Flavor 2014: Feb. 14 , 2014, at Singapore

U(3) Family Symmetry and Visible Gauge Boson Effects

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Based on YK and T.Yamashita, PLB 711, 384 (2012)

YK, PRD 87, 016016 (2013)

Contents

1 Theoretical aspects of our model

Mt. Fuji in winter

1.1 Why not consider a family gauge symmetry?

- The degree of freedom "families" is the last one which has still not been accepted as a gauge symmetry in SM, in spite of many pioneer works, e.g. Yanagida, Wilczek-Zee, ...
- If the family gauge symmetry is absent, the CKM mixing $V_{CKM} = U_{Lu}^{\dagger} U_{Ld}$ is observable, while the quark mixing matrices U_u and U_d are not observable! I think that a theory which includes such unobservable quantities is incomplete.

I believe that family gauge bosons really exist. Then, I think that the gauge bosons should be observed by terrestrial experiments.

1.2 Our gauge boson model: How different from other models

(i) U(3) family symmetry

charged lepton mass relation YK (1982)

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$

Ma, PLB (2007)

Sumino, JHEP (2009)

Sumino's cancellation mechanism

Y.Sumino, PLB671, 477 (2009)

(ii) Inverted mass hierarchy

YK&Yamashita, PLB671, 477(2012) $M_{33} \ll M_{22} \ll M_{11}$

(iii) Family number violation appears only in the quark sector

$$\mathcal{H}_{fam} = \frac{g_F}{\sqrt{2}} \left[(\bar{e}_i \gamma_\mu e_j) + (\bar{\nu}_i \gamma_\mu \nu_j) + U^{*d}_{ik} U^d_{jl} (\bar{d}_k \gamma_\mu d_l) + U^{*u}_{ik} U^u_{jl} (\bar{u}_k \gamma_\mu u_l) \right] (A_i{}^j)^\mu$$

1.3 Sumino mechanism

Y. Sumino, PLB 671, 477 (2009)

2009: Sumino has seriously taken why the mass formula

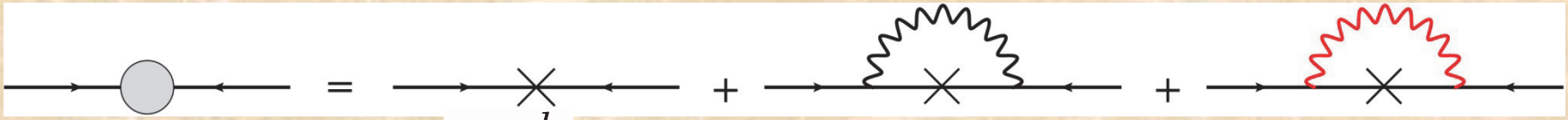
$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$

YK (1982)

is so remarkably satisfied with the pole masses, while if we take the running masses, the agreement is somewhat spoiled. The deviation comes from the $\log(m_{ei}^2/\mu^2)$ factor in the QED radiative correction. If the logarithmic term is absent, the formula can be invariant.

Sumino's basic idea

$M_{ii}^2 \propto m_{ei}$



$m_i(\mu)$ $\quad$ m_i^{pole} $\quad$ $e^2 \log m_i^2$ $\quad$ $-g^2 \log M_{ii}^2$

Note that the cancellation is done only for $\log(m_{ei}^2/\mu^2)$ term. Therefore, photon and family gauge bosons can still contribute to the evolution of the charged lepton masses.

1.4 Masses of the gauge bosons

We consider scalars $(3, 3^*)$ of $U(3)\times U(3)'$ with $\Lambda \ll \Lambda'$

There are no (6,1) and/or (3, 1) in this model.

Family symmetry is broken by VEV of $\langle \Phi_i{}^\alpha \rangle = \delta_i{}^\alpha v_i$

so that the gauge boson masses are given by

$$\mathcal{H}_{mass} = \frac{1}{2}\mathrm{Tr}\left[g_A^2 \Phi\Phi^\dagger AA + g_B^2 \Phi^\dagger\Phi BB - 2g_A g_B \Phi^\dagger A\Phi B\right].$$

In the limit of massive B, the masses of A are given by

$$M^2(A_i{}^j) = \frac{1}{2}g_A^2(|v_i|^2 + |v_j|^2)$$ **+(scalars for q & l)**

dominant negligibly small

Therefore, the gauge boson masses satisfy

$$2M_{ij}^2 = M_{ii}^2 + M_{jj}^2$$

$$\mathcal{H}_{fam} = \frac{g_F}{\sqrt{2}}\left[(\bar{e}_i\gamma_\mu e_j) + (\bar{\nu}_i\gamma_\mu \nu_j) + U_{ik}^{*d}U_{jl}^d(\bar{d}_k\gamma_\mu d_l) + U_{ik}^{*u}U_{jl}^u(\bar{u}_k\gamma_\mu u_l)\right](A_i{}^j)^\mu$$

YK an d T.Yamashita, PLB 711, 384 (2012)

2 Phenomenological aspects

Osaka Castle

2.1 Constraint form K^0-$\bar{K}^0$ mixing

The most severe constraint on the family gage boson masses comes from the observed K^0-$\bar{K}^0$ mixing:

From the observed value $\Delta m_K^{obs} = (3.484 \pm 0.006) \times 10^{-18}$ TeV

and an estimate in SM $\Delta m_K^{SM} \sim 2 \times 10^{-18}$ TeV

and assuming $U_d \simeq V_{CKM}$, we obtain $\tilde{M}_{22} \sim 340$ TeV

where $\tilde{M}_{ij}$ is defined as $\tilde{M}_{ij} \equiv M_{ij}/g_F/\sqrt{2}$

The value 340 TeV was considerably reduced compared with past models, but it is still too large to observe the effects!

We must try to reduce the value $\tilde{M}_{22} \sim 340$ TeV

or we have to change the model.

In this talk, from the phenomenological point of view, I will introduce another scenario.

2.2 Rare decays of ps-mesons

In order to consider a scale of family gauge boson masses, apart from the observed constraint from the K^0-$\bar{K}^0$ mixing, let us see the present status of rare K and B decay searches:

$$\tilde{M}_{ij} \equiv \frac{M_{ij}}{g_F/\sqrt{2}}$$

	Input	Output [TeV]	
$Br(K^+ \to \pi^+ e^- \mu^+)$	$< 1.3 \times 10^{-11}$	$\tilde{M}_{12}$	> 196
$Br(K_L \to \pi^0 e^\mp \mu^\pm)$	$< 7.6 \times 10^{-11}$	$\tilde{M}_{12}$	> 151
$Br(K_L \to \pi^0 \nu\bar{\nu})$	$< 2.6 \times 10^{-8}$	$\tilde{M}_{12}$	> 17.5
$Br(K^+ \to \pi^+ \nu\bar{\nu})$	$= (1.7 \pm 1.1) \times 10^{-10}$	$\tilde{M}_{12}$	$\sim 250^*$
$Br(B^+ \to K^+ \mu^\pm \tau^\mp)$	$< 7.7 \times 10^{-5}$	$\tilde{M}_{23}$	> 4.11
$Br(B^+ \to K^+ \nu\bar{\nu}))$	$< 1.3 \times 10^{-5}$	$\tilde{M}_{23}$	> 5.4
$Br(B^0 \to K^0 \nu\bar{\nu})$	$< 5.6 \times 10^{-6}$	$\tilde{M}_{23}$	> 6.7
$Br(B^0 \to \pi^0 \nu\bar{\nu})$	$< 2.2 \times 10^{-4}$	$\tilde{M}_{31}$	> 4.8

* We have used $Br(K^+ \to \pi^+ \nu\bar{\nu})_{SM} = (0.80 \pm 0.11) \times 10^{-10}$

Ishidori, et al. (05); Buras, et al. (08)

Roughly speaking, we can see that

$\tilde{M}_{12} \geq 2.5 \times 10^2$ TeV $\quad \tilde{M}_{23} \geq 7$ TeV

This is in favor of the inverted mass picture of A_j^i

3 Speculation about the mass values

Entertainment district in Osaka

3.1 We want a model with visible gauge bosons, but …

In the inverted mass hierarchy model (K-Y model), the mass spectrum is given by

$$M_{33} : M_{23} : M_{22} : M_{31} : M_{12} : M_{11}$$

$$= 1 : \frac{a}{\sqrt{2}}\sqrt{1+\frac{1}{a^2}} : a : \frac{b}{\sqrt{2}}\sqrt{1+\frac{1}{b^2}} : \frac{b}{\sqrt{2}}\sqrt{1+\frac{a^2}{b^2}} : b$$

$$\simeq 1 : \frac{a}{\sqrt{2}} : a : \frac{b}{\sqrt{2}} : \frac{b}{\sqrt{2}} : b \qquad (1 \ll a^2 \ll b^2)$$

where

$$a \equiv M_{22}/M_{33}, \quad b \equiv M_{11}/M_{33}$$

We cannot simultaneously assign 340 TeV to $\tilde{M}_{22}$

and 250 TeV to $\tilde{M}_{12}$

We have to search another model.

3.2 K^0-$\bar{K}^0$ mixing

The biggest obstacle comes from the estimate of K^0-$\bar{K}^0$ mixing.

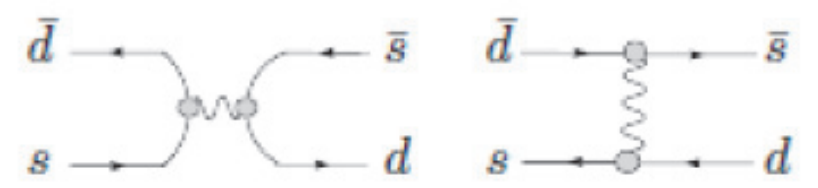
Effective current-current interactions with $\Delta N_{fam} = 2$ are given by

$$H^{eff} = \frac{1}{2} g_F^2 \left[\sum_i \frac{(\lambda_i)^2}{M_{ii}^2} + 2 \sum_{i<j} \frac{\lambda_i \lambda_j}{M_{ij}^2} \right] (\bar{q}_k \gamma_\mu q_l)(\bar{q}_k \gamma^\mu q_l)$$

where $\lambda_1 = U_{1k}^* U_{1l}, \quad \lambda_2 = U_{2k}^* U_{2l}, \quad \lambda_3 = U_{3k}^* U_{3l}$

For example, for K^0-$\bar{K}^0$ mixing, λ_i are given by

$$\lambda_1 = U_{11}^{d*} U_{12}^d, \quad \lambda_2 = U_{21}^{d*} U_{22}^d, \quad \lambda_3 = U_{31}^{d*} U_{32}^d$$



3.3 Harmless condition

The effective coupling constants are given by

$$G^{eff} = \frac{1}{2} g_F^2 \left[\frac{\lambda_1^2}{M_{11}^2} + \frac{\lambda_2^2}{M_{22}^2} + \frac{\lambda_3^2}{M_{33}^2} + 2\left(\frac{\lambda_1\lambda_2}{M_{12}^2} + \frac{\lambda_2\lambda_3}{M_{23}^2} + \frac{\lambda_3\lambda_1}{M_{31}^2} \right) \right]$$

Here, note that λ_i satisfy a unitary triangle condition

$$\lambda_1 + \lambda_2 + \lambda_3 = 0$$

Therefore, if we assume that the gauge boson masses satisfy

$$\frac{2}{M_{ij}^2} = \frac{1}{M_{ii}^2} + \frac{1}{M_{jj}^2}$$

the G^{eff} becomes exactly zero independently of the values of λ_i

For convenience, we call the above relation

harmless condition of the family gauge bosons to P^0-$\bar{P}^0$ mixing.

There is no theoretical basis for this harmless relation at present.

Nevertheless, we have optimistic view of future.

We will discuss possible phenomena under the harmless condition.

3.4 Harmless condition and mass spectrum

Let us discuss possible family gauge boson masses apart from the Sumino's cancelation condition for the time being.

(We regard the parameters a and b as free parameters..)

The harmless conditions lead to the following mass spectrum

$$M_{33} : M_{23} : M_{31} : M_{22} : M_{12} : M_{11}$$

$$= 1 : \sqrt{\frac{2}{1+1/a^2}} : \sqrt{\frac{2}{1+1/b^2}} : a : \sqrt{\frac{2}{1+a^2/b^2}}\, a : b$$

where the parameters a and b are defined by

$$a \equiv M_{22}/M_{33}, \quad b \equiv M_{11}/M_{33}$$

For a case of $1 \ll a^2 \ll b^2$, the relation leads to

$$M_{33} : M_{23} : M_{31} : M_{22} : M_{12} : M_{11} \simeq 1 : \sqrt{2} : \sqrt{2} : a : \sqrt{2}a : b$$

c.f. K-Y model : $M_{33} : M_{23} : M_{22} : M_{31} : M_{12} : M_{11}$

$$\simeq 1 : \frac{a}{\sqrt{2}} : a : \frac{b}{\sqrt{2}} : \frac{b}{\sqrt{2}} : b$$

3.5 Numerical speculation

We can expect visible family gauge boson effects.

However, regrettably, at present, we have no reliable input values of family gauge boson masses.

As temporary values, we use

$\tilde{M}_{23} \sim 7$ TeV from a lower limit of rare B decays

$\tilde{M}_{12} \sim 250$ TeV from $Br(K^+ \to \pi^+ \nu\bar{\nu})$

Then, masses of family gauge bosons are speculated as follows:

$$\tilde{M}_{33} \sim 5 \text{ TeV}, \quad \tilde{M}_{23} \simeq \tilde{M}_{31} \sim 7 \text{ TeV},$$

$$\tilde{M}_{22} \sim 177 \text{ TeV}, \quad \tilde{M}_{12} \sim 250 \text{ TeV}, \quad \tilde{M}_{11} \sim ? \text{ TeV}$$

Note that $\tilde{M}_{ij}$ is not the real mass value M_{ij}:

e.g. if $g_F/\sqrt{2} \sim 0.5$, $\tilde{M}_{33} \sim 5$ TeV means $M_{33} \sim 2.5$ TeV

Therefore, direct observations of $A_3{}^3$, $A_2{}^3$ and $A_1{}^3$ at 14 TeV LHC are within our reach.

4 Visible gauge boson effects

Monument at Osaka International Exposition Memorial Park

4.1 Visible effects: Overview

(i) Deviation from the e-μ-τ universality

I will skip this topic: see YK, PRD 87, 016016 (2013)

(ii) Lepton number violating rare decays of K and B

As seen in the previous Table, those are soon within our reach.

(iii) μ-e conversion ($\mu N \to eN$)

This experiment is powerful to determining the value of $\tilde{M}_{12}$

(The details are given in the next slide.)

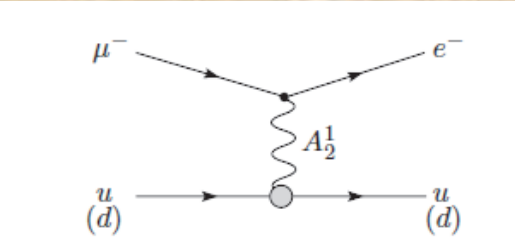
(iv) Direct production at 14 TeV LHC

e.g. $pp \to A_3^3 + \bar{b}b + X \to \tau^-\tau^+ + X$

$pp \to A_2^3 + \bar{s}b + X \to \mu^-\tau^+ + X$

$pp \to A_1^3 + \bar{d}b + X \to e^-\tau^+ + X$

4.2 μ-e conversion



Past experiment: SINDRUM (2006)

$$R(A_u) \equiv \frac{\sigma(\mu^- + Au \to e^- + Au)}{\sigma(\mu \text{ capture})} < 7 \times 10^{-13}$$

We tentatively give a rough estimate

$$R_q \equiv \frac{\sigma(\mu^- + q \to e^- + q)}{\sigma(\mu^- + u \to \nu_\mu + d)} \simeq \left(\frac{|U_{11}^{q*} U_{21}^q|}{|V_{ud}|} \frac{g_F^2/2}{M_{12}^2} \frac{M_W^2}{g_w^2/8} \right)^2$$

$$R_d \sim 3.8 \times 10^{-14} \qquad (\tilde{M}_{12} \simeq 250 \text{ TeV})$$

Near future: COMET (Coherent Muon to Electron Transition)

$$R \sim 3 \times 10^{-15} \ (2017)$$

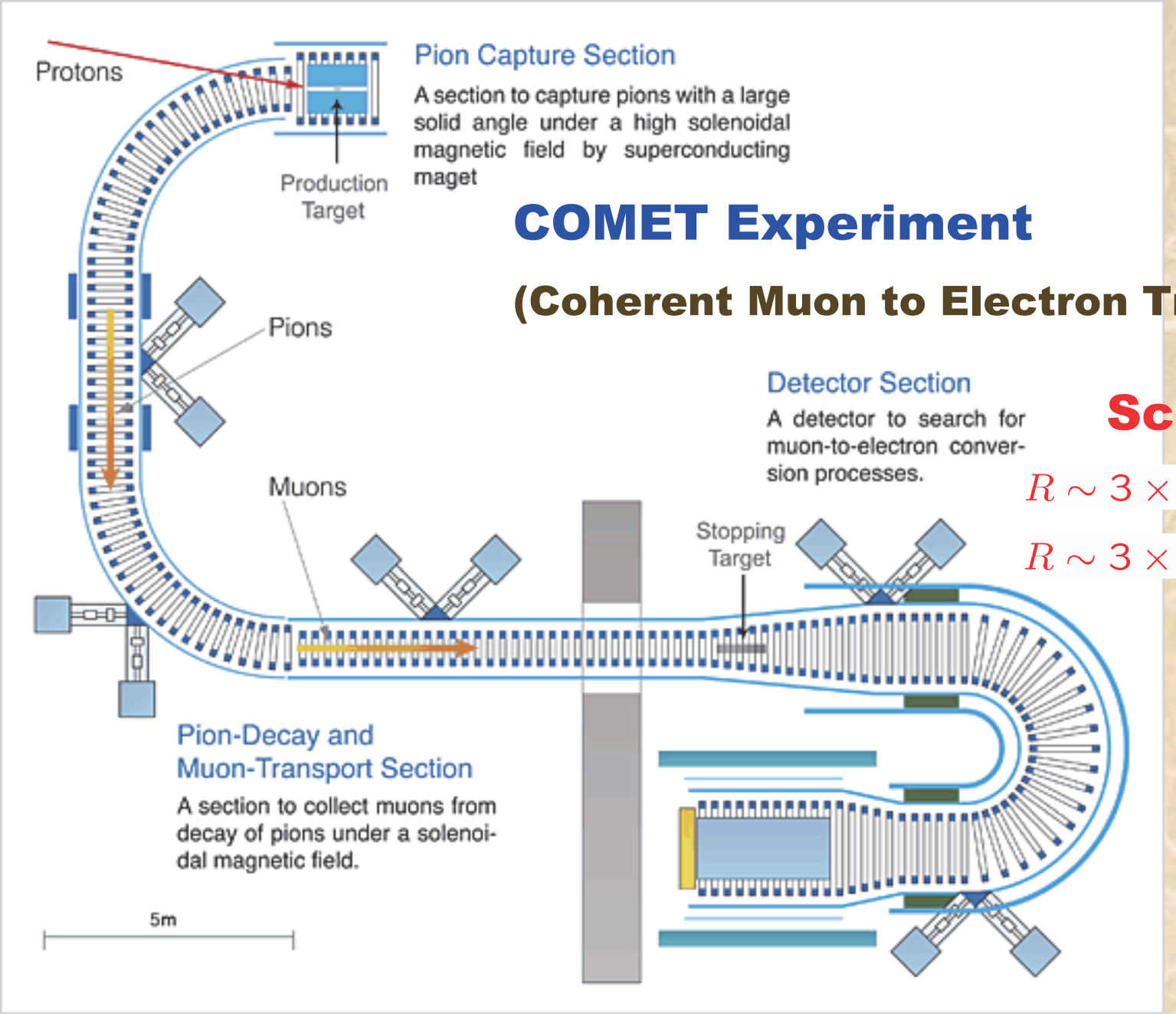
Note that

our gauge boson model allows μ-e conversion,

while the model highly suppresses $\mu \to e + \gamma$

COMET Experiment

(Coherent Muon to Electron Transition)



Schedule

$R \sim 3 \times 10^{-15}$ (2017)

$R \sim 3 \times 10^{-17}$ (2021)

図1 COMET全体図

quoted from http://www-kuno.phys.sci.osaka-u.ac.jp/research/index.html

4.4 Direct production of A_3^3 and so on

$$pp \to A_3^3 + \bar{b}b + X \to \tau^-\tau^+ + X$$

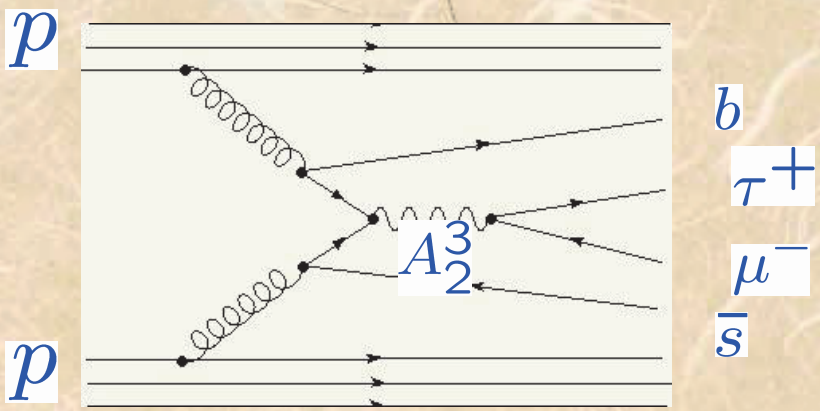
$$pp \to A_2^3 + \bar{s}b + X \to \mu^-\tau^+ + X$$

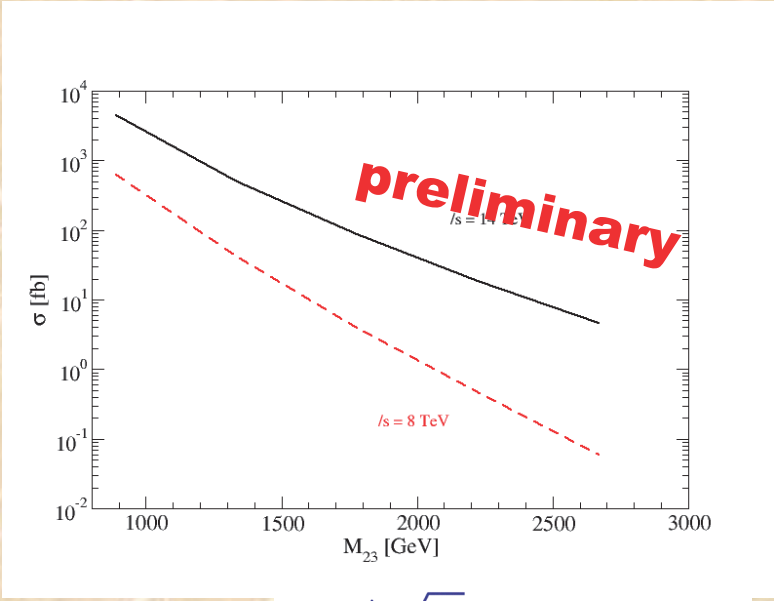
$$pp \to A_1^3 + \bar{d}b + X \to e^-\tau^+ + X$$

at 14 TeV LHC

$\sigma(pp \to A_2^3) \sim 40$ fb

at $M_{23} = 2$ TeV





$g_F/\sqrt{2} = 0.5322$

illustrated by H.Yokoya

Branching ratios

$$Br(A_i^j \to \ell_i \bar{\ell}_j) = 2/15 = 13.3\%$$

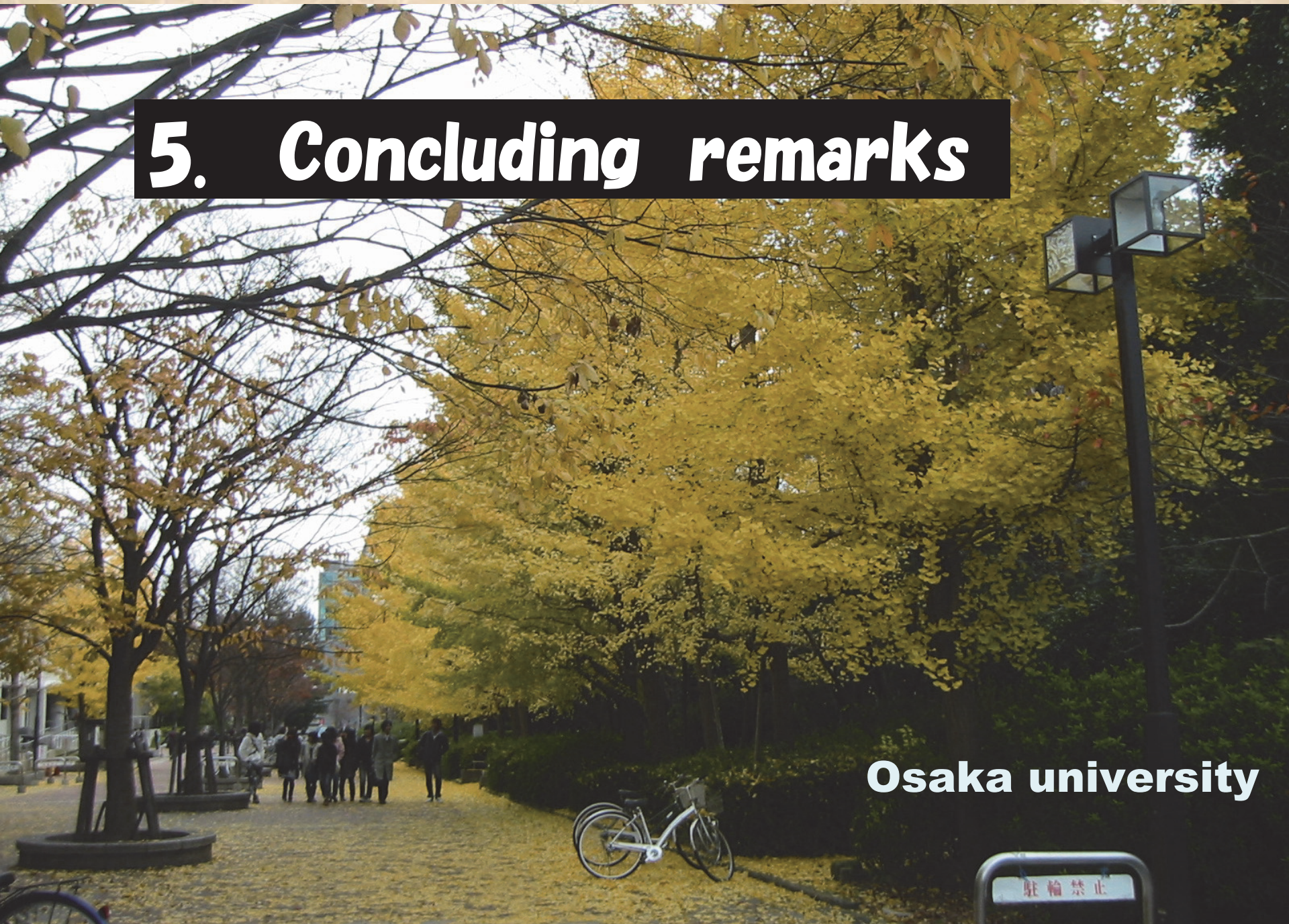
$$Br(A_i^j \to \nu_i \bar{\nu}_j) = 1/15 = 6.7\%$$

If ν are Dirac type,

$$Br(A_i^j \to \nu_i \bar{\nu}_j) = 2/16 = 12.5\%$$

We can determine whether ν is Majorana or Dirac.

5. Concluding remarks



Osaka university

We have investigated phenomenology of family gauge bosons based on a model:

$$\tilde{M}_{ij} \equiv M_{ij}/g_F/\sqrt{2}$$

(i) U(3) family gauge symmetry

(ii) Family number violation appears **only in the quark sector** through $U_u \neq 1$ and $U_d \neq 1$.

From the phenomenological interest, we investigate a case with the inverted mass hierarchy $M_{33}^2 \ll M_{22}^2 \ll M_{11}^2$ under an ansatz, harmless condition

$$\frac{2}{M_{ij}^2} = \frac{1}{M_{ii}^2} + \frac{1}{M_{jj}^2}$$

Theoretical derivation of the condition is a future task to us.

Then, we speculate the family gauge boson masses

$$\tilde{M}_{33} \sim 5 \text{ TeV}, \quad \tilde{M}_{23} \simeq \tilde{M}_{31} \sim 7 \text{ TeV},$$

$$\tilde{M}_{22} \sim 177 \text{ TeV}, \quad \tilde{M}_{12} \sim 250 \text{ TeV}, \quad \tilde{M}_{11} \sim ? \text{ TeV}$$

where we have used $\tilde{M}_{12} \simeq 250$ TeV and $\tilde{M}_{23} \sim 7$ TeV as input values.

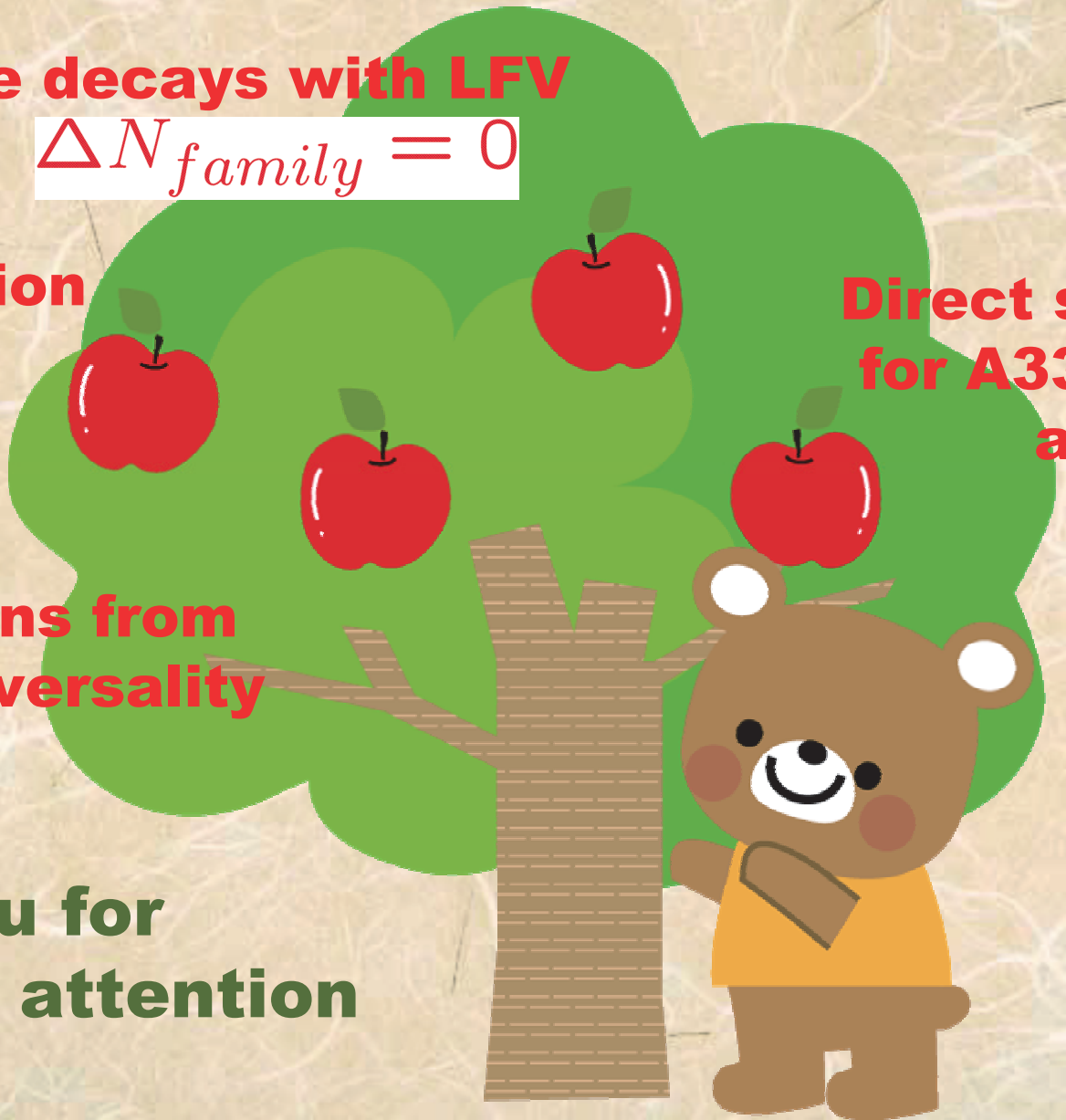
We can expect fruitful phenomenology

Rare decays with LFV
but $\Delta N_{family} = 0$

μ-e conversion

Direct search for A33, A31 at LHC

Deviations from e-μ-τ universality



Thank you for your kind attention

Backup Slides

2.2 Why "inverted" ?

Problems of the Sumino model:

(i) Sumino model is not anomaly free model

(ii) Effective current-current interactions with $\Delta N_f = 2$ appear

(iii) The vertex type diagram does not work in a SUSY model.

Sumino model	**Our model**
Y. Sumino,	YK and T.Yamashita
PLB 671, 477 (2009)	PLB 711, 381 (2012)

The origin of minus sign in order to cancel the QED correction

$(\psi_L, \psi_R) = (3, 3^*)$	$(\psi_L, \psi_R) = (3, 3)$
$M_{ij}^2 \equiv m^2(A_i^j) = k(m_{ei} + m_{ej})$	$M_{ij}^2 \equiv m^2(A_i^j) = k\left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}}\right)$
$+\log M_{ii}^2 = -\frac{1}{2}\log m_{ei}^2 + \log k$	$\log M_{ii}^2 = -\frac{1}{2}\log m_{ei}^2 + \log k$
Normal mass hierarchy	**Inverted** mass hierarch

Addendum to the Charged lepton mass relation

On this occasion, I would like to give a comment on the charged lepton mass formula:

Sometimes, people call the mass formula

"empirical mass formula".

But, I do not like that the formula is called as "empirical".

The formula was derived from a U(3) family model by assuming an ideal mixing between octet and singlet of U(3).

YK, LNC 34, 201 (1982), PL 120B, 161 (1983)

Regrettably, at that time, the formula was in poor agreement with experimental value of the tau mass.

It was 10 years after that the precise value of the tau mass was reported: ARGUS, BES, CLEO (1992).

If my aim was to search for an empirical formula, I would have proposed another formula.

Of course, the excellent agreement after 1992 is out of my anticipation, because a formula derived on the basis of a "model" is always only approximately satisfied.

How about deviation from the e-μ universality in the tau decays

From the branching ratios (PDG2012)

$$Br(\tau^- \to \mu^- \bar{\nu}_\tau \nu_\tau) = (17.41 \pm 0.04)\%$$

$$Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = (17.83 \pm 0.04)\%$$

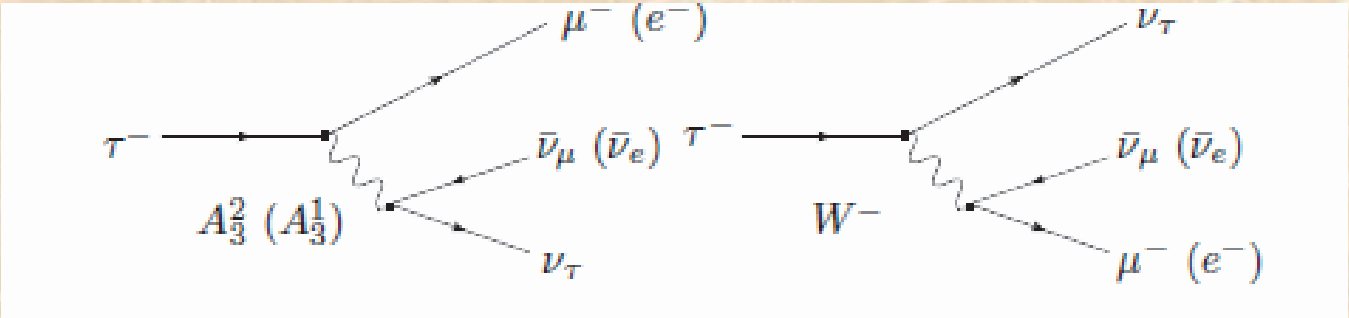
we obtain

$$R_{amp} \equiv \frac{1+\varepsilon_\mu}{1+\varepsilon_e} = \sqrt{\frac{Br(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{f(m_e/m_\tau)}{f(m_\mu/m_\tau)}} = 1.0020 \pm 0.0016.$$

where $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 12x^4 \log x^2$

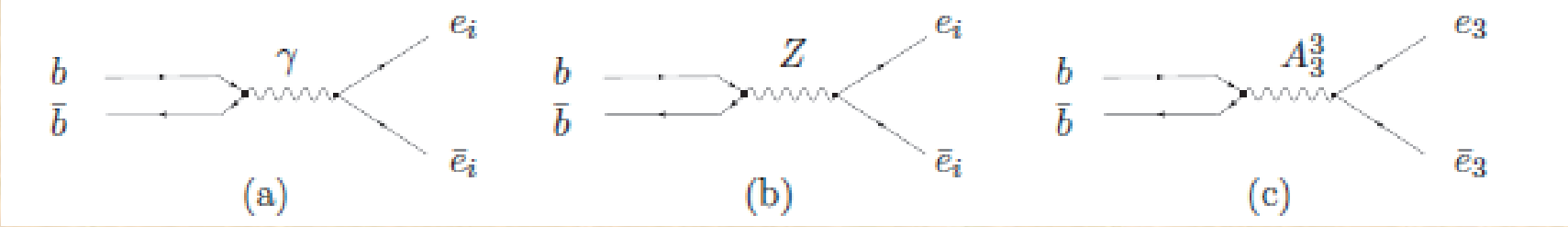
Therefore, we obtain $\varepsilon \equiv \varepsilon_\mu - \varepsilon_e = 0.0020 \pm 0.0016$

$$\tilde{M}_{23} = (6.4^{+7.9}_{-1.7})\ \text{TeV}$$



How about deviations from the e−μ−τ universality in Υ decays

We consider that the deviation from the e−μ−τ universality in the Υ decays is caused by the family gauge boson exchange terms



Under an approximation of neglecting family mixing in the quark sector, the b quark interacts only with A33 boson, so that the deviation parameter is given by

$$\varepsilon_\tau = \frac{g_F^2}{e^2/3}\frac{M_\Upsilon^2}{M_{33}^2}$$

$$\tilde{M}_{33} = (0.28^{+0.32}_{-0.06})\ \text{TeV}$$

Estimate of $\tilde{M}_{22}$ from Δm_K

- For convenience, we assume $U_d \simeq V_{CKM}$

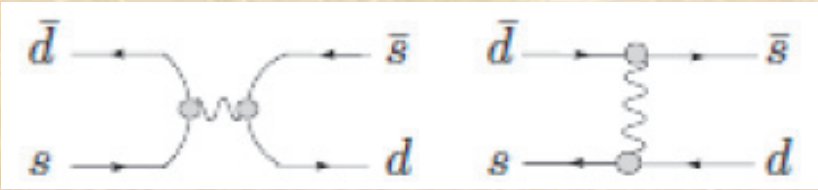
Then we obtain
$$|\lambda_1| = |U_{11}^* U_{12}| = 0.2195$$
$$|\lambda_2| = |U_{21}^* U_{22}| = 0.2192$$
$$|\lambda_3| = |U_{31}^* U_{32}| = 0.00035 \quad ,$$

so that we can regard those as $\lambda_3 \simeq 0, \quad \lambda_1 \simeq -\lambda_2$.

Therefore, we obtain

$$G^{eff} = \frac{1}{2} g_F^2 \left[\frac{\lambda_1^2}{M_{11}^2} + \frac{\lambda_2^2}{M_{22}^2} + \frac{\lambda_3^2}{M_{33}^2} + 2\left(\frac{\lambda_1\lambda_2}{M_{12}^2} + \frac{\lambda_2\lambda_3}{M_{23}^2} + \frac{\lambda_3\lambda_1}{M_{31}^2} \right) \right] \simeq \frac{1}{2} g_F^2 \frac{\lambda_2^2}{M_{22}^2}$$

- Under the vacuum-insertion approximation, we obtain



$$\Delta m_K = \frac{1}{6} G^{eff} f_K^2 m_K (1 + 2S_K)$$

$$S_K = \frac{m_K^2}{(m_s + m_d)^2}$$