}/m_{e3}}$$

v.s. QED

small enough

Sumino mechanism in SUSY

Y. Koide & T.Y. (2012)

- model

	ℓ	e^c	H_d	L	$\bar{L}$	E^c	$\bar{E}^c$	Φ	$\bar{\Phi}$	Ψ	$\bar{\Psi}$	θ_Φ	Θ_A	Θ_B	S	θ_S
$SU(2)_L$	**2**	**1**	**2**	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$U(3)$	**3**	**3***	**1**	**1**	**1**	**1**	**1**	**3**	**3***	**3**	**3***	**1**	**8 + 1**	**1**	**1**	**1**
$U(3)'$	**1**	**1**	**1**	**3**	**3***	**3***	**3**	**3***	**3**	**3***	**3**	**1**	**1**	**8 + 1**	**1**	**1**
$U(1)_S$	0	0	−1	1	−1	0	−1	1	1	0	0	−2	−1	−1	1	−1
$U(1)_R$	1	1	0	1	1	1	1	0	0	0	0	2	2	2	0	2

$$W_Y = y_\ell \ell_i \bar{\Phi}^i_\alpha \bar{L}^\alpha + y_{Hd} L_\alpha H_d E^{c\alpha} + y_e \bar{E}^c_\alpha \Phi^\alpha_j e^{cj} + M_E E^{c\alpha} \bar{E}^c_\alpha + M_L L_\alpha \bar{L}^\alpha$$

$$\begin{aligned}
W_{br} &= \mu_S S \theta_S - \varepsilon \mu_S^2 \theta_S \\
W_\Phi &= \lambda_1 \Phi^\alpha_i \bar{\Phi}^i_\alpha \theta_\Phi - \lambda_2 S^2 \theta_\Phi \\
W_{\Phi\Psi} &= (\lambda_A \bar{\Psi}^i_\alpha \Phi^\alpha_j + \bar{\lambda}_A \bar{\Phi}^i_\alpha \Psi^\alpha_j)(\Theta_A)^j_i \\
&\quad + (\lambda'_A \bar{\Psi}^i_\alpha \Phi^\alpha_i + \bar{\lambda}'_A \bar{\Phi}^i_\alpha \Psi^\alpha_i - \mu_A S)(\Theta_A)^j_j \\
&\quad + (\lambda_B \Phi^\alpha_i \bar{\Psi}^i_\beta + \bar{\lambda}_B \Psi^\alpha_i \bar{\Phi}^i_\beta)(\Theta_B)^\beta_\alpha \\
&\quad + (\lambda'_B \Phi^\alpha_i \bar{\Psi}^i_\alpha + \bar{\lambda}'_B \Psi^\alpha_i \bar{\Phi}^i_\alpha - \mu_B S)(\Theta_B)^\beta_\beta
\end{aligned}$$

fix VEVs

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vEv relations in SUSY

- SUSY: easy to analyze up to ~~SUSY~~
  - radiative corrections are suppressed
  - scalar potential V is non-negative

⇨ $V = 0$ is minimum (no need to examine m^2)

⇔ $W_i = 0$ for all i

- superfield formalism in superspace $(x, \theta, \bar{\theta})$

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

  - superpotential $W(\Phi) = \frac{1}{2} M^{ij} \Phi_i \Phi_j + \frac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k$

⇨ $\mathcal{L}_{\text{int}} = -V_{\text{scalar}}(\phi) - W^{ij}(\phi)\psi_i\psi_j$

$$W^i = \frac{dW}{d\Phi_i}$$

$$V_{\text{scalar}}(\phi) = |F_i|^2 + \cdots$$

$$F_i = -W_i(\phi)^*$$

VEV relations in SUSY

- VEV relations
  - when $W=\sum_i \Theta_I C_I(\Phi)$, $\Theta_I=0$ & $C_I=0$ give $V=0$

    (other vacua may exist)

  ⇨ $V=0$ is minimum (no need to examine m^2)

  ⇔ $W_i=0$ for all i

  ex) $W=\Theta(\mu A-\lambda BC)$ ⇨ $\mu \langle A \rangle = \lambda \langle B \rangle \langle C \rangle$

  - on this vacuum, $\Theta_I \Theta_J C_{IJ}(\Phi)$ has no effect
  - the above form can be realized

    by for instance $U(1)_R$ or $U(1)_{\text{anomalous}}$

Sumino mechanism in SUSY

Y. Koide & T.Y. (2012)

- model

	ℓ	e^c	H_d	L	$\bar{L}$	E^c	$\bar{E}^c$	Φ	$\bar{\Phi}$	Ψ	$\bar{\Psi}$	θ_Φ	Θ_A	Θ_B	S	θ_S
$SU(2)_L$	**2**	**1**	**2**	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$U(3)$	**3**	**3***	**1**	**1**	**1**	**1**	**1**	**3**	**3***	**3**	**3***	**1**	**8 + 1**	**1**	**1**	**1**
$U(3)'$	**1**	**1**	**1**	**3**	**3***	**3***	**3**	**3***	**3**	**3***	**3**	**1**	**1**	**8 + 1**	**1**	**1**
$U(1)_S$	0	0	−1	1	−1	0	−1	1	1	0	0	−2	−1	−1	1	−1
$U(1)_R$	1	1	0	1	1	1	1	0	0	0	0	2	2	2	0	2

$$W_Y = y_\ell \ell_i \bar{\Phi}^i_\alpha \bar{L}^\alpha + y_{Hd} L_\alpha H_d E^{c\alpha} + y_e \bar{E}^c_\alpha \Phi^\alpha_j e^{cj} + M_E E^{c\alpha} \bar{E}^c_\alpha + M_L L_\alpha \bar{L}^\alpha$$

$$\left.\begin{aligned}
W_{br} &= \mu_S S \theta_S - \varepsilon \mu_S^2 \theta_S \\
W_\Phi &= \lambda_1 \Phi^\alpha_i \bar{\Phi}^i_\alpha \theta_\Phi - \lambda_2 S^2 \theta_\Phi \\
W_{\Phi\Psi} &= \left(\lambda_A \bar{\Psi}^i_\alpha \Phi^\alpha_j + \bar{\lambda}_A \bar{\Phi}^i_\alpha \Psi^\alpha_j\right)(\Theta_A)^j_i \\
&\quad + \left(\lambda'_A \bar{\Psi}^i_\alpha \Phi^\alpha_i + \bar{\lambda}'_A \bar{\Phi}^i_\alpha \Psi^\alpha_i - \mu_A S\right)(\Theta_A)^j_j \\
&\quad + \left(\lambda_B \Phi^\alpha_i \bar{\Psi}^i_\beta + \bar{\lambda}_B \Psi^\alpha_i \bar{\Phi}^i_\beta\right)(\Theta_B)^\beta_\alpha \\
&\quad + \left(\lambda'_B \Phi^\alpha_i \bar{\Psi}^i_\alpha + \bar{\lambda}'_B \Psi^\alpha_i \bar{\Phi}^i_\alpha - \mu_B S\right)(\Theta_B)^\beta_\beta
\end{aligned}\right\}\ \text{fix VEVs}$$

Summary and discussion

- SUSY Sumino mech.

Y. Koide & T.Y. (2012)

  - ✓ less anomalous
  - ✓ inverted mass hierarchy
  - ➢ incomplete cancellation

    ⇦ VEV other than $v_i \propto m_{ei}^{-1/2}$?

- questions in this talk

  Q1) Should $<\Phi>$ be diag. w/o loss of generality?

  Q2) Is wf renorm. really flavor dependent?

  Q3) Is $le^c H \times \Phi\Phi$ induced against chiral symm.?