Flavon VEV scales in U(3)×U(3)' model

Hiroyuki Nishiura (Osaka Inst. of Tech.)
西浦宏幸（大阪工大）

In collaboration with Y. Koide

Key word:
Mass matrix model with seesaw form, Unified description of M_f based on m_{ei}

Mini-Workshop in Osaka 2016 on "Quarks, leptons and Family Gauge Bosons"
26-27 Dec. 2016, at Osaka University (Toyonaka Campus)

Contents

1. Introduction

The purpose of the present paper is to fix VEV scales of flavons from the observed data.

Mass matrices based on $\mathrm{U}(3) \times \mathrm{U}(3')$:

Flavons	VEV forms
$\Phi_f \ : \ (\mathbf{3}, \mathbf{3}^*)$	$\langle \Phi_f \rangle = v_\Phi \, \mathrm{diag}(z_1 e^{i\phi_1^f}, z_2 e^{i\phi_2^f}, z_3 e^{i\phi_3^f})$
$\hat{S}_f \ : \ (\mathbf{1}, \mathbf{8}+\mathbf{1})$	$\langle \hat{S}_f \rangle = v_S(\mathbf{1} + b_f X_3)$
$E \ : \ (\mathbf{8}+\mathbf{1}, \mathbf{1})$	$\langle \hat{E}_\circ{}^\circ \rangle = v_E \mathbf{1}$

$$X_3 = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\mathbf{1} = \mathrm{diag}(1,1,1)$$

We shall see that $\phi_i^f = 0$ for $f = d, e, \nu$ in later.
Therefore, we hereafter denote $(\phi_1^u, \phi_2^u, \phi_3^u)$ as (ϕ_1, ϕ_2, ϕ_3) simply.

We will give $b_e = 0$, i.e. $\hat{S}_e = v_S \mathbf{1}$. Thereby we obtain the charged lepton mass matrix $\hat{M}_e \propto \mathrm{diag}(z_1^2, z_2^2, z_3^2)$, from which we get a relation $z_i = \frac{\sqrt{m_{ei}}}{\sqrt{m_e + m_\mu + m_\tau}}$.

· Dirac mass matrices take a seesaw form common to all flavors.

$$(\hat{M}_f)_i{}^j \propto \langle \Phi_f \rangle_i{}^\alpha \langle \hat{S}_f^{-1} \rangle_\alpha{}^\beta \langle \bar{\Phi}_f \rangle_\beta{}^j, \tag{1.1}$$

VEV scales of flavons:

The new model in this paper is characterized as follows:

(i) There are no Yukawaons, although the flavons Φ_f and $\hat{S}_f$ still play an essential role in the new model, too.

(ii) There are three type of VEV scales: (We consider $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3$.)

$$\langle \hat{S}_f \rangle_{\bullet}^{\ \bullet} \sim \Lambda_1, \quad \langle \Phi_f \rangle_{\circ}^{\ \bullet} \sim \Lambda_2, \quad \langle E_{\circ\circ} \rangle \sim \Lambda_3. \tag{1.7}$$

We define Λ_1 as $\Lambda_1 = v_S$ in Eq.(1.2), Λ_2 as $\Lambda_2 = v_\Phi$ (except for $f = \nu$) in Eq.(1.4) and Λ_3 as $v_E = \Lambda_3$ which appears in Eq.(3.1) later. Here and hereafter, in order to be easy to see a scale of flavons, we will sometimes show the indexes of U(3) and U(3)′, by "$\circ$" and "$\bullet$", instead of $i, j, k, \cdots$" and "$\alpha, \beta, \gamma, \cdots$", respectively.

We fix those scales by the observed data as shown later:

$$\begin{aligned} \Lambda_3 &= 8.883 \text{ TeV}, \\ \Lambda_2 &= 1.306 \times 10^4 \text{ TeV}, \\ \Lambda_1 &= 1.332 \times 10^7 \text{ TeV}. \end{aligned}$$

(iii) Majorana mass matrix of the left-handed neutrinos is given by a seesaw mechanism

$$(M_\nu^{Maj})_{\circ\circ} = (\hat{M}_\nu)_\circ{}^\circ \langle \bar{Y}_R^{-1} \rangle_{\circ\circ} (\hat{M}_\nu^T \rangle^\circ{}_\circ . \quad (1.8)$$

In our model, in oder to make the model anomaly free, the right-handed neutrinos ν_R belong to triplet of U(3) as well as ν_L, so that the right-handed neutrino Majorana mass matrix $(M_R)^{ij}$ are given by VEV of a flavon $\bar{Y}_R$, $\langle \bar{Y}_R \rangle^{ij}$.

In the present model, a scale of the right-handed neutrino Majorana mass matrix is considerably small differently from the conventional neutrino seesaw, because our flavon seems to be $\langle \bar{Y}_R \rangle^{\circ\circ} \sim \Lambda_3$. This is somewhat troublesome.

2. 1 Seesaw-type mechanism in the Dirac mass matrix

We consider hypothetical fermions F_α ($\alpha = 1,2,3$), which belong to $(\mathbf{1},\mathbf{3})$ of U(3)×U(3)′, in addition to quarks and leptons f_i ($i = 1,2,3$) which belong to $(\mathbf{3},\mathbf{1})$. We assume that the VEV form (1.1) originates in the following 6×6 mass matrix model:

$$(\bar{f}^i_L \;\; \bar{F}^\alpha_L)\begin{pmatrix} (0)_i^{\ j} & (\Phi_f)_i^{\ \beta} \\ (\bar{\Phi}_f)_\alpha^{\ j} & -(S_f)_\alpha^{\ \beta} \end{pmatrix}\begin{pmatrix} f_{Rj} \\ F_{R\beta} \end{pmatrix}. \tag{2.1}$$

Here, $F_{L(R)}$ are heavy fermions with $(\mathbf{1},\mathbf{1},\mathbf{3})$ of SU(2)$_L$×U(3)×U(3)′. On the other hand, f_R are right-handed quarks and leptons, $f_R = (u,d,\nu,e^-)_R$, while f_L are not physical fields. They are given by the following combinations:

$$f_L \equiv (f_u, f_d, f_\nu, f_e)_L \equiv \left(\frac{1}{\Lambda_H}H_u^\dagger q_L, \frac{1}{\Lambda_H}H_d^\dagger q_L, \frac{1}{\Lambda_H}H_u^\dagger \ell_L, \frac{1}{\Lambda_H}H_u^\dagger \ell_L, \frac{1}{\Lambda_H}H_d^\dagger \ell_L\right) \tag{2.2}$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \ell_L = \begin{pmatrix} \nu_L \\ e^-_L \end{pmatrix}, \quad H_u = \begin{pmatrix} H_u^0 \\ H_u^- \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix}. \tag{2.3}$$

In other words, the matrix Eq.(2.1) denotes would-be Yukawa coupling constatns.

Taking a seesaw-like approximation with $\Lambda_2 \ll \Lambda_1$, the mass matrix (2.1) leads to the following Dirac mass matrices of quarks and leptons:

$$(\hat{M}_f)_i^{\ j} \simeq \frac{\langle H_{u/d}\rangle}{\Lambda_H} \langle \Phi_f \rangle_i^{\ \alpha} \langle (\hat{S}_f)^{-1} \rangle_\alpha^{\ \beta} \langle \bar{\Phi}_f \rangle_\beta^{\ j}. \tag{2.5}$$

We search VEV scales of flavons by assuming that our flavon VEV scale is given by one of Λ_1, Λ_2 and Λ_3, except for $H_{u/d}$. As we stated in Sec.1, we put $1/(\Lambda_1)^n$ for superpotential term with dimension $(3 + n)$. However, we assume that these rules are exceptional for the factor $H_{u/d}/\Lambda_H$. The VEV of H_u and H_d are fixed as $v_{Hu} = v_{Hd} \equiv v_H \equiv \frac{1}{\sqrt{2}} \times 246$ GeV. (The reason of $v_{Hu} = v_{Hd}$ will be given in Eq.(4.6) later.) We will take Λ_H as a different value from Λ_1, Λ_2 and Λ_3. That is, we consider that the additional factor i $H_{u/d}/\Lambda_H$ in Eq.(2.5) is out of our U(3)×U(3)′ scenario. For the time being, we do not discuss the mechanism which gives the factor $H_{u/d}/\Lambda_H$.

Effective Hamiltonian for quarks and leptons, after U(3) and U(3)′ breaking, is

$$\mathcal{H}_Y = (\bar{\nu}_L)^i(\hat{M}_\nu)_i^{\ j}(\nu_R)_j + (\bar{e}_L^i(\hat{M}_e)_i^{\ j}(e_R)_j + y_R(\bar{\nu}_R)^i(Y_R)_{ij}(\nu_R^c)^j$$

$$+(\bar{u}_L)^i(\hat{M}_u)_i^{\ j}(u_R)_j + (\bar{d}_L)^i(\hat{M}_d)_i^{\ j}(d_R)_j. \quad (2.4)$$

Of course, $(\hat{M}_f)_i^{\ j}$ do not mean flavons. Note that quarks and leptons f_i are no more U(3) family triplet in the exact meaning, and they are mixing states between f and F. However, we will still use the index of U(3) family for them.

2. 2 R-charge assignment

We assume that R charges are still conserved under the block diagonalization of (2.1), so that we consider an R charge relation

$$R(\hat{S}_f) = R(\Phi_f) \equiv r_f. \tag{2.7}$$

(Here, we have taken R charge of the Higgs fields as $R(H_{u/d}) = 0$.) Then, the Dirac mass matrix $\hat{M}_f$ has effectively a R charge r_f (although $\hat{M}_f$ is not a flavon). Note that if there is a flavon combination $(\Phi_{f'}\bar{\Phi}_{f''})_i^{\ j}/\Lambda_1$ with $r_{f'} + r_{f''} = r_f$, the combination can couple to $\bar{f}_L^j f_{Ri}$ as well as $(\hat{M}_f)_i^{\ j}$ with the same energy scale $(\Lambda_2)^2/\Lambda_1$. Besides, if $(\bar{\Phi}_f\Phi_{f'}))$ and another $(\bar{\Phi}_{f''}\Phi_{f'''}))$ have the same R charge, those can have the same VEV structure. This is an unwelcome situation, because we want that Φ_u and Φ_ν have somewhat different VEV structures from Φ_e and Φ_d as we state later. A simple way to avoid appearance of such unwelcome combinations is to take R charge difference among Φ_f's completely different from each other: $r_e - r_u = 2/3$, $r_d - r_e = 3/3$, $r_\nu - r_d = 4/3$, i.e.

$$(r_u, r_e, r_d, r_\nu) = \left(\frac{1}{3}, 1, 2, \frac{10}{3}\right). \tag{2.8}$$

The reason $r_e = 1$ is as follows: Only when $r_e = 1$. we can write a superpotential for $\hat{S}_e$,

$$W_{Se} = \lambda_1^S [(\hat{S}_e)_\alpha^{\ \beta} (\hat{S}_e)_\beta^{\ \alpha}] + \lambda_2^S [(\hat{S}_e)_\alpha^{\ \alpha}][(\hat{S}_e)_\beta^{\ \beta}]. \tag{2.9}$$

Supersymmetric vacuum condition leads to $\hat{S}_e = v_S \mathbf{1}$, so that we obtain $b_e = 0$. This means that the charged lepton mass matrix $\hat{M}_e$ is proportional to $\mathrm{diag}(z_1^2, z_2^2, z_3^2)$. Therefore, our parameters z_i are given by (1.6). The phase parameters ϕ_i^e do not have physical meaning because $\langle \Phi_e \rangle$ is commutable with $\langle \hat{S}_e \rangle = v_S \mathbf{1}$, so that we can hereafter put $\phi_i^e = 0$, i.e. $\langle \Phi_e \rangle = v_e\, \mathrm{diag}(z_1, z_2, z_3)$.

Table 1: R charges and VEV scales of leading flavons. Transformation property under U(3)×U(3)′ is indicated by "∘" and "•", respectively.

flavon	$(\Phi_u)_{\circ}^{\bullet}$	$(\Phi_e)_{\circ}^{\bullet}$	$(\Phi_d)_{\circ}^{\bullet}$	$(\Phi_\nu)_{\circ}^{\bullet}$	$(\hat{S}_u)_{\bullet}^{\bullet}$	$(\hat{S}_e)_{\bullet}^{\bullet}$	$(\hat{S}_d)_{\bullet}^{\bullet}$	$(\hat{S}_\nu)_{\bullet}^{\bullet}$
R	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{6}{3}$	$\frac{10}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{6}{3}$	$\frac{10}{3}$
Scale	Λ_2	Λ_2	Λ_2	Λ_3	Λ_1	Λ_1	Λ_1	Λ_2

$E_{\circ\circ}$	$\hat{E}_{\circ}^{\circ}$	$E_{\circ\bullet}$	$E_{\bullet\bullet}$	$\hat{E}_{\bullet}^{\bullet}$	$(\hat{Y}_{eu})_{\bullet}^{\bullet}$	$(\bar{S}'_u)^{\circ\bullet}$	$(\bar{Y}_R)^{\circ\circ}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{5}{3}$	$\frac{5}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{13}{3}$
Λ_3	Λ_3	Λ_2	Λ_1	Λ_1	Λ_1	Λ_2	Λ_3

$(\bar{\Theta}_d)_{\bullet}^{\circ}$	$\bar{\Theta}_u)_{\bullet}^{\bullet}$	$(\bar{\Theta}_\nu)_{\bullet}^{\circ}$	$(\hat{\Theta}_\nu)_{\bullet}^{\bullet}$	$(\hat{\Theta}_\phi)_{\bullet}^{\bullet}$	$(\Theta_R)_{\circ\circ}$	$(\hat{\Theta}_{eu})_{\bullet}^{\bullet}$	$(\Theta'_{Su})_{\bullet\circ}$	$(\hat{\Theta}_E)_{\bullet}^{\bullet}$	$(\hat{\Theta}_E)_{\circ}^{\circ}$
$-\frac{5}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$	$\frac{2}{3}$	-3	$\frac{2}{3}$	-1	$-\frac{4}{3}$	$\frac{2}{3}$
0	0	0	0	0	0	0	0	0	0

2. 3 VEV relation among $¥Phi_f$

A VEV of Φ_d is given by the following superpotential

$$W_d = \left\{(\Phi_d)_i^{\ \beta}(\hat{E})_\beta^{\ \alpha} + \lambda_d \frac{1}{\Lambda_1}(\Phi_e)_i^{\ \beta}(\hat{S}_e)_\beta^{\ \gamma}(\hat{E})_\gamma^{\ \alpha}\right\}(\bar{\Theta}_d)_\alpha^{\ i}, \tag{2.10}$$

Here and hereafter, we assume that the Θ fields always take $\langle\Theta\rangle = 0$. Therefore, from $\partial W_d/\partial\Theta_d = 0$, we obtain

$$(\Phi_d)_i^{\ \beta}(\hat{E})_\beta^{\ \alpha} = -\lambda_d \frac{1}{\Lambda_1}(\Phi_e)_i^{\ \beta}(\hat{S}_e)_\beta^{\ \gamma}(\hat{E})_\gamma^{\ \alpha}. \tag{2.11}$$

Therefore, when we take $(\hat{E})_\gamma^{\ \alpha} = \Lambda_1$, we obtain the VEV $\langle\Phi_d\rangle$ with the same order as $\langle\Phi_e\rangle$, i.e. $\langle\Phi_d\rangle = v_d\,\mathrm{diag}(z_1, z_2, z_3)$ with $\phi_i^d = 0$.

However, for Φ_u with $R = 1/3$, we have to consider another mechanism,

$$W_u = \left\{\frac{\lambda_{u1}}{\Lambda_1}(\bar{\Phi}_u)_\alpha^{\ k}(\Phi_u)_k^{\ \gamma}(\hat{E})_\gamma^{\ \beta} + \lambda_{u2}(\bar{\Phi}_d)_\alpha^{\ k}(\Phi_d)_k^{\ \beta}\right\}(\hat{\Theta}_u)_\beta^{\ \alpha}. \tag{2.12}$$

Here, we have introduced a new flavon $\hat{E}_{\bullet}^{\ \bullet}$ with $R = 5/3$ and a VEV form $\langle \hat{E} \rangle = v_E \mathbf{1}$. (For the flavon E together with other similar flavons E, we will summarize in Appendix C.) Since this potential gives a relation between $\Phi_u \bar{\Phi}_u$ and $\Phi_d \bar{\Phi}_d$, $\langle \Phi_u \rangle$ can have phase factors against real VEV matrix $\langle \Phi_d \rangle$. Hereafter, we simply denote ϕ_i^u as ϕ_i:

$$\langle \Phi_u \rangle = v_u \,\mathrm{diag}(z_1 e^{i\phi_1}, z_2 e^{i\phi_2}, z_3 e^{i\phi_3}). \tag{2.13}$$

Meanwhile, in this model, flavnos E with VEV matrix form $v_E \mathrm{diag}(1, 1, 1)$ frequently appear. For E with the scale Λ_3 and $R = 2/3$, we consider

$$W_E = \lambda_1 \mathrm{Tr}[\hat{E}_{\circ}^{\ \circ} E_{\circ\circ} \bar{E}^{\circ\circ}] + \lambda_2 \mathrm{Tr}[\hat{E}_{\circ}^{\ \circ}] \mathrm{Tr}[E_{\circ\circ} \bar{E}^{\circ\circ}]. \tag{2.14}$$

SUSY vacuum condition leads to

$$\hat{E}_{\circ}^{\ \circ} = v_E \mathbf{1}, \quad E_{\circ\circ} \bar{E}^{\circ\circ} = (v_E)^2 \mathbf{1}. \tag{2.15}$$

For $E_{\circ\circ}$, we take a special solution $E_{\circ\circ} = v_E \mathbf{1}$ in the relation $E_{\circ\circ} \bar{E}^{\circ\circ} = (v_E)^2 \mathbf{1}$.

For $\hat{E}_{\bullet}{}^{\bullet}$ and $E_{\bullet\bullet}$, there is not such a simple superpotential. We have to assume the following superpotential

$$W_E = \lambda_1 \mathrm{Tr}[\hat{E}_{\bullet}{}^{\bullet}\hat{E}_{\bullet}{}^{\bullet}(\hat{\Theta}_{-4/3})_{\bullet}{}^{\bullet}] + \lambda_2 \mathrm{Tr}[E_{\bullet\bullet}\bar{E}^{\bullet\bullet}(\hat{\Theta}_{-4/3})_{\bullet}{}^{\bullet}] + \ldots, \qquad (2.16)$$

where $(\hat{\Theta}_{-4/3})_{\bullet}{}^{\bullet}$ has $R = -4/3$. We also assume

$$W_E = \mathrm{Tr}[\hat{E}_{\circ}{}^{\circ}E_{\circ\bullet}\bar{E}^{\bullet\circ}(\hat{\Theta}_{2/3})_{\circ}{}^{\circ}] + \cdots , \qquad (2.17)$$

where $(\hat{\Theta}_{2/3})_{\circ}{}^{\circ}$ has $R = 2/3$.

Finally, let us discuss a relation of $\langle\Phi_\nu\rangle$. We cannot consider the similar mechanism as $\langle\Phi_f\rangle$ using $\hat{E}$, since we need to give a small scale compared with $\langle\Phi_e\rangle \sim \Lambda_2$ in order to satisfy the seesaw approximation (1.7). (Since the scale Λ_2 means the maximal scale of a flavon with $A_\circ{}^\bullet$, the requirement $\langle\Phi_\nu\rangle < \Lambda_2$ is no problem.) Therefore, we assume the following superpotential term:

$$W_\nu = \left\{\mu_\nu(\Phi_\nu)_\circ{}^\bullet + \lambda_\nu \frac{1}{(\Lambda_1)^2}(\Phi_e)_\circ{}^\bullet \bar{E}^{\bullet\bullet} E_{\bullet\circ} \bar{E}^{\circ\bullet}\right\}(\bar{\Theta}_\nu)_\bullet{}^\circ, \qquad (2.18)$$

where new flavons $E_{\circ\circ}$ and $E_{\bullet\bullet}$ with VEV form $v_E\mathbf{1}$ has R charges

$$R(E_{\circ\circ}) = \frac{2}{3}, \quad R(E_{\bullet\bullet}) = \frac{5}{3}. \qquad (2.19)$$

Therefore, a VEV scale of $(\Phi_\nu)_\circ{}^\bullet$ with $R = 10/3$ is given by

$$||\Phi_\nu|| = \frac{(\Lambda_2)^3}{\mu_\nu \Lambda_1}, \qquad (2.20)$$

Hereafter, for convenience, we denote a VEV scale of a flavon A as a notation $||A||$. Also, in the estimation of VEV scales for simplicity , we put $\lambda = 1$ for all dimensionless coeffiients λ in superpotentials (2.9), (2.10), and so on.

We denote Eq.(2.20) as

$$\langle \Phi_\nu \rangle = \xi_\nu \langle \Phi_e \rangle, \quad \xi_\nu = \frac{(\Lambda_2)^2}{\mu_\nu \Lambda_1}. \tag{2.21}$$

We have obtained $b_e = 0$, i.e. $\langle \hat{S}_e \rangle = v_S \mathbf{1}$ by assigning $r_e = 1$ as shown in Eq.(2.9). However, since $\hat{S}_\nu$ has $R = 10/3$, we cannot assert $b_\nu = 0$ by means of a similar way to Eq.(2.9). We want a similar VEV structure of $\hat{M}_\nu$ to $\hat{M}_e$ except for its VEV scale. Therefore, we assume a superpotential similar to ((2.18):

$$W_{S\nu} = \left\{ \mu_{S\nu} (\hat{S}_\nu)_\bullet^{\ \bullet} + \lambda_\nu \frac{1}{(\Lambda_1)^2} (\hat{S}_e)_\bullet^{\ \bullet} \bar{E}^{\bullet\bullet} E_{\bullet\circ} \bar{E}^{\circ\bullet} \right\} (\hat{\Theta}_\nu)_\bullet^{\ \bullet}. \tag{2.22}$$

Then, we obtain a result similar to (2.21):

$$\langle \hat{S}_\nu \rangle = \xi_{S\nu} \langle \hat{S}_e \rangle, \quad \xi_{S\nu} = \frac{(\Lambda_2)^2}{\mu_{S\nu} \Lambda_1}, \tag{2.23}$$

so that we can obtain $\hat{M}_\nu$ with the same form as $\hat{M}_e$:

$$\hat{M}_\nu = \xi_M \hat{M}_e, \quad \xi_M = (\xi_\nu)^2 (\xi_S)^{-1} = \frac{\mu_{S\nu} (\Lambda_2)^2}{(\mu_\nu)^2 \Lambda_1}. \tag{2.24}$$

2. 4 Parameter values in the quark sector

As far as quark sector is concerned, the VEV matrices are exactly the same as those in the previous paper, although the model is completely different from the previous one.

We choose parameter values (in the parameter fitting, we have used running masses at $\mu = m_Z$ at input quark and lepton masses) as

$$b_u = -1.011, \quad b_d = -3.522\, e^{i\ 17.7^\circ}, \tag{2.21}$$

and

$$(\tilde{\phi}_1, \tilde{\phi}_2) = (-176.05^\circ,\ -167.91^\circ), \tag{2.22}$$

where we have fitted (ϕ_1, ϕ_2, ϕ_3) as $(\tilde{\phi}_1, \tilde{\phi}_2,\ 0)$ without losing generality. Then, we can obtain reasonable quark masses and CKM mixing:
Y. Koide and H. Nishiura, Phys.Rev. **D 92** (2015) 111301(R).

Parameter fitting (quark mass ratios, CKM, MNS, and ratios of neutrino mass squared differences)

- u-quark mass ratios, $r_{12}^u \equiv \sqrt{\frac{m_u}{m_c}} = 0.045,\ r_{23}^u \equiv \sqrt{\frac{m_c}{m_t}} = 0.060$

  fix $b_u = -1.011$

- d-quark mass ratios, $r_{12}^d \equiv \frac{m_d}{m_s} = 0.053,\ r_{23}^d \equiv \frac{m_s}{m_b} = 0.019\text{(Xing) or } 0.031\text{(F-K)}$

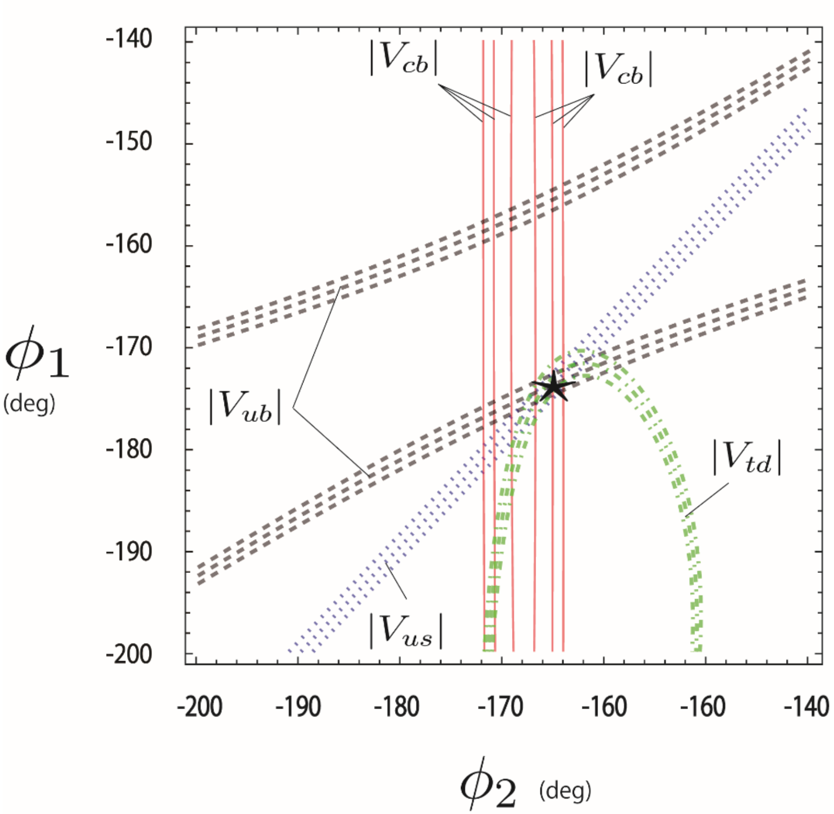
  fix $(b_d, \beta_d) = (-3.3522, 17.7°)$

Then, $r_{12}^u = 0.061, \quad r_{23}^u = 0.060, \quad r_{12}^d = 0.049, \quad r_{23}^d = 0.027$

- The CKM mixings matrix elements fix (ϕ_1, ϕ_2)

- The MNS mixings matrix and R_ν are functions of ξ_R

3. CKM fitting



$(\phi_1, \phi_2) = (-176.05°, -167.91°)$
is consistent with all the observed values:

$$|V_{us}| = 0.22536 \pm 0.00061,$$
$$|V_{cb}| = 0.0414 \pm 0.0012,$$
$$|V_{ub}| = 0.00355 \pm 0.00015,$$
$$|V_{td}| = 0.00886^{+0.00033}_{-0.00032},$$
$$\delta^q_{CP} = 69.4° \pm 3.4°.$$

Fig 1. Contour curves in the (ϕ_1, ϕ_2) parameter plane of the observed CKM mixing matrix elements of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and $|V_{td}|$. We draw the three contour curves, which corresponds to the center, upper, and lower values of the observed constraints for each the CKM mixing matrix elements, with taking $b_u = -1.011$, and $b_d = -3.3522$, $\beta_d = 17.7°$. We find that the parameter set arround $(\phi_1, \phi_2) = (-176.05°, -167.91°)$ indicated by a star (★) is consistent with all the observed values

3 Mas Relation between phase parameters $¥phi_i$ and charged lepton masses m_{ei}s

The parameters (ϕ_1, ϕ_2, ϕ_3) were typical family-number dependent parameters. Now we try to describe (ϕ_1, ϕ_2, ϕ_3) in terms of charged lepton masses m_{ei} and two family-number independent parameters. We assume the following superpotential

$$W_\phi = \left\{ \frac{\lambda_1^\phi}{\Lambda_1} \left[(\bar{\Phi}_u)_\bullet{}^\circ E_{\circ\circ} (\bar{E}')^{\circ\bullet} + h.c. \right] + \frac{\lambda_2^\phi}{(\Lambda_1)^2} (\bar{\Phi}_u)_\bullet{}^\circ (E')_{\circ\bullet} (\bar{E}')^{\bullet\circ} (\Phi_u)_\circ{}^\bullet \right.$$

$$\left. + \frac{\lambda_2^\phi}{(\Lambda_1)^3} (\bar{\Phi}_u)_\bullet{}^\circ (\Phi_u)_\circ{}^\bullet (\bar{\Phi}_u)_\bullet{}^\circ (\Phi_u)_\circ{}^\bullet \right\} (\hat{\Theta}_\phi)_\bullet{}^\bullet, \qquad (3.1)$$

where $R(\Theta_\phi) = 2/3$. E_{ij} is given by a superpotential similar to (2.10), and $(E')_{i\alpha}$ was already defined in (2.18).

Note that the second term $(\bar{\Phi}_u)_\bullet{}^\circ (E')_{\circ\bullet} (\bar{E}')^{\bullet\circ} (\Phi_u)_\circ{}^\bullet$ in Eq.(3.1), additional terms are allowed: $(\bar{\Phi}_u)_\bullet{}^\circ (\Phi_u)_\circ{}^\bullet (E')_{\bullet\circ} (\bar{E}')^{\circ\bullet}$, $(\bar{\Phi}_u)_\bullet{}^\circ (E')_{\circ\bullet} (\Phi_u^T)^\bullet{}_\circ (\bar{E}')^{\circ\bullet}$, $(E')_{\bullet\circ} (\bar{\Phi}_u^T)^\circ{}_\bullet \bar{E}^{\bullet\circ} (\Phi_u)_\circ{}^\bullet$, and $(E')_{\bullet\circ} (\bar{E}')^{\circ\bullet} (\bar{\Phi}_u)_\bullet{}^\circ (\Phi_u)_\circ{}^\bullet$. Here, some remarks are in order: (i) we regard those additional terms as "substantially same terms", (ii) but, we count a flavon $A^\dagger$ as a different field from A, and (iii) the coefficient λ is defined as follows: the coefficient λ is a coefficient with a factor $1/n$ for sum of n substantially same terms. For example, in the second term in (3.1). the factor λ_2^ϕ is defined as one for sum of the five terms with $1/5$. However, for simplicity, hereafter, we denote only representative one even if there are many similar terms, and give the coefficient λ instead of λ/n. Also note that the $h.c$ term in the first term is different from the first one according to our counting rule.

Here, some remarks are in order: (i) we regard those additional terms as "substantially same terms", (ii) but, we count a flavon $A^\dagger$ as a different field from A, and (iii) the coefficient λ is defined as follows: the coefficient λ is a coefficient with a factor $1/n$ for sum of n substantially same terms. For example, in the second term in (3.1). the factor λ_2^ϕ is defined as one for sum of the five terms with $1/5$. However, for simplicity, hereafter, we denote only representative one even if there are many similar terms, and give the coefficient λ instead of λ/n. Also note that the $h.c$ term in the first term is different from the first one according to our counting rule.

The flavon $\hat{\Theta}_\phi$ in (3.1) has VEV value of zero. The SUSY vacuum condition $\partial W_\phi / \partial \Theta_\phi = 0$ leads to a condition

$$2c_1 z_i \cos\phi_i = c_2 z_i^2 + c_3 z_i^4, \tag{3.2}$$

where parameters c_1, c_2 and c_3 are family-number independent parameters and they have scales

$$c_1 = \lambda_1 \frac{(\Lambda_2)^2}{(\Lambda_1)^2} \Lambda_1 \Lambda_3, \quad c_2 = \lambda_2 \frac{(\Lambda_2)^4}{(\Lambda_1)^2}, \quad c_3 = \lambda_3 \frac{(\Lambda_2)^4}{(\Lambda_1)^2}. \tag{3.3}$$

When we denote these parameters ϕ_i as

$$\begin{aligned} \phi_1 &= \phi_0 + \tilde{\phi}_1, \\ \phi_2 &= \phi_0 + \tilde{\phi}_2, \\ \phi_3 &= \phi_0, \end{aligned} \tag{3.4}$$

the parameter ϕ_0 is unobservable in the CKM parameter fitting, while it is not unobservable in the U(3)×U(3)′ model.

From the input values $(\tilde{\phi}_1,\ \tilde{\phi}_2) = (-176.05^\circ,\ -167.91^\circ)$ given in (2.22), we obtain

$$\phi_0 = 86.69^\circ, \quad \frac{c_2}{c_1} = 1.368, \quad \frac{c_3}{c_2} = -0.967. \tag{3.5}$$

which leads to

$$(\phi_1,\ \phi_2,\ \phi_3) = (-89.36^\circ,\ -87.25^\circ,\ 86.69^\circ). \tag{3.6}$$

Thus, the family-number dependent parameters (ϕ_1, ϕ_2, ϕ_3) can be reduced into family-number independent parameters $(c_2/c_1, c_3/c_2)$. (Note that the parameter ϕ_0 is not unobservable any longer in this model.)

Note that the numerical results (3.5) suggests $|c_3/c_2| = 1$ if we take $\lambda_2 = \lambda_3$. Then, it is natural that we consider $\lambda_1 = \lambda_2 = \lambda_3$. Only the ratio c_2/c_1 is not one. When we put the ratio as $c_2/c_1 \equiv \rho \simeq 1.37$, we obtain

$$R_\Lambda \equiv \frac{\Lambda_2}{\Lambda_1} = \rho \frac{\Lambda_3}{\Lambda_2}. \tag{3.7}$$

We will see $R_\Lambda \sim 10^{-3}$ later.

Hereafter, for convenience, we put $\lambda = 1$ for all dimensionless coeffiients λ in superpotentials (2.9), (2.10), and so on.

4 Slight deviation of the scales in the lepton sector

4.1 $¥hat{M}_{lepton}$ versus $¥hat{M}_{quark}$

Let us comment on the ratio v_{Hu}/v_{Hd}. Usually, it is understood that the value $\tan\beta \equiv v_{Hu}/v_{Hd}$ is $\tan\beta \sim 10$. However, from (2.6), our quark masses are predicted as

$$\begin{aligned}(m_u, m_c, m_t) &= (0.0003964,\ 0.106411,\ 29.743)m_{0u},\\ (m_d, m_s, m_b) &= (0.0007249,\ 0.01467,\ 0.5365)m_{0d},\end{aligned} \tag{4.1}$$

where m_{0f} are defined by

$$m_{0f} = \frac{v_{Hf}}{\Lambda_H}\frac{(v_f)^2}{v_{Sf}} = \frac{v_{Hf}}{\Lambda_H}\rho\Lambda_3. \tag{4.2}$$

In the last term in Eq.(4.2), we have used the relation (3.7). The numerical values in (4.1) are eigenvalues of the dimensionless matrix

$$Z(\mathbf{1} + b_f X_3)^{-1} Z, \tag{4.3}$$

$$Z = \mathrm{diag}(z_1, z_2, z_3), \tag{4.4}$$

and we have used input values $b_u = -1.011$ and $b_d = -3.3522e^{i\,17.7^\circ}$ respectively.

The prediction (4.1) can give reasonable values for mass ratios up- to down-quark

$$\frac{m_t}{m_b} = 55.44, \tag{4.5}$$

if we take $m_{0u} = m_{0d}$. The value (4.5) roughly agrees with the observed value $(m_t/m_b)^{obs} \simeq 59.41$. Therefore, in the present model, we can regard

$$\frac{m_{0u}}{m_{0d}} = 1, \quad i.e \quad v_{Hu} = v_{Hd}. \tag{4.6}$$

Also, since $m_\tau = (z_3)^2 m_{0e}$ ($z_3 = 0.97170$), we obtain

$$m_{0e} = 1.8499 \text{ GeV}, \tag{4.7}$$

so that we get

$$\frac{m_{0u}}{m_{0e}} = 3.121. \tag{4.8}$$

This suggests

$$\frac{m_{0u}}{m_{0e}} = 3. \tag{4.9}$$

We consider that such the factor 3 originates in a slight deviation between flavon VEV scales in the lepton sector and the quark sector.

4.2 Slight deviation of the scale $¥|¥Phi_{lepton}¥|$ from $¥|¥Phi_{quark}¥|$

In this subsection. we try to understand Eq.(4.9) from slight scale deviation of Φ_ℓ and $\hat{S}_\ell$ from Φ_q and $\hat{S}_q$ ($\ell = e, \nu$ and $q = u, d$):

$$\|(\Phi_\ell)_\circ{}^\bullet]] = \eta_\Phi \Lambda_2, \quad [[(\hat{S}_\ell)_\bullet{}^\bullet\| = \eta_S \Lambda_1, \tag{4.10}$$

against $\|(\Phi_q)_\circ{}^\bullet\| = \Lambda_2$ and $\|(\hat{S}_q)_\bullet{}^\bullet\| = \Lambda_1$. Here, we have consider that factors η_Φ and η_S are orders of one in contrast to factors ξ_ν and $\xi_{S\nu}$ with orders of 10^{-3} as shown later.

The modification (4.10) gives

$$\|(\hat{M}_\ell)_\circ{}^\circ\| = (\eta_\Phi)^2 (\eta_S)^{-1} \|(\hat{M}_q)_\circ{}^\circ\|, \tag{4.11}$$

so that (4.10) demands

$$\frac{(\eta_\Phi)^2}{\eta_S} = \frac{1}{3}, \quad i.e. \quad 3(\eta_\Phi)^2 = \eta_S . \tag{4.12}$$

On the other hand, the VEV relation (2.10) does not hold unless $\|(\Phi_\ell)_\circ{}^\bullet\| \cdot \|(\hat{S}_\ell)_\bullet{}^\bullet\| = \Lambda_2 \Lambda_1$. Therefore, we put additional relation

$$\eta_\Phi \eta_S = 1, \tag{4.13}$$

so that we obtain

$$3(\eta_\Phi)^3 = 1 \quad \Rightarrow \quad \eta_\Phi = \frac{1}{\eta_S} = \frac{1}{3^{1/3}}. \tag{4.14}$$

5 Flavons in the neutrino sector

In this section, we discuss Majonara mass matrix of the right-handed neutrinos ν_R, Y_R. Note that the mass matrix $\bar{Y}_R$ is $(\mathbf{6}^*, \mathbf{1})$ of U(3)×U(3)′, i.e. $(\bar{Y}_R)^{\circ\circ}$.

In the present neutrino mass matrix model, the Dirac neutrino mass matrix $\hat{M}_\nu$ is given by Eq.(2.5), i.e.

$$(\hat{M}_\nu)_\circ{}^\circ = \frac{\langle H_u \rangle}{\Lambda_H} (\Phi_\nu)_\circ{}^\bullet (\hat{S}_\nu^{-1})_\bullet{}^\bullet (\bar{\Phi}_\nu)_\bullet{}^\circ . \tag{5.1}$$

Since we pay attention to VEV scales of flavons, it is important whether those flavons belong to U(3) or U(3)′. From Eq.(5.1), we obtain

$$\|(\hat{M}_\nu)_\circ{}^\circ\| = \xi_M \frac{\langle H_u \rangle}{\Lambda_H} \frac{(\Lambda_2)^2}{\Lambda_1}, \tag{5.2}$$

where ξ_M is defined by (2.20) (also see (5.28) later). Since $\|\hat{M}_e\|$ is given by the order of $(\langle H_d \rangle/\Lambda_H)(\Lambda_2)^2/\Lambda_1$ and we consider an additional seesaw (1.8), a scale ratio of the scales $\|M^{Maj}\|/\|\hat{M}_e\|$ is given by

$$R_{\nu/e} \equiv \frac{\|M_\nu^{Maj}\|}{\|\hat{M}_e\|} = (\xi_M)^2 \frac{v_H)}{\Lambda_H} \frac{(\Lambda_2)^2}{\Lambda_1} \frac{1}{\|\bar{Y}_R\|}, \tag{5.3}$$

where $v_{Hu} = v_{Hd} \equiv v_H$. In order to estimate the ratio (5.3), we have to build model for $\bar{Y}_R$.

5.1 VEV structure of Majorana mass matrix $¥bar{Y}_RS

Majorana mass matrix of the left-handed neutrinos is given by a seesaw mechanism

$$(M_\nu^{Maj})_{\circ\circ} = (\hat{M}_\nu)_\circ{}^\circ \langle \bar{Y}_R^{-1} \rangle_{\circ\circ} (\hat{M}_\nu^T)^\circ{}_\circ . \tag{1.8}$$

We denote the form $\langle \bar{Y}_R \rangle$ by the following terms

$$\mu_R \langle \bar{Y}_R \rangle^{\circ\circ} = \mu_R \left[(\bar{Y}_R^{1st})^{\circ\circ} + (\bar{Y}_R^{2nd})^{\circ\circ} \right], \tag{5.4}$$

with $\|(\bar{Y}_R^{1st})\| \gg \|(\bar{Y}_R^{2nd})\|$. Here, in Eq.(5.4) , we denote the first and second terms in $\bar{Y}_R$ as $(\bar{Y}_R^{1st})$ and $(\bar{Y}_R^{2nd})$, which do not mean two new flavons.

$$\mu_R (\bar{Y}_R^{1st})^{\circ\circ} = \frac{1}{2} \frac{1}{(\Lambda_1)^2} \left\{ (\bar{\Phi}_e^T)^\circ{}_\bullet \bar{E}^{\bullet\bullet} (\hat{Y}_{eu})_\bullet{}^\bullet (\bar{\Phi}_u)_\bullet{}^\circ + (transposed) \right\}, \tag{5.7}$$

Here we define a new flavon $\hat{Y}_{eu}$:

$$\mu_{eu} (\hat{Y}_{eu})_\bullet{}^\bullet = (\bar{\Phi}_e)_\bullet{}^\circ (\Phi_u)_\circ{}^\bullet . \tag{5.6}$$

$$\mu_R(\bar{Y}_R^{2nd})^{\circ\circ} = \frac{1}{2}\frac{1}{(\Lambda_1)^2}\left\{(\bar{\Phi}_d)^{\circ}{}_{\bullet}(\bar{\Phi}_u^T)^{\bullet}{}_{\circ}(\bar{S}'_u)^{\circ\bullet}(\bar{\Phi}_u)_{\bullet}{}^{\circ} + (transposed)\right\}. \tag{5.10}$$

Here we introduce a new flavon whose VEV is proportional to $\langle \hat{S}_u^{-1}\rangle$:

$$W_{S'_u} = \left\{\bar{E}^{\circ\bullet}(\hat{E})_{\bullet}{}^{\bullet} + (\bar{S}'_u)^{\circ\bullet}(\hat{S}_u)_{\bullet}{}^{\bullet}\right\}(\Theta'_{Su})_{\bullet\circ}, \tag{5.9}$$

where, for simplicity, we have drop the coefficients λ_1 and λ_2 although we suppose $\lambda_1 \simeq \lambda_2 \simeq 1$.

Then, we estimate

$$\mu_R\|(\bar{Y}_R^{1st})^{\circ\circ}\| = (\eta_\Phi)^2\frac{\Lambda_1}{\mu_{eu}}\frac{(\Lambda_2)^4}{(\Lambda_1)^2}, \tag{5.8}$$

$$\mu_R\|(\bar{Y}_R^{2nd})\| = \frac{(\Lambda_2)^4}{(\Lambda_1)^2}. \tag{5.11}$$

Therefore, we obtain the ratio

$$R_{2/1} \equiv \frac{\|\bar{Y}_R^{2nd}\|}{\|\bar{Y}_R^{1st}\|} = \frac{1}{(\eta_\Phi)^2}\frac{\mu_{eu}}{\Lambda_1} \equiv \xi_R. \tag{5.12}$$

5.2 Parameter $¥xi_R$

In our model, after parameters $(b_u e^{\beta_u}, b_d)$ and $(\tilde{\phi}_1, \tilde{\phi}_2)$ defined in (1.2) and (3.4) are fixed by quark masses and CKM quark mixing fitting, neutrino masses and PMNS lepton mixing matrix are predicted only by one parameter ξ_R,

Parameter fitting for neutrino data is done under the following dimensionless rexpression:

$$\tilde{M}_\nu^{Maj} = (\xi_M)^4 Z^2 \tilde{Y}_R^{-1} Z^2, \tag{5.13}$$

where

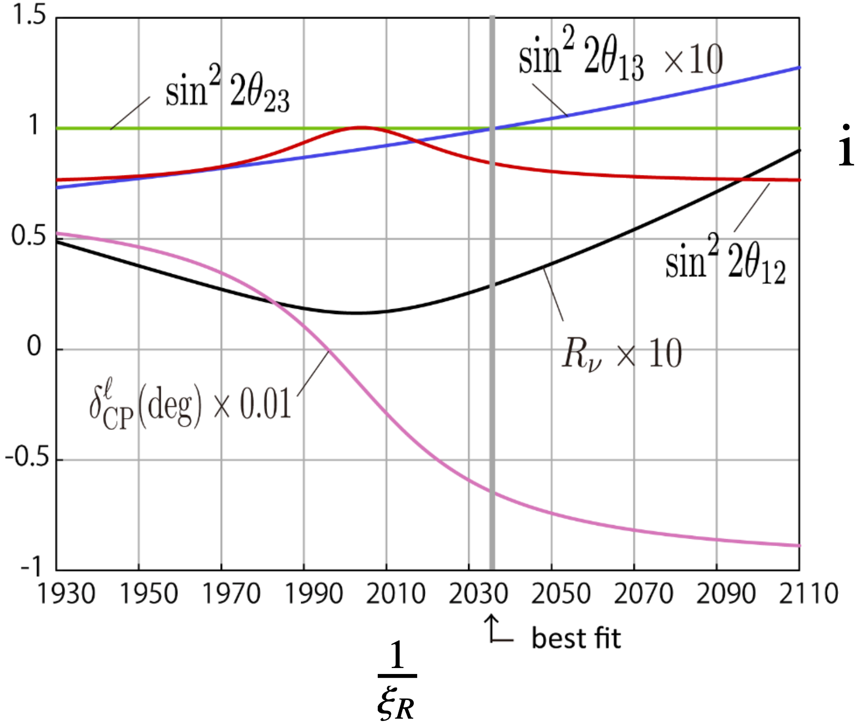
$$\tilde{Y}_R = Z^4 + \xi_R Z^2 P (\mathbf{1} + b_u X_3)^{-1} P^\dagger Z, \tag{5.14}$$

$$\begin{aligned} Z &= \mathrm{diag}(z_1, z_2, z_3), \\ P &= \mathrm{diag}(e^{i\tilde{\phi}_1}, e^{i\tilde{\phi}_2}, 1). \end{aligned} \tag{5.15}$$

(Z and P do not mean new flavons. Those are noting but dimensionless 3×3 matries.) The parameter ξ_R corresponds to the ratio $R_{2/1}$ defined in (5.12). (The parameter fitting is practically the same as one in the previous model [**?**]. However, note that the present parameter ξ_R corresponds to $1/\xi_R$ in the previous model.) The best fitting value of ξ_R is

$$\xi_R = 0.9806 \times 10^{-3}. \tag{5.17}$$

MNS and neutrino mass fitting



$$\frac{1}{\xi_R} = 2039.6 \qquad (\xi_R = 0.9806 \times 10^{-3}).$$

is consistent with all the observed values.

$$\sin^2 2\theta_{12} = 0.846 \pm 0.021,$$
$$\sin^2 2\theta_{23} = 0.999^{+0.001}_{-0.018},$$
$$\sin^2 2\theta_{13} = 0.093 \pm 0.008,$$
$$R_\nu \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = (3.09 \pm 0.15) \times 10^{-2}.$$

Fig. 2 ξ_R dependence of the lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared difference ratio R_ν. We draw curves of those as functions of ξ_R for the case of $b_u = -1.011$ and $(\phi_1, \phi_2) = (-176.05°, -167.91°)$.

Y. Koide and H. Nishiura, Phys.Rev. **D 92** (2015) 111301(R).

Predictions

- b_u, (b_d, β_d), (ϕ_1, ϕ_2), ξ_R have been fixed, so we predict

Table 1: Predicted values vs. observed values.

	$\lvert V_{us}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{ub}\rvert$	$\lvert V_{td}\rvert$	δ^q_{CP}	r^u_{12}	r^u_{23}	r^d_{12}	r^d_{23}
Pred	0.2257	0.03996	0.00370	0.00917	81.0°	0.061	0.060	0.049	0.027
Obs	0.22536	0.0414	0.00355	0.00886	69.4°	0.045	0.060	0.053	0.019
	±0.00061	±0.0012	±0.00015	$^{+0.00033}_{-0.00032}$	±3.4°	$^{+0.013}_{-0.010}$	±0.005	$^{+0.005}_{-0.003}$	$^{+0.006}_{-0.006}$
	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$	R_ν $[10^{-2}]$	δ^ℓ_{CP}	$m_{\nu 1}$ [eV]	$m_{\nu 2}$ [eV]	$m_{\nu 3}$ [eV]	$\langle m \rangle$ [eV]
Pred	0.8254	0.9967	0.1007	3.118	−68.1°	0.038	0.039	0.063	0.021
Obs	0.846	0.999	0.093	3.09	-	-	-	-	$< O(10^{-1})$
	±0.021	$^{+0.001}_{-0.018}$	±0.008	±0.15					

- The CP violating phase parameters δ^q_{CP} and δ^ℓ_{CP} in the standard expression of V_{CKM} and U_{PMNS} are predicted as $\delta^q_{CP} \simeq 81°$ and $\delta^\ell_{CP} = -68°$ respectively, i.e. $\delta^\ell_{CP} \sim -\delta^q_{CP}$. We also predit $\langle m \rangle = 0.021$ [eV]

Prediction of the Jarlskog invariant in CKM : $J = 3.21 \times 10^{-5}$

Observed value: $J = \left(3.06^{+0.21}_{-0.20}\right) \times 10^{-5}$

5.3 Scales of $¥mu$ parameters$¥xi_R$

Now, we have four flavon mass parameters μ_ν, $\mu_{S\nu}$, μ_{eu} and μ_R defined by (2.14), (2.18), (5.6) and (5.7), respectively. (For the time being, we do not discuss μ_H in (2.2).) So far, we have considered three VEV scale Λ_1, Λ_2 and Λ_3 as shown in(1.7) and also in Table 1. Therefor,let us put the following selection rules for

$$\mu_A ||A|| = ||G||, \tag{5.18}$$

where G is a combination of some flavons including factor $1/(\Lambda_1)^n$. (i) We assume that our parameters μ_ν, $\mu_{S\nu}$, μ_{eu} and μ_R are given by some of those three scales Λ_1, Λ_2 and Λ_3. (ii) As a result of our selection $\mu_A = \Lambda_a$ $(a = 1, 2, 3)$, if we get a scale $||A||$ whose value is larger than the maximally allowed scale of $||A||$, e.g. Λ_1 for $||(\hat{Y}_{eu})_\bullet{}^\bullet||$, Λ_2 for $||(\Phi_\nu)_\circ{}^\bullet||$, and Λ_3 for $||(\bar{Y}_R)^{\circ\circ}||$, then, the choice of μ_A is ruled out, because the scale $||A|| \leq ||A||_{max}$. (iii) If the choice $\mu_A = \Lambda_a$ gives a VEV value $||A||$ which is two rank lower compared with $||A||_{max}$, the case is also ruled out.

For example, let us see the case (5.6):

$$\mu_{eu}\|(\hat{Y}_{eu})_{\bullet}^{\ \bullet}\| = \eta_\Phi(\Lambda_2)^2 = \eta_\Phi\rho\Lambda_1\Lambda_3. \tag{5.19}$$

If we tale $\mu_{eu} = \Lambda_3$, we obtain $[\hat{Y}_{eu}] = \eta_\Phi\rho\Lambda_1$ from Eq.(5.19). Since $\|(\hat{Y}_{eu})_{\bullet}^{\ \bullet}\| \leq \Lambda_1$, the case $\mu_{eu} = \Lambda_3$ is not rulded out by the rule (ii). However, the case give $R_{2/1} = (\eta_\Phi)^{-2} \sim O(1)$ from (5.12). This contradicts with the parameter fitting result (5.17). Therefore, we rule out this case $\mu_{eu} = \Lambda_3$. On the other hand, if we take $\mu_{eu} = \Lambda_1$, we obtain $[\hat{Y}_{eu}] = \rho\Lambda_3$. However, the value is two rank small compared with the maximal value Λ_1, so that we rule out the case $\mu_{eu} = \Lambda_1$ by the rule (iii). As a result, we choose $\mu_{eu} = \Lambda_2$:

$$\mu_{eu} = \Lambda_2, \quad i.e. \quad \|(\hat{Y}_{eu})_{\bullet}^{\ \bullet}\| = \eta_\Phi\Lambda_2. \tag{5.20}$$

Then, from the relation (5.12), we obtain

$$\xi_R \equiv R_{2/1} = \frac{1}{(\eta_\Phi)^2}\frac{\Lambda_2}{\Lambda_1}. \tag{5.21}$$

Similarly, in the relation (2.19):

$$\mu_\nu \|(\Phi_\nu)_\circ^{\ \bullet}\| = \eta_\Phi \frac{(\Lambda_2)^2}{\Lambda_1} \Lambda_2 = \eta_\Phi \rho \Lambda_3 \Lambda_2 = \rho \Lambda_3 \|(\Phi_e)_\circ^{\ \bullet}\|, \tag{5.22}$$

if we take take $\mu_\nu = \Lambda_3$, we obtain $\|(\Phi_\nu)_\circ^{\ \bullet}\| = \Lambda_2$, i.e. $\xi_\nu = \rho > 1$. The result is not our desired one, because our aim is to understand tiny neutrino masses by $\xi_\nu \ll 1$. Therefore, we choose $\mu_\nu = \Lambda_2$, and we get

$$\|(\Phi_\nu)_\circ^{\ \bullet}\| = \eta_\Phi \rho \Lambda_3 = \rho \frac{\Lambda_3}{\Lambda_2} \eta_\Phi \Lambda_2 = \frac{\Lambda_2}{\Lambda_1} \|(\hat{S}_e)_\bullet^{\ \bullet}\|, \tag{5.23}$$

so that

$$\xi_\nu = \frac{\Lambda_2}{\Lambda_1} = (\eta_\Phi)^2 \xi_R. \tag{5.24}$$

from (5.12).

Similarly, when we choose $\mu_{S\nu} = \Lambda_2$ in Eq.(2.23), from

$$\mu_{S\nu}(\hat{S}_\nu)_\bullet^{\ \bullet} = \eta_S \Lambda_1 \frac{(\Lambda_2)^2}{\Lambda_1} = \eta_S \rho \Lambda_1 \Lambda_3, \tag{5.25}$$

we obtain

$$\|(\hat{S}_\nu)_\bullet^{\ \bullet}\| = \eta_S \frac{\Lambda_2}{\Lambda_1}\Lambda_1 = \frac{\Lambda_2}{\Lambda_1}\|\hat{S}_e\|, \tag{5.26}$$

so that

$$\xi_{S\nu} = \frac{\Lambda_2}{\Lambda_1} = (\eta_\Phi)^2 \xi_R. \tag{5.27}$$

Therefore, from the relation (2.26), we obtain

$$\xi_M = (\xi_\nu)^2 (\xi_{S\nu})^{-1} = (\eta_\Phi)^2 \xi_R. \tag{5.28}$$

Finally, we discuss a scale of μ_R. Now, from (2.14), by regarding Y_R as $Y_R \simeq Y_R^{1st}$, we can write

$$\mu_R \|(\bar{Y}_R)^{\circ\circ}\| = \frac{1}{(\Lambda_1)^2}(\eta_\Phi)^2 \Lambda_2 \Lambda_1 \Lambda_2 \Lambda_2 = \rho(\eta_\Phi)^2 \Lambda_2 \Lambda_3. \tag{5.29}$$

If we take $\mu_R = \Lambda_2$, we obtain

$$\|(\bar{Y}_R)^{\circ\circ}\| = \rho(\eta_\Phi)^2 \Lambda_3. \tag{5.30}$$

Since $\|(\bar{Y}_R)^{\circ\circ}\|$ cannot have a larger scale than Λ_3, we have a constraint

$$\rho(\eta_\Phi)^2 \leq 1, \quad i.e. \ \rho \leq (\eta_\Phi)^{-2} = 3^{2/3}. \tag{5.31}$$

Considering the relation (4.14), it is likely that the value of ρ is given in unit of $3^{1/3}$. Comparing the fitting value $\rho = 1.37$ with the value $(\eta_\Phi)^{-1} = 3^{1/3} = 1.44$, we regard the value of ρ as

$$\rho = \frac{1}{\eta_\Phi} = 3^{1/3}. \tag{5.32}$$

(Recall that in our scenario, we have already regarded $|c_3/c_2| = 0.967$ as $|c_3/c_2| = 1$, $m_{0u}/m_{0e} = 3.12$ as $m_{0u}/m_{0e} = 3$, and so on, approximately.)

6 Estimate of flavon scales

Similarly to Eq.(4.1), from the diagonalization of (5.13) with the parameter values (2.21), (2.22) and (5.16), we obtain

$$(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) = (0.30892, 0.31689, 0.51026)\, m_{0\nu}. \tag{6.1}$$

Here, we have defined m_{0R} by

$$m_{0\nu} = (\xi_M)^2 \frac{(m_{0e})^2}{m_{0R}}. \tag{6.2}$$

From the observed value $\Delta m^2_{32} = 0.000244$ eV^2 [**?**] and $\Delta m^2_{21}/\Delta m^2_{32} = 0.0309$ [**?**], we estimate $m_{\nu 3} = 0.063$ eV, and we obtain

$$m_{0\nu} = 1.235 \times 10^{-13} \text{ TeV}. \tag{6.3}$$

Then, from Eq.(6.2), we obtain ,

$$m_{0R} = (\xi_M)^2 \frac{(m_{0e})^2}{m_{0\nu}} = (\xi_M)^2 \times 2.7710 \times 10^7 \text{ TeV}. \tag{6.4}$$

From Eq.(5.28), we estimate

$$m_{0R} = (\eta_\Phi)^4 (\xi_R)^2 \times 2.7710 \times 10^7 \text{ TeV} = 6.158 \text{ TeV}. \quad (6.5)$$

Therefore, from (5.30) with (5.32), i.e. $\|Y_R\| = \eta_\Phi \Lambda_3$, we obtain

$$\Lambda_3 = (\eta_\Phi)^{-1} m_{0R} = 8.883 \text{ TeV}. \quad (6.6)$$

In conclusion, we obtain

$$\begin{aligned} \Lambda_3 &= 8.883 \text{ TeV}, \\ \Lambda_2 &= \rho\Lambda_3/\xi_R = 1.306 \times 10^4 \text{ TeV}, \\ \Lambda_1 &= \Lambda_2/\xi_R = 1.332 \times 10^7 \text{ TeV}. \end{aligned} \quad (6.7)$$

Finally, we estimate the value of Λ_H defined in (2.5). From (2.5), we use a relation

$$m_{0e} = \|\Psi_e(\hat{S}_e)^{-1}\bar{\Phi}_e\| = \frac{v_{Hd}}{\Lambda_H}(\eta_\Phi)^2(\eta_S)^{-1}\frac{(\Lambda_2)^2}{\Lambda_1} = \frac{v_H}{\Lambda_H}(\eta_\Phi)^2(\eta_S)^{-1}\rho\Lambda_3, \quad (6.8)$$

where $m_{0e} = 1.8499 \times 10^{-3}$ TeV, $v_H = 173.9 \times 10^{-3}$ TeV, and $(\eta_\Phi)^2(\eta_S)^{-1}\rho = 3^{-2/3}$, so that we obtain

$$\Lambda_H = 401.4 \text{ TeV}. \quad (6.9)$$

The result (6.9) gives $(v_H)/\Lambda_H = 0.4332 \times 10^{-3}$.

6. Summary

· We have constructed a mass matrix model for quarks and leptons (yukawaon model + seesaw model). We use only the observed values of charged lepton masses (m_e, m_μ, m_τ) as family number-dependent input parameters.

· the VEV scales of flavons $(\mathbf{8}+\mathbf{1},\mathbf{1})$, $(\mathbf{3},\mathbf{3}^*)$, and $(\mathbf{1},\mathbf{8}+\mathbf{1})$ of the model which is newly reconstructed without changing the previous phenomenological success of parameter fitting for masses and mixings of quarks and leptons By using results of the previous parameter fitting, we conclude that VEVs of flavons $(\mathbf{8}+\mathbf{1},\mathbf{1})$, $(\mathbf{3},\mathbf{3}^*)$, and $(\mathbf{1},\mathbf{8}+\mathbf{1})$ are of the orders of 10 TeV, 10^4 TeV, and 10^7 TeV, respectively. This result is consistent with speculation in Sumino's U(3)×O(3) model.

· Our predictions are reasonable values as shown in Table 1. Especially, the CP violating phase parameters δ^q_{CP} and δ^ℓ_{CP} in the standard expression of V_{CKM} and U_{PMNS} are predicted as $\delta^q_{CP} \simeq 81.0^\circ$ and $\delta^\ell_{CP} = -68.1^\circ$ respectively, i.e. $\delta^\ell_{CP} \sim -\delta^q_{CP}$. We also predit $\langle m \rangle = 0.021$ [eV]