The 1st workshop on Intensity Frontier of Muon Fundamental Science
KEK, June 10-11, 2010

New U(3) Family Gauge Symmetry and Muonium into Antimuonium Conversion

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1. Why do we need a U(3) family gauge symmetry?

Motivation

- **Investigations of mass spectra have always provided promising clues for solving problems in physics:**

  Balmer formula ⟹ Bohr theory

  Gell-Mann-Okubo mass formula ⟹ quark model

Therefore, we may expect that investigation of the quark and lepton mass spectra and mixings will also provide a promising clue to new fundamental physics.

Starting point of a story

- **Related to this topic, we know an empirical mass formula** [Y.K. LNC 34, 201 (1982); PLB 120, 161 (1983)]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$

which is remarkably satisfied with an accuracy of 10^{-5} for the observed charged lepton masses (pole masses):

$$K^{pole} = \frac{2}{3} \times (0.999989 \pm 0.000014)$$

Mystery of the charged lepton mass relation

- However, in conventional mass matrix models,

"mass" means not “pole mass”, but “running mass”.

The formula is only valid with an accuracy of 10^{-3} for the running masses.

$$K^{pole} = \frac{2}{3} \times (0.999989 \pm 0.000014)$$

$$K^{run}_{\mu=M_Z} = \frac{2}{3} \times (1.00189 \pm 0.00002)$$

- This has been a mysterious problem for long years.

Sumino mechanism

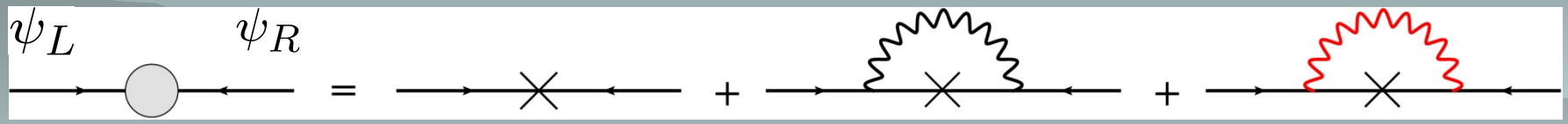
Recently, a possible solution of this problem has been proposed by Sumino [Y.S., PLB 671, 477 (2009); JHEP 0905, 075 (2009)].

The deviation of $K^{run}(\mu)$ from K^{pole} is caused by a term $m_i \log(\mu/m_i)$ in the running mass terms.

He considers that a flavor symmetry is gauged, and in the running mass terms of the charged lepton e_i, the term $m_i \log(\mu/m_i)$ from the radiative correction due to photon is exactly canceled by those by flavor gauge bosons.

(This does not always mean $m_{ei}(\mu) = m_{ei}^{pole}$.)

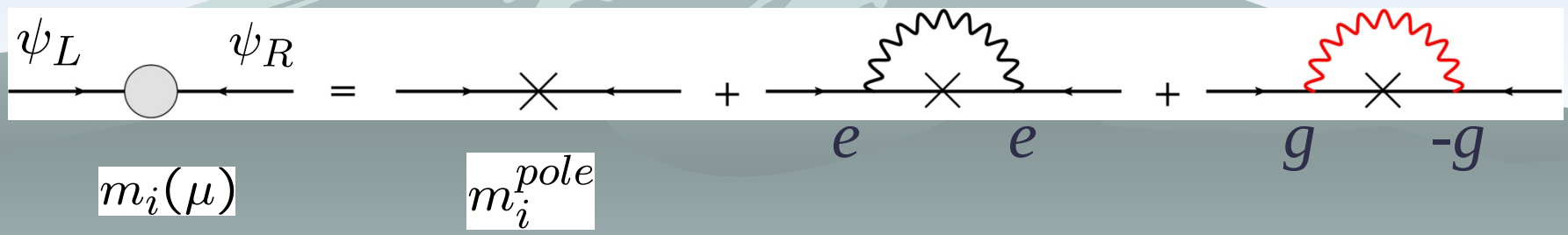
photon new family gauge bosons



$m_i(\mu)$ m_i^{pole}

What is essential for the cancellation?

In order that the Sumino mechanism (cancellation mechanism) works correctly, a left-handed filed ψ_L and its right-handed partner ψ_R must be assigned to 3 and 3* of U(3), respectively.



$$(\psi_L, \psi_R) = (3, 3^*)$$

New assignment is quite natural!

Even if we are apart from the Sumino mechanism, the assignment $(\psi_L, \psi_R) = (3, 3^*)$ seems to be quite natural from a point of view of a grand unification (GUT) scenario, too.

If we adopt the conventional assignment $(\psi_L, \psi_R) = (3, 3)$ for an SU(5) GUT model, we must consider $(5^*_L, 3) + (10_L, 3^*)$ of $\mathrm{SU}(5)\times\mathrm{U}(3)_{fam}$, while we can consider $(5^*_L + 10_L, 3)$ in the new assignment.

(Also, we can consider $(16_L, 3)$ in an SO(10) GUT model under the new assignment.)

Then, what happens?

Conventional U(3)	New U(3)
$(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3})$	$(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$
$(J_\mu)_i{}^j = \bar{\psi}_L^j \gamma_\mu \psi_{Li} + \bar{\psi}_R^j \gamma_\mu \psi_{Ri}$	$(J_\mu)_i{}^j = \bar{\psi}_L^j \gamma_\mu \psi_{Li} - \bar{\psi}_{Ri} \gamma_\mu \psi_R^j$
$(J_\rho)_1^2 = \bar{\mu}_L \gamma_\rho e_L + \bar{\mu}_R \gamma_\rho e_R$	$(J_\rho)_1^2 = \bar{\mu}_L \gamma_\rho e_L - \bar{e}_R \gamma_\rho \mu_R$

We can expect a muon number violation process

$e^- + e^- \to \mu^- + \mu^-$ **from the effective interaction**

$$\mathcal{L}^{eff} = \frac{G_{M\bar{M}}}{\sqrt{2}} [\bar{\mu}\gamma_\rho(1-\gamma_5)e]\,[\bar{\mu}\gamma^\rho(1+\gamma_5)e] + h.c.$$

Also we may expect flavor violation processes with $|\Delta N_i| = 2$

(N_i **is an individual family number) such as**

$u + e^- \to c + \mu^-$, $u + u \to t + t$, **and so on.**

$$\frac{G_{M\bar{M}}}{\sqrt{2}} = \frac{g_f^2}{4M^2(A_1^2)}$$

In the conventional models [*e.g.* the bilepton model (Frampton(1992))], g_f and M are free parameters.

In contrast to the conventional models, the present model has the following constraints on the parameters:

(1) The coupling constant g_f is related to the electric coupling constant e as

$$\frac{1}{4}g_f^2 = e^2 = g_W^2 \sin^2\theta_W$$

in order to work the Sumino mechanism correctly.

(2) The gauge boson masses $M(A_i^j)$ are related to the charged lepton masses m_{ei} as $M^2(A_i^j) = M_0(m_{ei} + m_{ej})$ **.**

In the Sumino model, since the energy scale of the effective theory is assumed as $\Lambda \sim 10^3$ TeV, we suppose $M(A_1^3) \sim 10^{2-3}$ TeV and $M(A_1^2) \sim 10^{1-2}$ TeV **.**

It is convenient to describe our predictions in terms of a factor $G_{M\bar{M}}/G_F$. In the present model, the factor $G_{M\bar{M}}/G_F$ is given as

$$\frac{G_{M\bar{M}}}{G_F} = 4\sin^2\theta_W \left(\frac{M_W}{M(A_1^2)}\right)^2 = \frac{5.98}{(M(A_1^2)[\mathrm{TeV}])^2}$$

Thus, our predictions can be given only in terms of the gauge boson masses $M(A_i^j)$.

Besides, we conjecture that $M(A_1^2) \sim 10^{1-2}\ \mathrm{TeV}$

2. Constraints from Kaon rare decays

Current form in down-quark sector

In general, the family gauge current forms in the quark sectors depend on quark flavor mixing matrices which are given in the diagonal basis of the charged lepton mass matrix M_e.

For example, when , for simplicity, we consider only *d-s* mixing

$$d^0 = d\cos\theta - s\sin\theta,\ \ s^0 = d\sin\theta + s\cos\theta$$

we can obtain a current form for the down-quark sector

$$(J_\mu^{(d)})_1^2 = \bar{s}_L^0\gamma_\mu d_L^0 - \bar{d}_R^0\gamma_\mu s_R^0$$

$$= \frac{1}{2}(\bar{s}\gamma_\mu d - \bar{d}\gamma_\mu s) - \frac{1}{2}(\bar{s}\gamma_\mu\gamma_5 d + \bar{d}\gamma_\mu\gamma_5 s)\cos 2\theta + \frac{1}{2}(\bar{s}\gamma_\mu\gamma_5 s - \bar{d}\gamma_\mu\gamma_5 d)\sin 2\theta$$

CP= - 1 **CP= +1** **CP= +1**

How contributes to Kaon decays?

$$(J_\mu^{(d)})_1^2 = \bar{s}_L^0 \gamma_\mu d_L^0 - \bar{d}_R^0 \gamma_\mu s_R^0$$

$$= \frac{1}{2}(\bar{s}\gamma_\mu d - \bar{d}\gamma_\mu s) - \frac{1}{2}(\bar{s}\gamma_\mu\gamma_5 d + \bar{d}\gamma_\mu\gamma_5 s)\cos 2\theta + \frac{1}{2}(\bar{s}\gamma_\mu\gamma_5 s - \bar{d}\gamma_\mu\gamma_5 d)\sin 2\theta$$

CP= - 1 **CP= +1** **CP= +1**

$K_S \to e^\pm + \mu^\mp$

$K^+ \to \pi^+ + e^\pm + \mu^\mp$

(Note that this term cannot contribute to $K_L \to e^\pm + \mu^\mp$ **in the limit of neglecting CP violation)**

Constraints on the masses

We can predict

$$R \equiv \frac{B(K^+ \to \pi^+ e^- \mu^+)}{B(K^+ \to \pi^0 \mu^+ \nu_\mu)} = \frac{(2g_f^2/M_{f12}^2)^2}{2|V_{us}|^2 (g_W^2/M_W^2/\sqrt{2})^2}$$

$$= 67.27 (M_W/M_f)^4$$

ΔN=0 mode	ΔN=2 mode
$K^+ \to \pi^+ e^- \mu^+$	$K^+ \to \pi^+ \mu^- e^+$
$B < 1.3 \times 10^{-11}$	$B < 5.2 \times 10^{-10}$
$M(A_1^2) > 51.9$ TeV	$M(A_1^2) > 20.6$ TeV
$M(A_1^3) > 213$ TeV	$M(A_1^2) > 84.6$ TeV

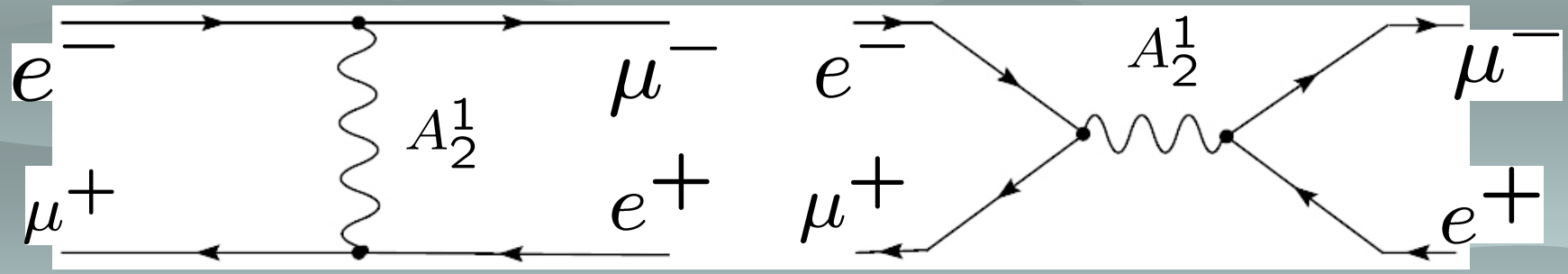
These results suggest that experimental observations of the flavor number violation effects soon become in our reach.

3. Muonium into antimuonium conversion

Mt.Fuji in Winter

Why important to investigate muonium-antimuonium conversion

Exactly speaking, the constraints from kaon rare decays may depend on the down-quark mixing structure and some another hadronic effects, although we suppose that such effects are small. We would like to test for the new gauge boson effects in the pure leptonic processes. So, the investigation of $M\bar{M}$ conversion is indispensable.



$$\mathcal{L}^{eff} = \frac{G_{M\bar{M}}}{\sqrt{2}} [\bar{\mu}\gamma_\rho(1-\gamma_5)e] [\bar{\mu}\gamma^\rho(1+\gamma_5)e] + h.c.$$

Conversion probability

The total conversion probability is given by

$$P_{M\bar{M}}(B) = \frac{\delta^2}{2[\delta^2 + (E_M - E_{\bar{M}})^2 + \lambda^2]}$$

where the mass matrix for $(M, \bar{M})$ **is given by**

$$M_{mass} = \begin{pmatrix} E_M & \frac{1}{2}\delta \\ \frac{1}{2}\delta & E_{\bar{M}} \end{pmatrix}$$

and

λ **: muon decay width**

$$\delta \propto \frac{G_{M\bar{M}}}{\sqrt{2}} \frac{1}{\pi a^3}$$

a: **the Bohr radius**

The effective Hamiltoinian in the present model is given as

$$\mathcal{L}^{eff} = \frac{G_{M\bar{M}}}{\sqrt{2}} [\bar{\mu}\gamma_\rho(1-\gamma_5)e] [\bar{\mu}\gamma^\rho(1+\gamma_5)e] + h.c.$$

The case (V-A)(V+A) has already calculated by Horikawa and Sasaki [PRD 53, 560 (1996)]:

$$(V-A)(V+A)$$

$$P_{M\bar{M}}(0) \simeq \frac{3}{2}\frac{\delta^2}{\lambda^2}$$

$$\delta = -8\frac{G_{M\bar{M}}}{\sqrt{2}}\frac{1}{\pi a^3}$$

$$\Rightarrow \; P_{M\bar{M}}(0) = 1.96 \times 10^{-5} \left(\frac{G_{M\bar{M}}}{G_F}\right)^2$$

c.f. $(V-A)(V-A)$

$$P_{M\bar{M}}(0) \simeq \frac{1}{2}\frac{\delta^2}{\lambda^2}$$

$$\delta = 16\frac{G_{M\bar{M}}}{\sqrt{2}}\frac{1}{\pi a^3}$$

$$\Rightarrow \; P_{M\bar{M}}(0) = 2.56 \times 10^{-5} \left(\frac{G_{M\bar{M}}}{G_F}\right)^2$$

We present an external magnetic field dependence by

$$P_{M\bar{M}}(B) \equiv P_{M\bar{M}}(0) S_B(B)$$

For example,

$(V-A)(V+A)$	c.f. $(V-A)(V-A)$
$S_B(0.1\mathrm{T}) = 0.78$	$S_B(0.1\mathrm{T}) = 0.35$
$S_B(100\mathrm{T}) = 0.67$	$S_B(100\mathrm{T}) = 0.0$

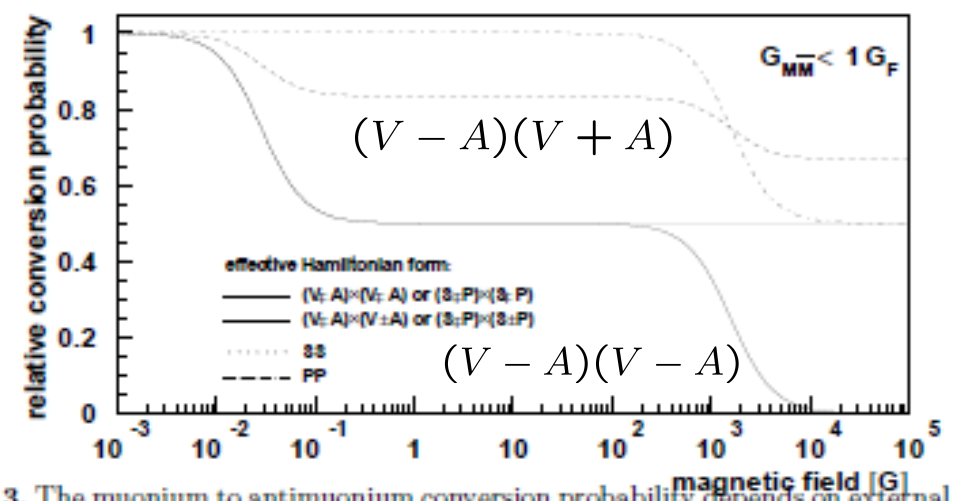


Fig. 3. The muonium to antimuonium conversion probability depends on external magnetic fields and the coupling type. Recent independent calculations were performed by Wong and Hou [16] and Horikawa and Sasaki [29].

quoted from L.Willmann and K.Jungmann, Lect.Notes Phys. 499,43 (1997)

Present experimental limit on the gauge boson mass

Experiment at PSI: L.Willmann, *et al.* PRL 82, 49 (1999)

$$P_{M\bar{M}}(0.1\mathrm{T}) \leq 8.3 \times 10^{-11}$$

This result gives

$$P_{M\bar{M}}(0) \leq 1.06 \times 10^{-10} \qquad [2.3 \times 10^{-10}]$$

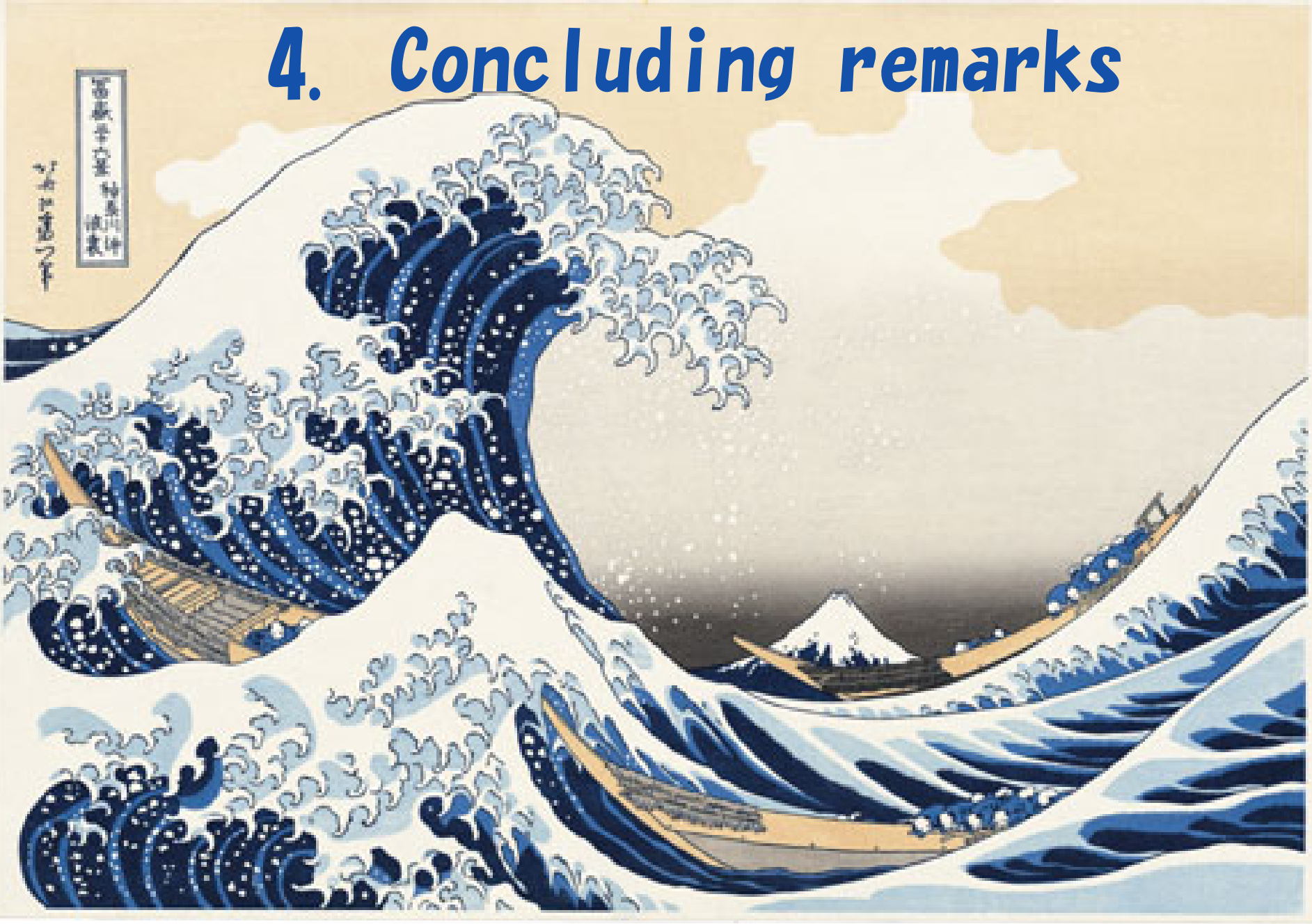
$$G_{M\bar{M}}/G_F \leq 2.3 \times 10^{-3} \qquad [3.0 \times 10^{-3}]$$

for *(V-A)(V+A)* [c.f. *(V-A)(V-A)*].

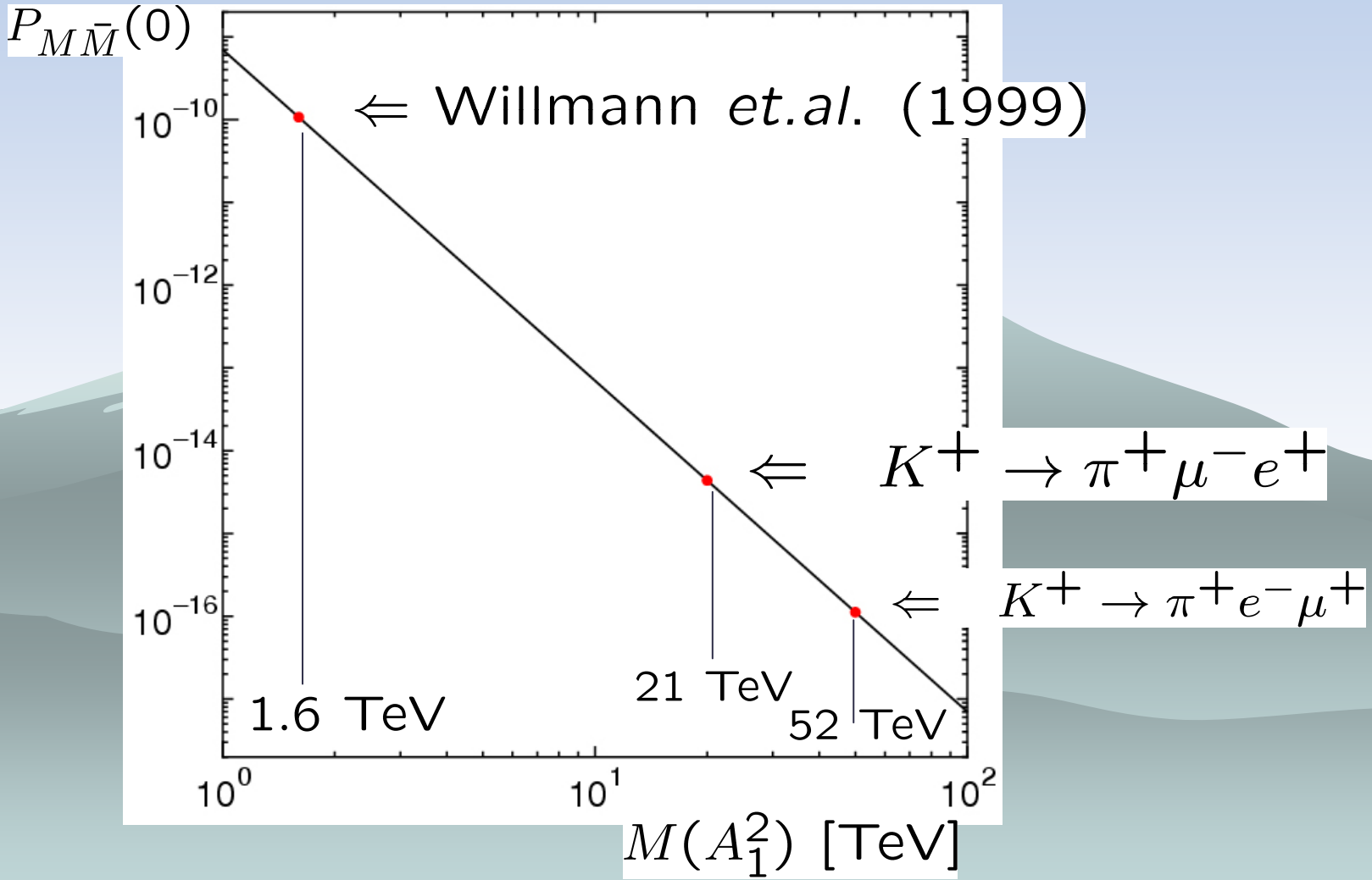
Therefore, **we obtain a lower limit of our gauge boson mass**

$$M(A_1^2) \geq 20 M_W = 1.6 \ \mathrm{TeV}$$

4. Concluding remarks



Prediction of conversion probability and limits from experimental data



why we need $M\bar{M}$ conversion experiment?

Although the present lower limit of $M(A_1^2) \geq 1.6$ TeV from $M\bar{M}$ conversion is considerably looser than those from rare kaon decays: $M(A_1^2) \geq 21$ TeV from $K^+ \to \pi^+\mu^-e^+$

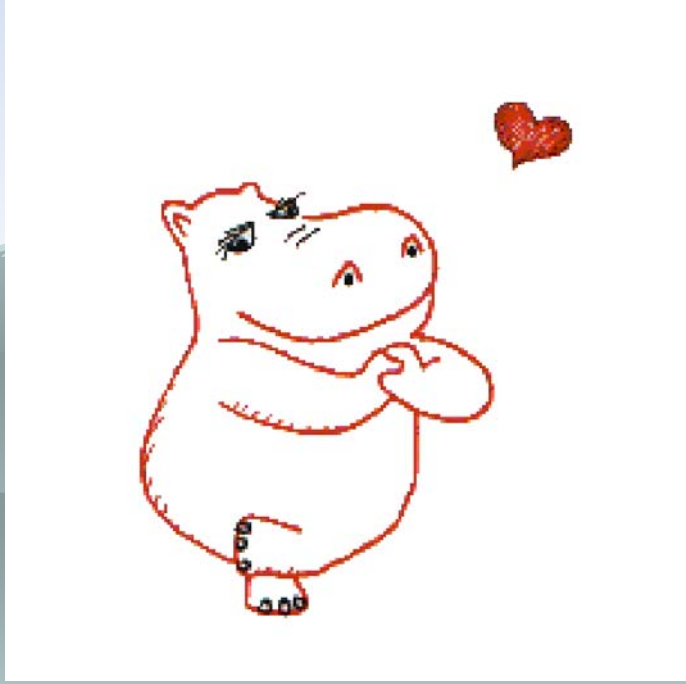
$M(A_1^2) \geq 52$ TeV from $K^+ \to \pi^+e^-\mu^+$

However, I would like to emphasize that the constraints from kaon rare decays may depend on the down-quark mixing structure and some another hadronic effects, so that the test for the new flavor gauge symmetry in $M\bar{M}$ conversion is still indispensable.

Besides, if we find a positive evidence in the $M\bar{M}$ conversion, and if we find that the mass $M(A_1^2)$ is lower than the lower limit of $M(A_1^2)$ from the kaon decays, we can obtain an important clue to the down-quark mixing U_d (not V_{CKM}!) .

Conclusion

We will be happy
if we can obtain 10^4 times muonium
as compared with that
in PSI experiment (1999).



Thank you

Appendix: Speculation on the order of Λ

For the scale Λ of the effective theory,

Sumino has speculated as follows:

Cancellation condition: $\alpha(m_\tau) = \frac{1}{4}\alpha_f(m(A_i^j))$

Definition for e and g_2: $e^2 = g_2^2 \sin^2\theta_W$

We know the fact $\sin^2\theta_W(M_Z) = 0.23119 \sim \frac{1}{4}$

The energy scale Λ which gives $e^2(m_\tau) = \frac{1}{4}g_2^2(\Lambda)$ is

$$\Lambda \sim 10^{2-3}\ \mathrm{TeV}$$

So, he speculated that $U(3)_f$ and $SU(2)_L$ will be unified at this scale.

$$\begin{array}{ccccc} U(1)_{em} & & SU(2)_L & & U(3)_f \\ 4e^2(m_\tau) & \simeq & g_2^2(\Lambda) & = & g_f^2(\Lambda) \end{array} \quad \text{at} \quad \Lambda \sim 10^{2-3}\ \mathrm{TeV}$$

Appendix: Masses of the charged leptons and family gauge bosons

In the Sumino model, the charged lepton mass spectrum is given by

$$(Y_e^{eff})_{ij} = \frac{1}{\Lambda^2}\sum_a \langle (\Phi_e)_{ia}\rangle \langle (\Phi_e)^T_{aj}\rangle$$

where Φ_e is a scalar with (3,3) of flavor U(3)$\times$ O(3) symmetry.

In other words, the VEV matrix $\langle \Phi_e \rangle$ is given by

$$\langle \Phi_e \rangle = \mathrm{diag}(v_1, v_2, v_3) \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$$

Gauge symmetry U(3) is broken by $\langle \Phi_e \rangle$, so that we obtain a relation $M^2(A_i^j) = M_0(m_{ei} + m_{ej})$