Broken SU(3) Flavor Symmetry and Tribimaximal Neutrino Mixing [1]

Yoshio Koide

IHERP, Osaka University,

1-16 Machikaneyama, Toyonaka, Osaka 560-0043, Japan

E-mail address: koide@het.phys.sci.osaka-u.ac.jp

Abstract

Recent work on a lepton mass matrix model based on an SU(3) flavor symmetry which is broken into S_4 is reviewed. The flavor structures of the masses and mixing are caused by VEVs of $SU(2)_L$-singlet scalars ϕ which are nonets ($\mathbf{8}+\mathbf{1}$) of the SU(3) flavor symmetry, and which are broken into $\mathbf{2}+\mathbf{3}+\mathbf{3}'$ and $\mathbf{1}$ of S_4. If we require the invariance under the transformation $(\phi^{(8)}, \phi^{(1)}) \to (-\phi^{(8)}, +\phi^{(1)})$ for the superpotential of the nonet field $\phi^{(8+1)}$, the model leads to a beautiful relation for the charged lepton masses. The observed tribimaximal neutrino mixing is understood by assuming two SU(3) singlet right-handed neutrinos $\nu_R^{(\pm)}$ and an SU(3) triplet scalar χ.

1 Introduction

The observed mass spectra and mixings of the fundamental particles will provide promising clues to unified understanding of the quarks and leptons, especially, to the understanding of the "flavor". In the present paper, we notice the following observed characteristic features in the lepton sector [1]:

(i) The observed charged lepton masses (m_e, m_μ, m_τ) satisfy the relation [2, 3]

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2, \tag{1.1}$$

with remarkable precision;

(ii) The observed neutrino mixing U_ν is approximately given by the so-called tribimaximal mixing [4]

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1.2}$$

which suggests that the mixing can be described by Clebsh-Gordan-like coefficients. Therefore, for a start, in the present paper, we investigate the lepton masses and mixings. The purpose of the present paper is to review a recent attempt [5] to investigate the mass relation (1.1) and the tribimaximal mixing.

In order to understand the relation (1.1), for example, we assume that there are three scalars ϕ_i $(i = 1, 2, 3)$, and the values of the charged lepton masses m_{ei} are proportional to the

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square of the vacuum expectation values (VEVs) $v_i = \langle\phi_i\rangle$ of the scalars ϕ_i, $m_{ei} = kv_i^2$ (in the Ref.[3, 6, 7], for instance, a seesaw type model $(M_e)_{ij} = \delta_{ij} v_i (M_E)^{-1} v_j$ has been assumed). We define singlet ϕ_σ and doublet (ϕ_π, ϕ_η) of a permutation symmetry S_3 [8] by

$$\begin{pmatrix} \phi_\pi \\ \phi_\eta \\ \phi_\sigma \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \tag{1.3}$$

from the three objects (ϕ_1, ϕ_2, ϕ_3), and we consider the following S_3 invariant scalar potential $V(\phi)$ [3, 9, 10]:

$$V(\phi) = m^2(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2) + \lambda_1(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2)^2 + \lambda_2 \phi_\sigma^2(\phi_\pi^2 + \phi_\eta^2). \tag{1.4}$$

The minimizing condition of the potential (1.4) leads to the relation

$$v_\pi^2 + v_\eta^2 = v_\sigma^2. \tag{1.5}$$

The relation (1.5) means

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2, \tag{1.6}$$

because

$$v_1^2 + v_2^2 + v_3^2 = v_\pi^2 + v_\eta^2 + v_\sigma^2 = 2v_\sigma^2 = 2\left(\frac{v_1 + v_2 + v_3}{\sqrt{3}}\right)^2. \tag{1.7}$$

Therefore, we can obtain the mass relation (1.1) from the assumption $m_{ei} \propto v_i^2$. Here, note that although the scalar potential (1.4) is invariant under the S_3 symmetry, but it is not a general one of the S_3 invariant form. As pointed out in Ref. [10], the scalar potential with a general form cannot lead to the relation (1.5). For the derivation of the VEV relation (1.5), it is essential to choose the specific form (1.4) of the S_3 invariant terms. Similar formulation is also possible for other discrete symmetries A_4 [11] and S_4 (see below). However, in such a symmetry, we still need an additional specific selection rule. What is the meaning of such a specific selection? In the present paper, we investigate this problem by assuming that the S_4 flavor symmetry is embedded into SU(3).

On the other hand, the observed tribimaximal mixing suggests the following scenario: From the definition (1.2), we can denote the fields (ψ_1, ψ_2, ψ_3) as

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = U_{TB} \begin{pmatrix} \psi_\eta \\ \psi_\sigma \\ \psi_\pi \end{pmatrix}. \tag{1.8}$$

The observed neutrino mixing (1.2) means that when the mass eigenstates of the charged leptons are given by the (ψ_1, ψ_2, ψ_3) basis, the mass eigenstates of the neutrinos are given by the

$(\psi_\eta, \psi_\sigma, \psi_\pi)$ basis. Therefore, the problem is to find a model where the charged lepton mass eigenstates are (e_1, e_2, e_3), while the neutrino mass eigenstates are given by $(\nu_\eta, \nu_\sigma, \nu_\pi)$ with the mass hierarchy $m_\eta^2 < m_\sigma^2 \ll m_\pi^2$ (or $m_\pi^2 \ll m_\eta^2 < m_\sigma^2$). In the S_4, we can define the same relation as (1.8).

Thus, the characteristic features (1.1) and (1.2) in the lepton sector may be understood from the language of S_4 (also S_3 or A_4). However, as seen from the above review, the characteristic features (1.1) and (1.2) cannot be understood from the S_4 symmetry only. We need some additional assumptions. In the present model, we will investigate these problems under an assumption that the present S_4 symmetry is embedded into an SU(3) symmetry [12]. In the next section, the singlet ϕ_σ and doublet (ϕ_π, ϕ_η) will be understood as members of a nonet scalar ϕ [**1**+**8** of SU(3)], and the VEV relation (1.5) will be derived by requiring that $W(\phi)$ is invariant under a Z_2 symmetry. In Sec.3, in order to give the charged lepton masses and tribimaximal neutrino mixing, we will discuss the effective Hamiltonian by assuming an Froggatt-Nelsen [13] type model. Finally, Sec.4 will be devoted to the summary and concluding remarks.

2 VEVs of SU(3) nonet scalars

The goal in the present section is to obtain the VEV relation (1.6) [i.e. (1.5)]. As seen in the previous section, in order to obtain the desirable results (1.5), we need assume an equal weight between the doublet and singlet terms of S_4. In the present paper, we assume that the S_4 symmetry is embedded into an SU(3) symmetry. The doublet (ϕ_π, ϕ_η) and singlet ϕ_σ of S_4 are embedded in the **6** and $(\mathbf{8}+\mathbf{1})$ of SU(3) [12]. In the present model [5], we assume that the doublet (ϕ_π, ϕ_η) and singlet ϕ_σ originate in SU(3) octet and singlet, respectively. The essential assumption in the present paper is that the fields ϕ_u and ϕ_d always appear in the theory with the form of the nonet of U(3):

$$\phi = \begin{pmatrix} \phi_1^1 & * & * \\ * & \phi_2^2 & * \\ * & * & \phi_3^3 \end{pmatrix}, \tag{2.1}$$

where

$$\begin{aligned} \phi_1^1 &= \tfrac{1}{\sqrt{3}}\phi_\sigma + \tfrac{2}{\sqrt{6}}\phi_\eta, \\ \phi_2^2 &= \tfrac{1}{\sqrt{3}}\phi_\sigma - \tfrac{1}{\sqrt{6}}\phi_\eta - \tfrac{1}{\sqrt{2}}\phi_\pi, \\ \phi_3^3 &= \tfrac{1}{\sqrt{3}}\phi_\sigma - \tfrac{1}{\sqrt{6}}\phi_\eta + \tfrac{1}{\sqrt{2}}\phi_\pi, \end{aligned} \tag{2.2}$$

and the index f $(f = u, d)$ has been dropped.

Although we have obtained the VEV (1.5) relation from the scalar potential (1.4) by calculating $\partial V/\partial\phi_a$ $(a = \pi, \eta, \sigma)$, in the present paper, we will obtain the relation (1.5) from an SU(3) invariant superpotential W (The derivation of the VEV relation (1.5) from a superpotential W has first been attempted by Ma [14]): The SU(3) invariant superpotential for the nonet fields ϕ_f $(f = u, d)$ are given by

$$W(\phi_f) = \frac{1}{2} m_f \mathrm{Tr}(\phi_f \phi_f) + \frac{1}{2\sqrt{3}} \lambda_f \mathrm{Tr}(\phi_f \phi_f \phi_f). \tag{2.3}$$

Since, in the next section, we want to assign charges $+1$ and -1 of a Z_3 symmetry to the fields ϕ_u and ϕ_d, respectively, we also assign the Z_3 charges $+1$ and -1 to the mass parameters m_u and m_d in Eq.(2.3), respectively. However, the U(3) invariant superpotential (2.3) cannot give the relation (1.5). As we show below, only when we drop the $\mathrm{Tr}[(\phi^{(8)})^3]$-term in the cubic terms $\mathrm{Tr}(\phi^3)$, we can obtain the VEV relation (1.5). Therefore, we introduce a Z_2 symmetry, and we assign the Z_2 parities -1 and $+1$ (the Z_2 charge $+1$ and 0) for the octet part $\phi^{(8)}$ and singlet part $\phi^{(1)}$ of the nonet field ϕ, respectively. The symmetry Z_2 breaks U(3) into SU(3). (In other words, in the present model, the flavor symmetry U(3) is explicitly broken from the beginning by the Z_2 symmetry.) Under the requirement of the Z_2 invariance, i.e. the invariance under the transformation

$$(\phi^{(8)}, \phi^{(1)}) \to (-\phi^{(8)}, +\phi^{(1)}), \tag{2.4}$$

the terms $\mathrm{Tr}(\phi^{(8)}\phi^{(8)}\phi^{(8)})$ are forbidden. Thus, the superpotential (2.3) with the Z_2 invariance leads to

$$\begin{aligned} W(\phi) &= \frac{1}{2}m\left[\mathrm{Tr}(\phi^{(8)}\phi^{(8)}) + \phi_\sigma^2\right] + \frac{1}{2}\lambda\phi_\sigma\left[\mathrm{Tr}(\phi^{(8)}\phi^{(8)}) + \frac{1}{3}\phi_\sigma^2\right] \\ &= \frac{1}{2}m\left(\phi_\sigma^2 + \phi_\pi^2 + \phi_\eta^2\right) + \frac{1}{2}\lambda\left[(\phi_\pi^2 + \phi_\eta^2)\phi_\sigma + \frac{1}{3}\phi_\sigma^3\right] + \cdots . \end{aligned} \tag{2.5}$$

From the superpotential (2.5) with the Z_2 invariance, we obtain the VEV relation (1.5) as follows: From the condition

$$\frac{\partial W}{\partial(\phi^{(8)})_i^j} = m(\phi^{(8)})_j^i + \lambda\phi_\sigma(\phi^{(8)})_j^i = 0, \tag{2.6}$$

we obtain

$$m + \lambda\phi_\sigma = 0, \tag{2.7}$$

for $(\phi^{(8)})_i^j \neq 0$. By eliminating m from Eq.(2.7) and the condition

$$\frac{\partial W}{\partial\phi_\sigma} = m\phi_\sigma + \frac{1}{2}\lambda\left[\mathrm{Tr}(\phi^{(8)}\phi^{(8)}) + \phi_\sigma^2\right] = 0, \tag{2.8}$$

we obtain the relation

$$\phi_\sigma^2 = \mathrm{Tr}(\phi^{(8)}\phi^{(8)}) = \phi_\pi^2 + \phi_\eta^2 + \cdots , \tag{2.9}$$

where "$\cdots$" denotes the contributions of **3** and **3′** of S_4.

The result (2.9) is still not our goal, because the relation contains the VEVs of the **3** and **3′** of S_4. So far, we have not discussed the splitting among the S_4 multiplets. Now, we bring a soft symmetry breaking of SU(3) into S_4 with an infinitesimal parameter ε into the mass term of $W(\phi)$ as

$$\mathrm{Tr}(\phi^{(8)}\phi^{(8)}) \Rightarrow \phi_\pi\phi_\pi + \phi_\eta\phi_\eta + (1+\varepsilon)\sum_{i\neq j}(\phi^{(8)})_i^j(\phi^{(8)})_j^i, \tag{2.10}$$

by hand. Recall that when we obtain the relation (2.7), we have assumed $(\phi^{(8)})_i^j \neq 0$. Now, the conditions (2.6) are modified into the following conditions:

$$[(1+\varepsilon)m + \lambda\phi_\sigma]\,(\phi^{(8)})^i_j = 0 \quad (i \neq j), \tag{2.11}$$

$$(m + \lambda\phi_\sigma)\,\phi_a = 0 \quad (a = \pi, \eta), \tag{2.12}$$

Therefore, we must take either $(\phi^{(8)})^i_j = 0$ $(i \neq j)$ or $\phi_a = 0$ $(a = \pi, \eta)$ for $\varepsilon \neq 0$. When we choose the solution

$$\langle(\phi^{(8)})^i_j\rangle = 0 \quad (i \neq j), \tag{2.13}$$

we can obtain the desirable relation (1.5). (However, it is possible that we can also take another solution with $\phi_\pi = \phi_\eta = 0$ and $(\phi^{(8)})^i_j \neq 0$. The VEV solutions are not unique. The result (1.5) is merely one of the possible solutions.)

Thus, we have obtained not only the desirable VEV relation (1.5), but also the results (2.13). It should be worthwhile noticing that if we have assume the superpotential (2.3) without requiring the Z_2 invariance, we could obtain neither (1.5) nor (2.13).

By the way, we know that the three masses in any sectors of quarks and leptons are completely different among them. Therefore, if we assume a flavor symmetry, the symmetry must finally be broken completely. Usually, a relation which we derive in the exact symmetry limit is only approximately satisfied under the symmetry breaking. Although we derive the VEV relation (1.5) under the S_4 symmetry, the problem is whether the VEV relation (1.5) which is obtained under the S_4 symmetry is spoiled or not when we introduce such a symmetry breaking. In Ref.[5], we will find that such a symmetry breaking term without spoiling the relation (1.5) is indeed possible.

3 Effective Hamiltonian

If we regard the scalars ϕ_u and ϕ_d as $SU(2)_L$ doublets, such a model with multi-Higgs doublets causes a flavor changing neutral current (FCNC) problem. Therefore, we must consider that the fields ϕ_u and ϕ_d are $SU(2)_L$ singlets. In the present paper, we assume a Froggatt-Nielsen [13] type model

$$H^{eff} = y_e \bar{\ell}_L H_L^d \frac{\phi_d}{\Lambda}\frac{\phi_d}{\Lambda}\frac{\xi}{\Lambda} e_R + y_\nu \bar{\ell}_L H_L^u \frac{\phi_u}{\Lambda}\frac{\chi}{\Lambda}\nu_R + y_R \bar{\nu}_R \Phi_R \nu_R^*, \tag{3.1}$$

where ℓ_{iL} are $SU(2)_L$ doublet leptons $\ell_{iL} = (\nu_{iL}, e_{iL})$, H_L^d and H_L^u are conventional $SU(2)_L$ doublet Higgs scalars, ϕ_f $(f = u, d)$, ξ and χ are $SU(2)_L$ singlet scalars, and Λ is a scale of the effective theory. We consider that $\langle\phi_f\rangle/\Lambda$, $\langle\xi\rangle/\Lambda$ and $\langle\chi\rangle/\Lambda$ are of the order of 1. The scalar Φ_R has been introduced in order to generate the Majorana mass M_R of the right-handed neutrino ν_R. As we note later, in the present model, the right-handed neutrinos $\nu_R = (\nu_R^{(+)} + \nu_R^{(-)})/\sqrt{2}$ are singlets of the SU(3) flavor. The role of $\xi = (\xi^{(+)} + \xi^{(-)})/\sqrt{2}$ and χ will be explained later. In order to understand the appearance of the combinations $H_L^d \phi_d \phi_d \xi$ and $H_L^u \phi_u \chi$, we assume two Z_3 symmetries (Z_3 and Z_3' in Table 1). Those quantum number assignments are given in Table 1. However, even with those quantum numbers, we cannot distinguish the state $\phi_f^\dagger$ from $\phi_f \phi_f$. For example, the interaction $\bar{\ell}_L H_d \phi_d^\dagger \xi e_R$ is possible in addition to the interaction

Table 1 SU(3) and S_4 assignments of the fields

Fields	$SU(2)_L$	SU(3)	S_4	Z_3	Z_3'	Z_2
ℓ_L	**2**	**3**	$\mathbf{3'}$	0	0	0
e_R	**1**	**3**	$\mathbf{3'}$	0	0	0
$\nu_R^{(\pm)}$	**1**	**1**	**1**	0	0	0/+1
ϕ_u	**1**	**1+8**	$\mathbf{1}+(\mathbf{2}+\mathbf{3}+\mathbf{3'})$	+1	+1	0/+1
ϕ_d	**1**	**1+8**	$\mathbf{1}+(\mathbf{2}+\mathbf{3}+\mathbf{3'})$	-1	-1	0/+1
$\xi^{(\pm)}$	**1**	**1**	**1**	0	-1	0/+1
χ	**1**	**3**	$\mathbf{3'}$	+1	-1	0
H_L^u	**2**	**1**	**1**	+1	0	0
H_L^d	**2**	**1**	**1**	-1	0	0
Φ_R	**1**	**1**	**1**	0	0	0

$\bar{\ell}_L H_d \phi_d \phi_d \xi e_R$. Although we have started from an SUSY scenario in the previous section, now, we have adopted an effective Hamiltonian which is not renormalizable. Therefore, in principle, the interaction $\bar{\ell}_L H_d \phi_d^\dagger \xi e_R$ cannot be ruled out. For the moment, in order to forbid such an undesirable term, we assume that the fields which can appear in the effective Hamiltonian are confined to holomorphic ones.

(a) Charged lepton sector

Recall that we have already assumed the invariance of the superpotential under the Z_2 transformation (2.4) in order to drop the cubic part of the octet $\phi^{(8)}$. Therefore, the term $\phi\phi$ means $\phi^{(8)}\phi^{(8)} + \phi^{(1)}\phi^{(1)}$ under the Z_2 invariance. However, in order to give $m_{ei} \propto \langle \phi_i^i \rangle^2$, what we want is not $\phi^{(8)}\phi^{(8)} + \phi^{(1)}\phi^{(1)}$, but $\phi^{(8)}\phi^{(8)} + \phi^{(1)}\phi^{(1)} + \phi^{(8)}\phi^{(1)} + \phi^{(1)}\phi^{(8)}$. In order to evade this problem, we introduce additional fields $\xi^{(+)}$ and $\xi^{(-)}$ whose Z_2 parity are +1 and -1, respectively. The effective interactions in the charged lepton sector are given by

$$H_e^{eff} = \frac{y_e}{\sqrt{2}} \bar{e}_L^i (\phi_d)_i^j (\phi_d)_j^k (\xi^{(+)} + \xi^{(-)}) e_{Rk}, \tag{3.2}$$

where we have dropped the Higgs scalar H_L^d since we discuss flavor structure only. The expression (3.2) becomes

$$H_e^{eff} = \frac{y_e}{\sqrt{2}} \bar{e}_L [(\phi_d^{(8)}\phi_d^{(8)} + \phi_d^{(1)}\phi_d^{(1)})\xi^{(+)} + (\phi_d^{(8)}\phi_d^{(1)} + \phi_d^{(1)}\phi_d^{(8)})\xi^{(-)}] e_R. \tag{3.3}$$

Since we have assumed that $\xi^{(+)}$ and $\xi^{(-)}$ appear symmetrically in the theory, we also assume

$$\langle \xi^{(+)} \rangle = \langle \xi^{(-)} \rangle \equiv v_\xi. \tag{3.4}$$

Then, we obtain the effective Hamiltonian for the charged leptons

$$H_e^{eff} = \frac{y_e v_d v_\xi}{\sqrt{2}\Lambda^3} \sum_i \bar{e}_L^i \langle (\phi_d^{(8+1)})_i^i \rangle^2 e_{Ri}, \tag{3.5}$$

where $v_d = \langle H_L^{d0} \rangle$. Since the fields $(\phi_d)_i^i$ are defined by Eq.(2.2), we can obtain the charged lepton mass relation (1.1) from the VEV relation (1.6).

(b) Neutrino sector

In the present model, the right-handed neutrinos $\nu^{(\pm)}$ are singlets of SU(3). Therefore, in the neutrino seesaw mass matrix $M_\nu = m_L^\nu M_R^{-1} (m_L^\nu)^T$, M_R is a 1×1 matrix and m_L^ν is a 3×1 matrix. In order to compensate for the absence of the conventional triplet neutrinos ν_R, a new scalar χ which is a triplet of SU(3) has been introduced. The neutrino Dirac mass terms are given by the following effective Hamiltonian

$$H_{Dirac}^{eff} = y_\nu \frac{v_u}{\Lambda^2} \bar{\nu}_L^i \langle (\phi_u)_i^j \rangle \langle \chi_j \rangle (\nu_R^{(+)} + \nu_R^{(-)}), \tag{3.6}$$

where $v_u = \langle H_L^{u0} \rangle$. It is likely that the scalar potential $V(\chi)$ for the SU(3) triplet χ has a specific VEV solution

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle \equiv v_\chi. \tag{3.7}$$

When we assume the VEVs (3.7), we obtain

$$H_{Dirac}^{eff} = y_\nu \frac{v_u v_\chi}{\sqrt{2}\Lambda^2} (\bar{\nu}_\eta \ \bar{\nu}_\sigma \ \bar{\nu}_\pi)_L \left[\begin{pmatrix} v_\eta \\ 0 \\ v_\pi \end{pmatrix} \nu_R^{(-)} + \begin{pmatrix} 0 \\ v_\sigma \\ 0 \end{pmatrix} \nu_R^{(+)} \right], \tag{3.8}$$

where $v_a = \langle \phi_{ua} \rangle$ $(a = \pi, \eta, \sigma)$ (for convenience, we have dropped the index u). Therefore, we obtain the effective neutrino mass matrix on the (η, σ, π) basis,

$$U_{TB}^T M_\nu U_{TB} \equiv M_\nu^{(\eta\sigma\pi)} = \frac{1}{M_R^{(-)}} \begin{pmatrix} v_\eta^2 & 0 & v_\pi v_\eta \\ 0 & 0 & 0 \\ v_\pi v_\eta & 0 & v_\pi^2 \end{pmatrix} + \frac{1}{M_R^{(+)}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & v_\sigma^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3.9}$$

where $M_R^{(\pm)} = y_R^{(\pm)} \langle \Phi_R \rangle$, and we have dropped the common factors $(y_\nu v_u v_\chi / \sqrt{2}\Lambda^2)^2$. By the way, the ratio v_π / v_η cannot be determined from the potential (2.6), and the ratio is determined by a soft S_4 symmetry breaking term W_{SB} which has been discussed in the previous section. We can choose a solution $v_\pi = 0$ in the superpotential $W(\phi_u)$ by adjusting the parameter β in W_{SB}, differently from the case of $W(\phi_d)$. Then, the neutrino mass matrix (3.9) becomes a diagonal form $D_\nu = (1/M_R^{(-)}) \mathrm{diag}(v_\eta^2, 0, 0) + (1/M_R^{(+)}) \mathrm{diag}(0, v_\sigma^2, 0)$. Since the mass matrix M_ν on the $(\nu_1, \nu_2, \nu_3) = (\nu_e, \nu_\mu, \nu_\tau)$ basis is given by

$$M_\nu = U_{TB} M_\nu^{(\eta\sigma\pi)} U_{TB}^T = U_{TB} D_\nu U_{TB}^T, \tag{3.10}$$

we can obtain the tribimaximal mixing

$$U_\nu = U_{TB}, \tag{3.11}$$

and the neutrino masses

$$m_{\nu 1} = k v_\eta^2, \quad m_{\nu 2} = k v_\sigma^2, \quad m_{\nu 3} = 0, \tag{3.12}$$

for the case of $M_R^{(+)} = M_R^{(-)} \equiv M_R$, where $k = (y_\nu v_u v_\chi)^2/2M_R\Lambda^4$ and $(\nu_\eta, \nu_\sigma, \nu_\pi)$ has been renamed (ν_1, ν_2, ν_3) according to the conventional naming.

However, since we have taken $v_\pi = 0$, the value of v_η satisfies $v_\eta^2 = v_\sigma^2$ from the relation (1.5), so that the result (3.12) gives $m_{\nu 1} = m_{\nu 2}$. The observed value [15] Δm^2_{solar} is small, but it is not zero. Therefore, we must consider a small deviation between the first and second terms in (3.9) (i.e. $M_R^{(+)} \neq M_R^{(-)}$). Since the value $M_R^{(-)}/M_R^{(+)}$ is free in the present model, we cannot predict an explicit value of the ratio $\Delta m^2_{solar}/\Delta m^2_{atm}$.

Since the present model gives an inverse hierarchy of the neutrino masses, the predicted effective electron neutrino mass

$$\langle m_{\nu_e} \rangle = \left| \sum_i U_{ei}^2 m_{\nu i} \right| \simeq |m_{\nu 1}| \simeq |m_{\nu 2}| \simeq \sqrt{\Delta m^2_{atm}} = 5.23^{+0.25}_{-0.40} \times 10^{-2}\,\mathrm{eV}, \tag{3.13}$$

where we have used the value [16] $\Delta m^2_{atm} = 2.74^{+0.44}_{-0.26} \times 10^{-3}\,\mathrm{eV}^2$. This value (3.13) is sufficiently sensitive to the next generation experiments of the neutrinoless double beta decay.

4 Summary

In conclusion, on the basis of the S_4 symmetry which is embedded into SU(3), we have investigated a lepton mass model with the effective Hamiltonian of the Froggatt-Nielsen type (3.1). We have assumed that the singlet and doublet of S_4 originate in the singlet and octet of SU(3), and we have obtained the VEV relation (1.5). In the derivation of the VEV relation (1.5), the essential assumptions for the superpotential $W(\phi_f)$ are the following two: (i) the scalar fields ϕ_f always appear in terms of the nonet form (2.1) of U(3); (ii) the superpotential $W(\phi_f)$ is invariant under the Z_2 transformation (2.4). Then, we have obtained not only the VEV relation (1.5), but also $\langle (\phi^{(8)})_i^j \rangle = 0$ $(i \neq j)$ for the other components of $\phi^{(8)}$ (i.e. $\langle \mathbf{3} \rangle = \langle \mathbf{3'} \rangle = 0$).

In the charged lepton sector, we have assumed the Frogatt-Nielsen-type effective Hamiltonian $\bar{e}_L^i (\phi_d)_i^j (\phi_d)_j^k e_{Rj}$. Since we have obtained the VEV of ϕ_i^j

$$\langle \phi_d \rangle = \mathrm{diag}\left(\frac{1}{\sqrt{3}} v_\sigma + \frac{2}{\sqrt{6}} v_\eta, \frac{1}{\sqrt{3}} v_\sigma - \frac{1}{\sqrt{6}} v_\eta - \frac{1}{\sqrt{2}} v_\pi, \frac{1}{\sqrt{3}} v_\sigma - \frac{1}{\sqrt{6}} v_\eta + \frac{1}{\sqrt{2}} v_\pi \right), \tag{4.1}$$

we can obtain

$$m_{ei} \propto \langle (\phi_d)_i^i \rangle^2 \equiv (v_i^i)^2, \tag{4.2}$$

so that the charged lepton masses m_{ei} satisfy the relation (1.1) under the definition (2.2). Here, we would like to emphasize that the result $\langle \phi_i^j \rangle = 0$ for $i \neq j$, Eq.(2.13), is essential to obtain Eq.(4.2) in the Froggatt-Nielsen-type model. In the Froggatt-Nielsen-type model, if we had considered a triplet scalar ϕ, we could obtain the result $m_{ei} \propto v_i^2$, because the effective interaction $\bar{e}_L \phi\phi e_R$ gives $\bar{e}_{Li} \phi_i \phi_j e_{Rj}$. On the other hand, if we had adopted a seesaw-type model, by assuming a triplet scalar ϕ whose effective interaction is given by

$$H_e = \sum_i \left(\bar{e}_{Li} \phi_i E_{Ri} + \bar{E}_{Li} \phi_i e_{Ri} + M_E \bar{E}_{Li} E_{Ri} \right), \tag{4.3}$$

we could automatically obtain a form

$$H_L^{eff} = \sum_i \frac{1}{M_E} \bar{e}_{Li} \phi_i^2 e_{Ri}, \tag{4.4}$$

through a seesaw mechanism [17]. However, it is not easy to obtain the VEV relation (1.6) for the triplet scalar ϕ_i. This is the main motive for introducing the nonet (not triplet) scalar ϕ.

For the neutrino sector, we have obtained the tribimaximal mixing (1.2) by introducing an SU(3) triplet scalar χ and the two SU(3) singlet right-handed neutrinos $\nu_R^{(\pm)}$ in addition to the nonet scalar ϕ_u. In the present model, the right-handed neutrinos $\nu_R^{(\pm)}$ are singlets of SU(3), the Majorana neutrino mass matrices $M_R^{(\pm)}$ have no flavor structure, (i.e. M_R are 1 matrices). Also note that the neutrino Dirac mass matrix m_L^ν is a 3×1 matrix. This plays an essential role to derivation of the tribimaximal mixing. For the neutrino mass spectrum, since the model gives $m_{\nu 1} = m_{\nu 2}$ in the limit of $M_R^{(+)} = M_R^{(-)}$, we must consider a small deviation $M_R^{(+)} \neq M_R^{(-)}$. Since the value of $M_R^{(-)}/M_R^{(+)}$ is a free parameter in the present model, we cannot predict the value $\Delta m^2_{solar}/\Delta m^2_{atm}$ at present, although the smallness of the ratio $\Delta m^2_{solar}/\Delta m^2_{atm}$ can be understood. Since the present model gives an inverse hierarchy of the neutrino masses, we can predict the effective electron neutrino mass $\langle m_{\nu_e} \rangle \simeq 0.05$ eV, which is sufficiently sensitive to the next generation experiments of the neutrinoless double beta decay.

The present model seems to provide suggestive hints on seeking for a model which leads to the tribimaximal mixing (1.2) and the charged lepton mass relation (1.1), although the model has still many points which should be improved. We summarize some of the future tasks:

(i) Seeking for a more natural mechanism which can provide $m_{ei} \propto v_i^2$, apart from a Froggatt-Nielsen-type model.

(ii) Seeking for a model which is completely predictable neutrino mass spectrum (in the present model, Δm^2_{21} was not predictable) together with the nearly tribimaximal mixing.

(iii) In the present model, the right-handed neutrino ν_R was a singlet of SU(3). However, it is likely that ν_R still a triplet.

(iv) Seeking for the origin of the symmetry breaking of SU(3) into S_4.

We hope that the present model will also provide a promising clue to the unified mass matrix model of the quarks and leptons.

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