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Can Family Gauge Bosons Be Visible by Terrestrial Experiments?

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Slides used at Flavor Physics 2014, Singapore, are reused
in part, but the present contents are substantially different from previous.

Contents

1. Why not consider family gauge symmetry?
2. If FGB have masses of the order of 1-10^2 TeV, ...
3. The biggest obstacle against such an optimistic view
4. An example of FGB model

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1. Why not consider a family gauge symmetry?

Welcome to our restaurant "New Physics"

We recommend our special menu " visible family gauge boson model" to you!

1.1 Basic standpoints

- The degree of freedom "families" is the last one which has still not been accepted as a gauge symmetry in SM. The idea of FGB is the most natural extension of SM.
- If the family gauge symmetry is absent, the CKM mixing $V_{CKM} = U_{Lu}^{\dagger} U_{Ld}$ is observable, while the quark mixing matrices U_u and U_d are not observable! I think that a theory which includes such unobservable quantities is incomplete.
- If FGB really exist, those should be particles which can be observed by terrestrial experiments.

1.2 Background Knowledge:
Sumino model and its extended one

The Sumino model shed a new light on the FGB model

Y. Sumino, PLB 671, 477 (2009)

Besides, we have its extended version:

YK and T.Yamashita, PLB 711, 384 (2012)

I will talk a possibility of the observation of FGB on the basis of those models.

Characteristics of new FGB model

(i) U(3) family symmetry: Number of FGB is 9

charged lepton mass relation YK (1982)

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$

Ma, PLB (2007)

Sumino, JHEP (2009)

Sumino's cancellation mechanism

Y.Sumino, PLB671, 477 (2009)

(ii) Inverted mass hierarchy

YK&Yamashita, PLB671, 477(2012)

$$M_{33} \ll M_{22} \ll M_{11}$$

(iii) Family number violation appears only in the quark sector

$$\mathcal{H}_{fam} = \frac{g_F}{\sqrt{2}}\left[(\bar{e}_i\gamma_\mu e_j) + (\bar{\nu}_i\gamma_\mu\nu_j) + U^{*d}_{ik}U^{d}_{jl}(\bar{d}_k\gamma_\mu d_l) + U^{*u}_{ik}U^{u}_{jl}(\bar{u}_k\gamma_\mu u_l)\right](A_i{}^j)^\mu$$

1.2.1 Sumino model

Y. Sumino, PLB 671, 477 (2009)

(i) **Family gauge bosons exist in the mass-eigen-states on the basis in which the charged lepton mass matrix is diagonal.**

(ii) **Up- and down quark mass matrices are, in general, not diagonal, so that the family gauge boson interactions are given by**

$$\mathcal{H}_{fam} = \frac{g_F}{\sqrt{2}} \left[(\bar{e}_i \gamma_\mu e_j) + (\bar{\nu}_i \gamma_\mu \nu_j) + U^{*d}_{ik} U^d_{jl} (\bar{d}_k \gamma_\mu d_l) + U^{*u}_{ik} U^u_{jl} (\bar{u}_k \gamma_\mu u_l) \right] (A_i{}^j)^\mu$$

(iii) **That is, family-number violation is caused only in the quark sector!**

(Exactly speaking, the above interaction form is one in K–Y model as we review later.)

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Motive of the Sumino model

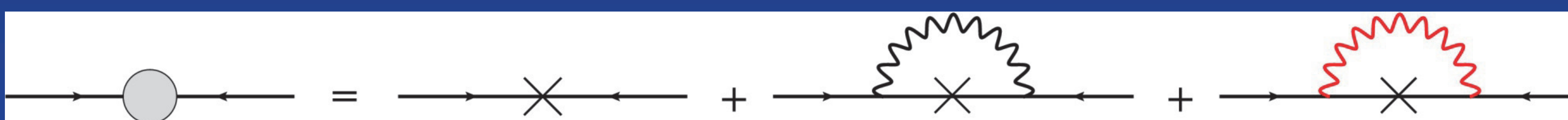
2009: Sumino has seriously taken why the mass formula

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$ YK (1982)

is so remarkably satisfied with the pole masses, while if we take the running masses, the agreement is somewhat spoiled. The deviation comes from the $\log(m_{ei}^2/\mu^2)$ factor in the QED radiative correction. If the logarithmic term is absent, the formula can be invariant.

Sumino's basic idea

$M_{ii}^2 \propto m_{ei}$



$m_i(\mu)$ $\quad$ m_i^{pole} $\quad$ $e^2 \log m_i^2$ $\quad$ $-g^2 \log M_{ii}^2$

Note that, in the Sumino model, not only the family gauge boson mass spectrum is related to the charged lepton mass spectrum, but also the coupling constant of the family gauge bosons is strictly fixed by the EW gauge coupling constant.

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1.2.2 Why "inverted" in K–Y model?

Problems of the Sumino model:

(i) Sumino model is not anomaly free model

(ii) Effective current-current interactions with $\Delta N_f = 2$ appear

(iii) The vertex type diagram does not work in a SUSY model.

Sumino model	**Our model**
Y. Sumino, PLB 671, 477 (2009)	YK and T.Yamashita PLB 711, 381 (2012)

The origin of minus sign in order to cancel the QED correction

Sumino model	Our model
$(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$	$(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3})$
$M_{ij}^2 \equiv m^2(A_i^j) = k(m_{ei} + m_{ej})$	$M_{ij}^2 \equiv m^2(A_i^j) = k\left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}}\right)$
$+\log M_{ii}^2 = -\frac{1}{2}\log m_{ei}^2 + \log k$	$\log M_{ii}^2 = -\frac{1}{2}\log m_{ei}^2 + \log k$
Normal mass hierarchy	**Inverted mass hierarchy**

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1.2.3 Masses of the gauge bosons

We consider scalars $(3, 3^*)$ of $U(3)\times U(3)'$ with $\Lambda \ll \Lambda'$

There are no (6,1) and/or (3, 1) in this model.

Family symmetry is broken by VEV of $\langle \Phi_i{}^\alpha \rangle = \delta_i{}^\alpha v_i$

so that the gauge boson masses are given by

$$\mathcal{H}_{mass} = \frac{1}{2}\mathrm{Tr}\left[g_A^2 \Phi\Phi^\dagger AA + g_B^2 \Phi^\dagger \Phi BB - 2 g_A g_B \Phi^\dagger A \Phi B \right].$$

In the limit of massive B, the masses of A are given by

$$M^2(A_i{}^j) = \frac{1}{2} g_A^2 (|v_i|^2 + |v_j|^2)$$ +(scalars for q & l)

dominant negligibly small

Therefore, the gauge boson masses satisfy

$$2M_{ij}^2 = M_{ii}^2 + M_{jj}^2$$

$$\mathcal{H}_{fam} = \frac{g_F}{\sqrt{2}} \left[(\bar{e}_i \gamma_\mu e_j) + (\bar{\nu}_i \gamma_\mu \nu_j) + U_{ik}^{*d} U_{jl}^{d} (\bar{d}_k \gamma_\mu d_l) + U_{ik}^{*u} U_{jl}^{u} (\bar{u}_k \gamma_\mu u_l) \right] (A_i{}^j)^\mu$$

YK an d T.Yamashita, PLB 711, 384 (2012)

2. If FGB have masses of the order of $1-10^2$ TeV, …

For the time being,
let us forget the theoretical basis, and
let us see what happen when FGB have masses of the order of $1\text{-}10^2$ TeV only from the phenomenological point of view.

2.1 Rare decays of ps-mesons

In order to consider a scale of family gauge boson masses, apart from the observed constraint from the K^0-$\bar{K}^0$ mixing, $*$ $\tilde{M}_{ij} \equiv \frac{M_{ij}}{g_F/\sqrt{2}}$ let us see the present status of rare K and B decay searches:

	Input	Output [TeV]	
$Br(K^+ \to \pi^+ e^- \mu^+)$	$< 1.3 \times 10^{-11}$	$\tilde{M}_{12}$	> 196
$Br(K_L \to \pi^0 e^\mp \mu^\pm)$	$< 7.6 \times 10^{-11}$	$\tilde{M}_{12}$	> 151
$Br(K_L \to \pi^0 \nu\bar{\nu})$	$< 2.6 \times 10^{-8}$	$\tilde{M}_{12}$	> 17.5
$Br(K^+ \to \pi^+ \nu\bar{\nu})$	$= (1.7 \pm 1.1) \times 10^{-10}$	$\tilde{M}_{12}$	$\sim 250^*$
$Br(B^+ \to K^+ \mu^\pm \tau^\mp)$	$< 7.7 \times 10^{-5}$	$\tilde{M}_{23}$	> 4.11
$Br(B^+ \to K^+ \nu\bar{\nu}))$	$< 1.3 \times 10^{-5}$	$\tilde{M}_{23}$	> 5.4
$Br(B^0 \to K^0 \nu\bar{\nu})$	$< 5.6 \times 10^{-6}$	$\tilde{M}_{23}$	> 6.7
$Br(B^0 \to \pi^0 \nu\bar{\nu})$	$< 2.2 \times 10^{-4}$	$\tilde{M}_{31}$	> 4.8

$*$ We have used $Br(K^+ \to \pi^+ \nu\bar{\nu})_{SM} = (0.80 \pm 0.11) \times 10^{-10}$

Ishidori, et al. (05); Buras, et al. (08)

Roughly speaking, we can see that

$\tilde{M}_{12} \geq 2.5 \times 10^2$ TeV $\quad$ $\tilde{M}_{23} \geq 7$ TeV

This is in favor of the inverted mass picture of Aj^i

$*$ Note that $\tilde{M}_{ij}$ is a direct quantity rather than M_{ij} for four fermion interactions.

- If we take $\tilde{M}_{12} \sim 250$ TeV (from $K^+ \to \pi^+ \nu \bar{\nu}$) ,
 we can observe $\mu^- N \to e^- N$
at the COMET experiment

- If $\tilde{M}_{33} \sim 5$ TeV , $\tilde{M}_{23} \sim 7$ TeV (from a lower limit of rare B decays)

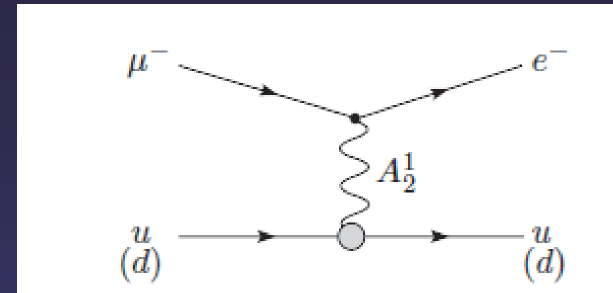
we will observe direct productions of these family gauge bosons at 14 TeV LHC

$$pp \to A_3^3 + \bar{b}b + X \to \tau^- \tau^+ + X$$

$$pp \to A_2^3 + \bar{s}b + X \to \mu^- \tau^+ + X$$

In the following slides, we will discuss more details of these phenomena.

2.2 μ-e conversion



Past experiment: SINDRUM (2006)

$$R(A_u) \equiv \frac{\sigma(\mu^- + Au \to e^- + Au)}{\sigma(\mu \text{ capture})} < 7 \times 10^{-13}$$

We tentatively give a rough estimate

$$R_q \equiv \frac{\sigma(\mu^- + q \to e^- + q)}{\sigma(\mu^- + u \to \nu_\mu + d)} \simeq \left(\frac{|U_{11}^{q*} U_{21}^{q}|}{|V_{ud}|} \frac{g_F^2/2}{M_{12}^2} \frac{M_W^2}{g_w^2/8} \right)^2$$

$R_d \sim 3.8 \times 10^{-14}$ $\quad$ $(\tilde{M}_{12} \simeq 250 \text{ TeV})$

Near future: COMET (Coherent Muon to Electron Transition)

$$R \sim 3 \times 10^{-15} \ (2017)$$

Note that

our gauge boson model allows μ-e conversion,
while the model highly suppresses $\mu \to e + \gamma$

COMET Experiment

(Coherent Muon to Electron Transition)

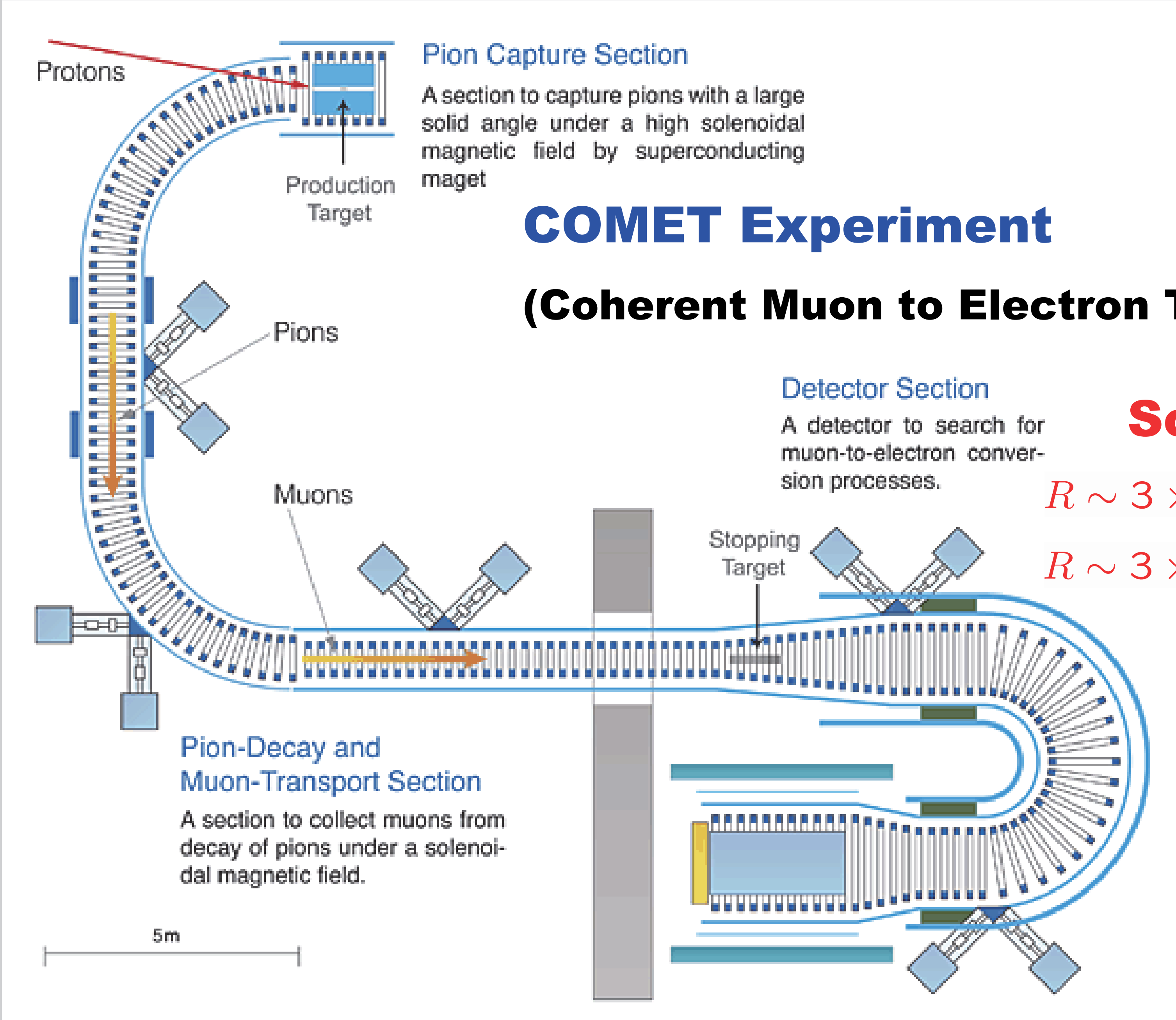


図1 COMET全体図

Schedule

$R \sim 3 \times 10^{-15}$ (2017)

$R \sim 3 \times 10^{-17}$ (2021)

quoted from http://www-kuno.phys.sci.osaka-u.ac.jp/research/index.html

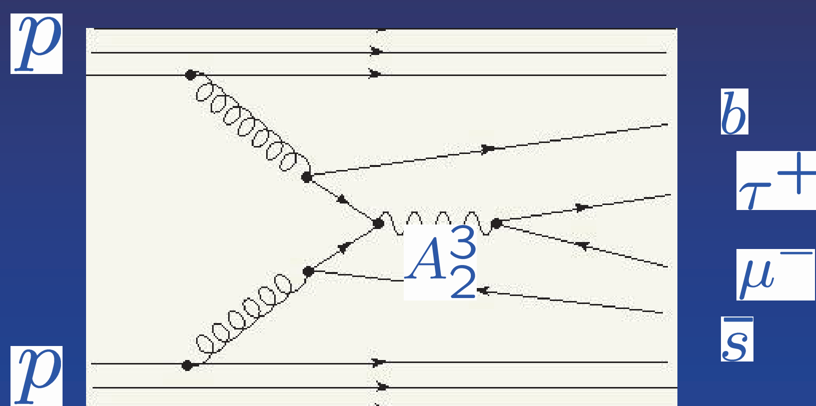
2.3 Direct production of A_3^3 and A_2^3

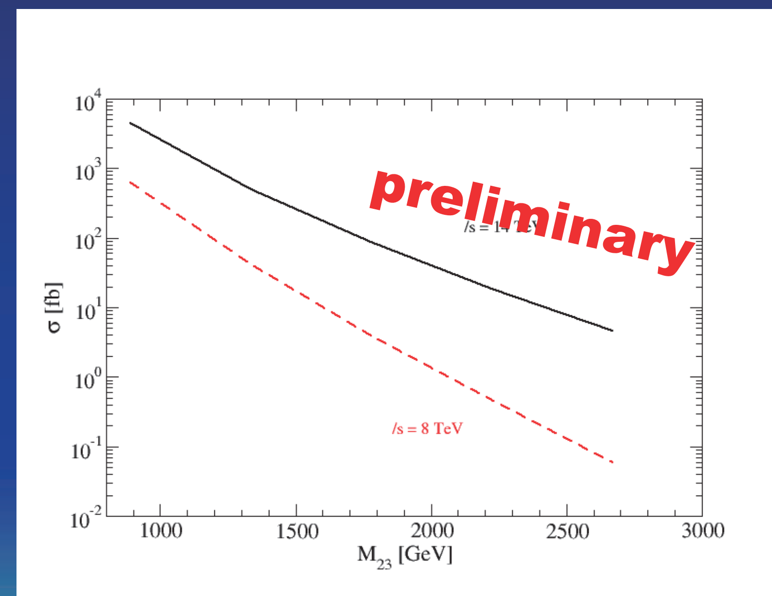
$$pp \to A_3^3 + \bar{b}b + X \to \tau^-\tau^+ + X$$

$$pp \to A_2^3 + \bar{s}b + X \to \mu^-\tau^+ + X$$

$\sigma(pp \to A_2^3) \sim 40$ fb

at $M_{23} = 2$ TeV





$g_F/\sqrt{2} = 0.5322$

illustrated by H.Yokoya

Branching ratios

$$Br(A_i^j \to \ell_i\bar{\ell}_j) = 2/15 = 13.3\%$$

$$Br(A_i^j \to \nu_i\bar{\nu}_j) = 1/15 = 6.7\%$$

If ν are Dirac type,

$$Br(A_i^j \to \nu_i\bar{\nu}_j) = 2/16 = 12.5\%$$

We can determine whether ν is Majorana or Dirac.

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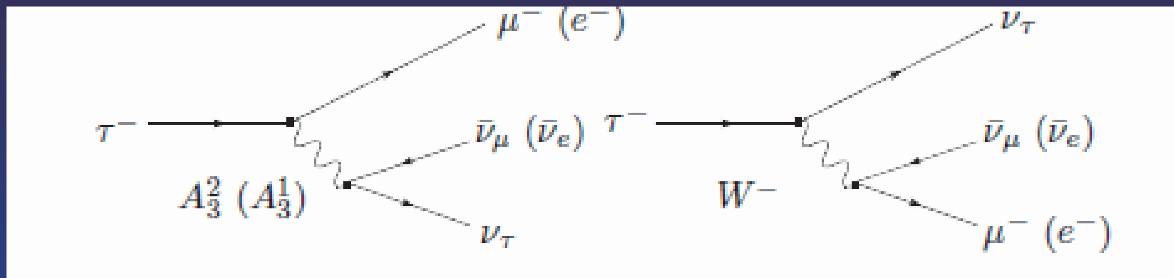
2.4 Deviations from the e−μ−τ universality

(i) Tau decays

YK, PRD 87, 016016 (2013)



From the branching ratios (PDG2012)

$$Br(\tau^- \to \mu^- \bar{\nu}_\tau \nu_\tau) = (17.41 \pm 0.04)\%$$

$$Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = (17.83 \pm 0.04)\%$$

we obtain

$$R_{amp} \equiv \frac{1+\varepsilon_\mu}{1+\varepsilon_e} = \sqrt{\frac{Br(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)\, f(m_e/m_\tau)}{Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau)\, f(m_\mu/m_\tau)}} = 1.0020 \pm 0.0016.$$

where $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 12x^4 \log x^2$

Therefore, we obtain $\varepsilon \equiv \varepsilon_\mu - \varepsilon_e = 0.0020 \pm 0.0016$

$$\tilde{M}_{23} = (5.2^{+6.4}_{-1.4})\ \text{TeV}$$

Because of a large error, the result should not be taken rigidly.

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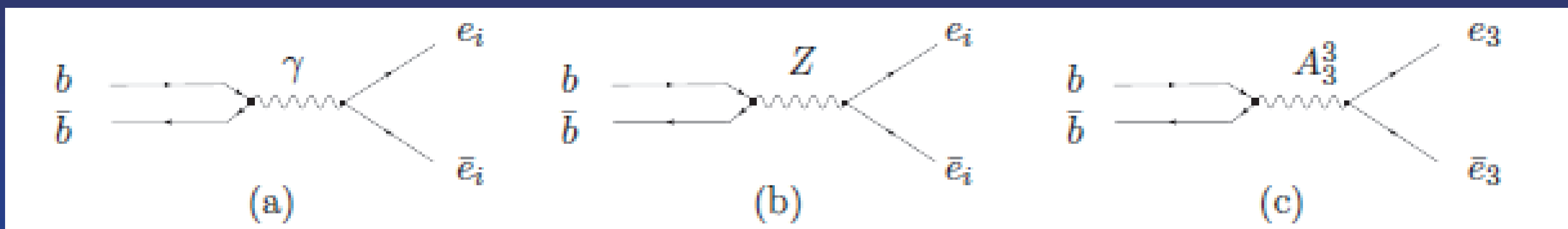
(ii) Upsilon decays

YK, PRD 87, 016016 (2013)

We consider that the deviation from the e–μ–τ universality in the Υ decays is caused by the family gauge boson exchange terms



Under an approximation of neglecting family mixing in the quark sector, the b quark interacts only with A33 boson, so that the deviation parameter is given by

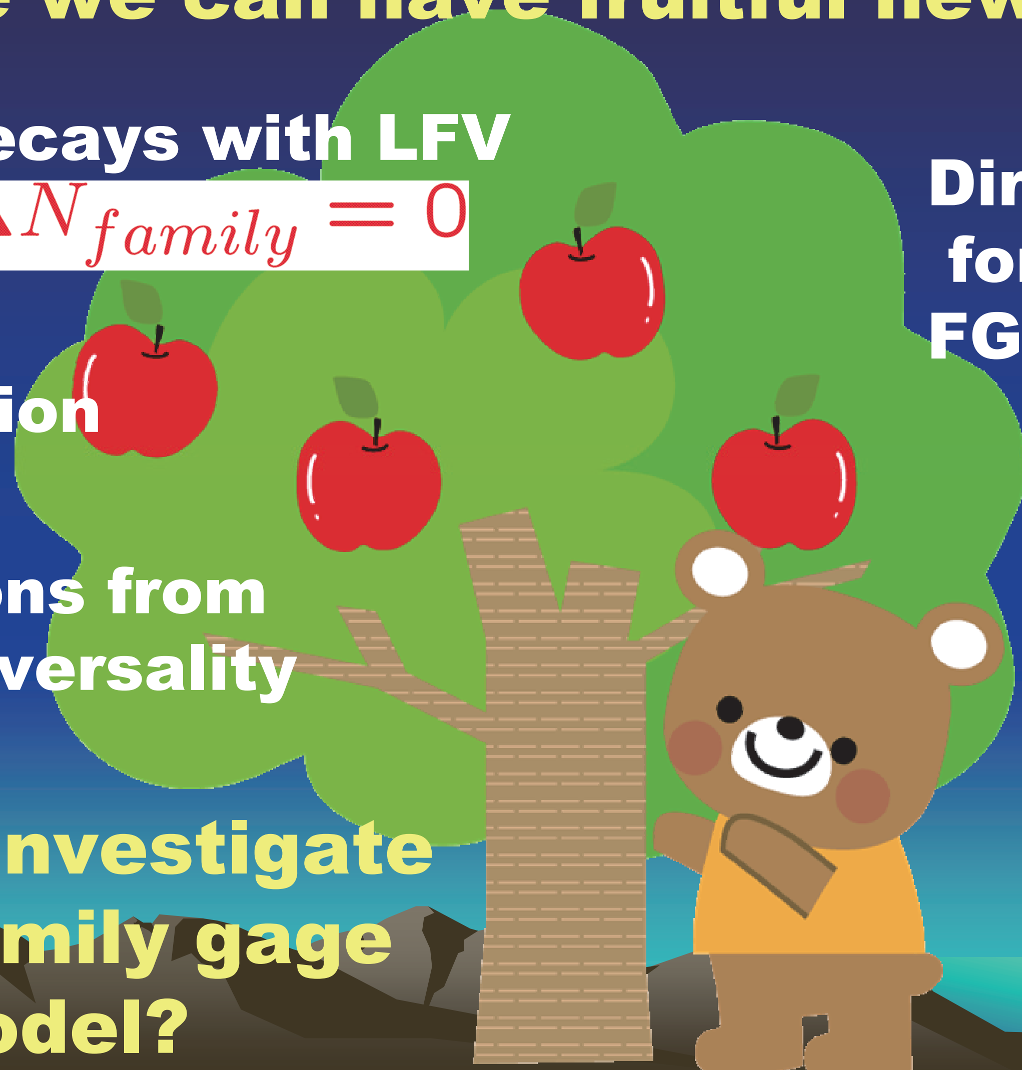
$$\varepsilon_\tau = \frac{g_F^2}{e^2/3} \frac{M_\Upsilon^2}{M_{33}^2}$$

$$\tilde{M}_{33} = (0.22^{+0.26}_{-0.05})\ \mathrm{TeV}$$

Because of a large error, the result should not be taken rigidly.

Summary of Sec.2:
We are very happy

because we can have fruitful new physics



Rare decays with LFV
but $\Delta N_{family} = 0$

Direct search
for the light
FGB at LHC

μ-e conversion

Deviations from
e-μ-τ universality

Why not investigate
such a family gage
boson model?

3 The biggest obstacle against such an optimistic view

The biggest obstacle is still a constraint from the observed K^0-$\bar{K}^0$ mixing, although the K-Y model considerably released the constraints compared with the conventional FGB models

3.1 K^0-$\bar{K}^0$ mixing

The biggest obstacle comes from the estimate of K^0-$\bar{K}^0$ mixing.

Effective current–current interactions with $\Delta N_{fam} = 2$ are given by

$$H^{eff} = \frac{1}{2} g_F^2 \left[\sum_i \frac{(\lambda_i)^2}{M_{ii}^2} + 2 \sum_{i<j} \frac{\lambda_i \lambda_j}{M_{ij}^2} \right] (\bar{q}_k \gamma_\mu q_l)(\bar{q}_k \gamma^\mu q_l)$$

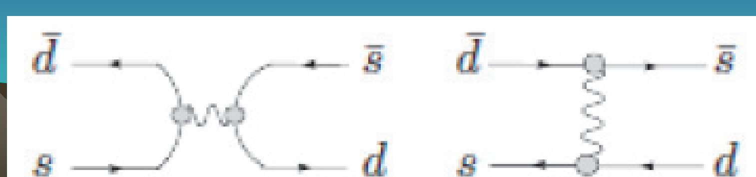
where $\lambda_1 = U_{1k}^* U_{1l}, \quad \lambda_2 = U_{2k}^* U_{2l}, \quad \lambda_3 = U_{3k}^* U_{3l}$

For example, for K^0-$\bar{K}^0$ mixing, λ_i are given by

$$\lambda_1 = U_{11}^{d*} U_{12}^d, \quad \lambda_2 = U_{21}^{d*} U_{22}^d, \quad \lambda_3 = U_{31}^{d*} U_{32}^d$$

Note that λ_i satisfy the unitary triangle relation

$$\lambda_1 + \lambda_2 + \lambda_3 = 0$$



Estimate of $\tilde{M}_{22}$ from Δm_K

- For convenience, we assume $U_d \simeq V_{CKM}$

Then we obtain $|\lambda_1| = |U_{11}^* U_{12}| = 0.2195$

$$|\lambda_2| = |U_{21}^* U_{22}| = 0.2192$$

$$|\lambda_3| = |U_{31}^* U_{32}| = 0.00035 ,$$

so that we can regard those as $\lambda_3 \simeq 0, \quad \lambda_1 \simeq -\lambda_2$.

Therefore, we obtain

$$G^{eff} = \frac{1}{2} g_F^2 \left[\frac{\lambda_1^2}{M_{11}^2} + \frac{\lambda_2^2}{M_{22}^2} + \frac{\lambda_3^2}{M_{33}^2} + 2 \left(\frac{\lambda_1 \lambda_2}{M_{12}^2} + \frac{\lambda_2 \lambda_3}{M_{23}^2} + \frac{\lambda_3 \lambda_1}{M_{31}^2} \right) \right] \simeq \frac{1}{2} g_F^2 \frac{\lambda_2^2}{M_{22}^2}$$

- Under the vacuum-insertion approximation, we obtain

$$\Delta m_K = \frac{1}{6} G^{eff} f_K^2 m_K (1 + 2 S_K) \qquad S_K = \frac{m_K^2}{(m_s + m_d)^2}$$

$$\Delta m_K^{obs} = (3.484 \pm 0.006) \times 10^{-18} \ \text{TeV}$$

$$\Delta m_K^{SM} \sim 2 \times 10^{-18} \ \text{TeV}$$

$$\tilde{M}_{22} \sim 340 \ \text{TeV}$$

3.2 Can we build a model with visible gauge bosons?

In the inverted mass hierarchy model (K-Y model), the mass spectrum is given by

$$M_{33} : M_{23} : M_{22} : M_{31} : M_{12} : M_{11}$$

$$= 1 : \sqrt{\frac{1+a^2}{2}} : a : \sqrt{\frac{1+b^2}{2}} : \sqrt{\frac{a^2+b^2}{2}} : b$$

$$\simeq 1 : \frac{a}{\sqrt{2}} : a : \frac{b}{\sqrt{2}} : \frac{b}{\sqrt{2}} : b$$

$$(1 \ll a^2 \ll b^2)$$

where

$$a \equiv \frac{M_{22}}{M_{33}} = \left(\frac{m_\tau}{m_\mu}\right)^{n/2}, \quad b \equiv \frac{M_{11}}{M_{33}} = \left(\frac{m_\tau}{m_e}\right)^{n/2}$$

(n=1 in the K-Y model)

When we assign $\tilde{M}_{22} \sim 340$ TeV

we obtain

n	$\tilde{M}_{33}$	$\tilde{M}_{12}$
1	1.65	4051
1/2	23.6	986

We cannot obtain visible values of $\tilde{M}_{33}$ and $\tilde{M}_{12}$

4. An example of FGB model

As far as we adhere to the Sumino's cancellation condition, we find that it is impossible to build a visible family gauge boson model.

Nevertheless, we never say "give up"!
We still believe "FGB exist!"

(The cancellation condition fixes not only the mass spectrum, but also the gauge coupling constant.
Too tight condition!)

4.1 Abondonment of Suminoism

Are you a believer of the charged lepton mass formula?

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$

If you are not so,
you are free from Sumino's cancellation condition,
and you can regard the parameters a and b as free parameters.

Then, you can obtain a visible FGB model.
However, regrettably, at present, we have no reliable input values of family gauge boson masses.

4.2 Again the K−Y model

- **We keep the mass relation**

$$M_{33} : M_{23} : M_{22} : M_{31} : M_{12} : M_{11}$$
$$= 1 : \sqrt{\frac{1+a^2}{2}} : a : \sqrt{\frac{1+b^2}{2}} : \sqrt{\frac{a^2+b^2}{2}} : b$$

but, let us regard a and b as free parameters.

- **Next, we suppose $b \simeq a$ on trial.**
- **Then, note that in the limit of b=a, Δm_K**

(and also Δm_D) becomes negligibly small because of $\lambda_3 \simeq 0, \quad \lambda_1 \simeq -\lambda_2$

Therefore, we can build a FGB model without using the input value $\tilde{M}_{22} \sim 340$ TeV

- In this case, we obtain

$$M_{33} : M_{23} : M_{22} : M_{31} : M_{12} : M_{11}$$
$$= 1 : \frac{a}{\sqrt{2}} : a : \frac{a}{\sqrt{2}} : a : a$$

- We still consider that gauge boson mass ratios are deeply related to the charged lepton mass ratios, for example,

$$M_{33} : M_{22} : M_{11} = \sqrt{m_e} : \sqrt{m_\mu} : \sqrt{m_\tau}$$

- Since we assumed $b \simeq a$, we speculate

$$a = \frac{M_{11}}{M_{22}} \simeq \frac{M_{11}}{M_{33}} \simeq \sqrt{\frac{M_{11}}{M_{22}}\frac{M_{11}}{M_{33}}} = \sqrt{\sqrt{\frac{m_\mu}{m_e}}\sqrt{\frac{m_\tau}{m_e}}} = 29.1$$

- If we regard $\tilde{M}_{12}$ as $\tilde{M}_{12} = 250$ TeV from $K^+ \to \pi^+ \nu \bar{\nu}$ we obtain (This is a temporal value. Do not take it rigidly.)

$\tilde{M}_{33} \sim 8.6$ TeV — A33: visible at LHC

$\tilde{M}_{23} \simeq \tilde{M}_{31} \sim 177$ TeV

$\tilde{M}_{22} \simeq \tilde{M}_{12} \simeq \tilde{M}_{11} \sim 250$ TeV — A12: visible at COMET

5. Concluding remarks

- We have discussed a possibility that family gauge bosons are visible by terrestrial experiments.
- The observed value of K^0-$\bar{K}^0$ is still the biggest obstacle against such a model.
- The Sumino model has suggested that we have a possibility to overcome such an obstacle.
- However, the original Sumino model is too tight, so that some improvement will be required.
- We have demonstrate a model with visible family gauge bosons. However, the purpose of the present work is not to give numerical predictions of FGB masses, but to demonstrate that we have still posses a possibility to overcome the K^0-$\bar{K}^0$ problem.

I still believe that family gauge bosons really exist, and they are visible by terrestrial experiments.

We must search for a more natural and reasonable model.

Thank you for your kind attention