Charged Lepton Mass Formula [1]
– Development and Prospect –

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Abstract

The recent devolopment on the charged lepton mass forumula $m_e+m_\mu+m_\tau = \frac{2}{3}\left(\sqrt{m_e}+\sqrt{m_\mu}\right.$ $\left.+\sqrt{m_\tau}\right)^2$ is reviewed. An S_3 or A_4 model will be promising for the mass relation.

1 Beginning of the charged lepton mass formula

It is generally considered that masses and mixings of the quarks and leptons will obey a simple law of nature, so that we expect that we will find a beautiful relation among those values. It is also considered that the mass matrices of the fundamental particles will be governed by a symmetry.

Our dream is to understand masses and mixings from a symmetry, i.e. not from adjustable parameters, but from Clebsch-Gordan-like coefficients.

For example, in 1971, the author[1] tried to understand the Cabibbo mixing from a U(3) symmetry from a composite model of quarks, the so-called Hiroshima model[2]. [3] The basic idea[1] to give the Cabibbo angle was analogy to the hadronic π^0-η^0-σ^0 mixing where we assume $(\nu_e, \nu_\mu, \ell) \sim \mathbf{3}$ of U(3), and 3 quarks are members of $\mathbf{8+1}$ of U(3). In the model, when we define the following lepton-antilepton states

$$\begin{aligned} \pi &= \frac{1}{\sqrt{2}}(\nu_e\overline{\nu}_e - \nu_\mu\overline{\nu}_\mu), \\ \eta &= \frac{1}{\sqrt{6}}(\nu_e\overline{\nu}_e + \nu_\mu\overline{\nu}_\mu - 2\ell\overline{\ell}), \\ \sigma &= \frac{1}{\sqrt{3}}(\nu_e\overline{\nu}_e + \nu_\mu\overline{\nu}_\mu + \ell\overline{\ell}), \end{aligned} \tag{1.1}$$

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[3] In the model, the quarks are composed as

$$\begin{aligned} u &= \langle(\nu_e\overline{\nu}_e\cos\theta + \nu_\mu\overline{\nu}_\mu\sin\theta\rangle)B\rangle, \\ c &= \langle(-\nu_e\overline{\nu}_e\sin\theta + \nu_\mu\overline{\nu}_\mu\cos\theta\rangle)B\rangle, \\ d &= \langle(e^-\overline{\nu}_e)B\rangle, \\ s &= \langle(\mu^-\overline{\nu}_\mu)B\rangle, \end{aligned}$$

where B is a hypothetical fermion with the baryon number. In the model, semileptonic decays $d \to u$ and $s \to u$ are understood from $e^- \to \nu_e$ with the factor $\cos\theta$ and from $\mu^- \to \nu_\mu$ with the factor $\sin\theta$, respectively. The nonleptonic decay $s \to d$ can be understood from $\mu^-\overline{\nu}_\mu \to e^-\overline{\nu}_e$ with the factor 1 without assuming the so-called penguin diagram.

$$
\begin{aligned}
\pi &= \tfrac{1}{\sqrt{2}}(\nu_e\overline{\nu}_e - \nu_\mu\overline{\nu}_\mu), \\
\omega &= \tfrac{1}{\sqrt{2}}(\nu_e\overline{\nu}_e + \nu_\mu\overline{\nu}_\mu), \\
\phi &= \tfrac{1}{\sqrt{3}}\ell\overline{\ell},
\end{aligned}
\tag{1.2}
$$

from the analogy with the hadoronic π-η-η' mixing, we consider that the physical up-quark u has the lepton-antilepton state

$$
\pi' = \frac{\sqrt{3}}{2}\pi + \frac{1}{2}\omega = \frac{\sqrt{3}+1}{2\sqrt{2}}\nu_e\overline{\nu}_e - \frac{\sqrt{3}-1}{2\sqrt{2}}\nu_\mu\overline{\nu}_\mu, \tag{1.3}
$$

so that the Cabibbo angle θ_C is given by

$$
\sin\theta_C = \frac{\sqrt{3}-1}{2\sqrt{2}}, \quad \cos\theta_C = \frac{\sqrt{3}+1}{2\sqrt{2}}. \tag{1.4}
$$

However, the derivation of (1.4) was strained and tricky. In 1978, Harari *et al.*[3] have proposed a model based on a permutation symmetry S_3 and they have successfully derived the relation (1.4) for the Cabibbo angle θ_C.

In 1982, the author[4] have proposed a charged lepton mass relation[4]

$$
m_e + m_\mu + m_\tau = \frac{2}{3}\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2. \tag{1.5}
$$

It is well-known that the observed charged lepton mass spectrum satisfies the relation with remarkable precision.

The mass formula (1.5) is invariant under any exchange $\sqrt{m_i} \leftrightarrow \sqrt{m_j}$. This suggests that a description by a permutation symmetry[6] S_3 may be useful for the mass matrix model. However, this formula was derived from an extension of the π-η-σ mixing model (composite model), and the author had never considered an S_3 symmetry although he had still assumed a U(3) symmetry until 1999[7].

In the present paper, we will demonstrate how the S_3 symmetry is promising to understand the masses and mixings. And, we will comment that an A_4 symmetry is also promising. Although we think that there are beautiful relations among quark and lepton masses and mixings, it is hard to see such a symmetry in the quark sector, because the original symmetry will be spoiled by the gluon cloud. In the present study, we will confine ourselves to the investigation of the lepton masses and mixings.

2 Leptons under S_3

Let us demonstrate that the S_3 symmetry is promising not only for understanding of the charged lepton mass relation (1.5), but also for that of the observed neutrino mixing, i.e. the so-called

[4] Exactly speaking, the expression (1.5) appeared in Ref. [5]. In Refs.[4], the formula has been given by the expression $m_{ei} = m_0(x_i + x_0)^2$, where $x_1 + x_2 + x_3 = 0$ and $x_0 = \sqrt{(x_1^2 + x_2^2 + x_3^2)/3}$.

tribimaximal mixing[8]

$$U_{TB} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{2.1}$$

2.1 Tribimaximal mixing

We define the doublet (ψ_π, ψ_η) and singlet ψ_σ of S_3

$$\begin{pmatrix} \psi_\pi \\ \psi_\eta \\ \psi_\sigma \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}. \tag{2.2}$$

If the neutrino mass eigenstates are $(\nu_\pi, \nu_\eta, \nu_\sigma)$ defined by the relation (2.2) with the mass hierarchy $m_{\nu\eta}^2 < m_{\nu\sigma}^2 < m_{\nu\pi}^2$ in contrast to the weak eigenstates $(\nu_1, \nu_2, \nu_3) = (\nu_e, \nu_\mu, \nu_\tau)$, then we can obtain the tribimaximal mixing (2.1) because of the relation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_\eta \\ \nu_\sigma \\ \nu_\pi \end{pmatrix}. \tag{2.3}$$

2.2 Charged lepton mass formula and Higgs potential

The mass formula (1.5) can be understood from a universal seesaw model[5] with 3-flavor scalars ϕ_i:

$$M_f = m_L^f M_F^{-1} m_R^f. \tag{2.4}$$

Here, for the charged lepton sector, we take

$$m_L^e = \frac{1}{\kappa} m_R^e = y_e \mathrm{diag}(v_1, v_2, v_3), \tag{2.5}$$

where the VEV $v_i = \langle \phi_i \rangle$ satisfy the relation

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}\left(v_1 + v_2 + v_3\right)^2. \tag{2.6}$$

The VEV relation (2.6) means

$$v_\pi^2 + v_\eta^2 = v_\sigma^2, \tag{2.7}$$

because

$$v_1^2 + v_2^2 + v_3^2 = v_\pi^2 + v_\eta^2 + v_\sigma^2 = 2v_\sigma^2 = 2\left(\frac{v_1 + v_2 + v_3}{\sqrt{3}}\right)^2. \tag{2.8}$$

The Higgs potential which gives the relation (2.7) is, for example, given by[11]

$$V = \mu^2 \sum_i (\overline{\phi}_i \phi_i) + \frac{1}{2}\lambda_1 \left[\sum_i (\overline{\phi}_i \phi_i) \right]^2$$

$$+\lambda_2 (\overline{\phi}_\sigma \phi_\sigma)(\overline{\phi}_\pi \phi_\pi + \overline{\phi}_\eta \phi_\eta) + V_{SB}, \tag{2.9}$$

where V_{SB} is a soft symmetry breaking term which does not affect the derivation of (2.7). Here, the existence of the S_3 invariant λ_2-term is essential for the derivation of the VEV relation (2.7). Note that the relation (2.7) can be obtained independently of the explicit values of the parameter λ_2.

2.3 Yukawa interaction form under S_3

The general form of the S_3 invariant Yukawa interaction is given by

$$H = \left(y_0 \frac{\bar{\psi}_\pi \psi_\pi + \bar{\psi}_\eta \psi_\eta + \bar{\psi}_\sigma \psi_\sigma}{\sqrt{3}} + y_1 \frac{\bar{\psi}_\pi \psi_\pi + \bar{\psi}_\eta \psi_\eta - 2\bar{\psi}_\sigma \psi_\sigma}{\sqrt{6}} \right) \phi_\sigma$$

$$+y_2 \left(\frac{\bar{\psi}_\pi \psi_\eta + \bar{\psi}_\eta \psi_\pi}{\sqrt{2}} \phi_\pi + \frac{\bar{\psi}_\pi \psi_\pi - \bar{\psi}_\eta \psi_\eta}{\sqrt{2}} \phi_\eta \right)$$

$$+y_3 \frac{\bar{\psi}_\pi \phi_\pi + \bar{\psi}_\eta \phi_\eta}{\sqrt{2}} \psi_\sigma + y_4 \bar{\psi}_\sigma \frac{\phi_\pi \psi_\pi + \phi_\eta \psi_\eta}{\sqrt{2}}. \tag{2.10}$$

For the charged lepton sector, we have already assumed the form

$$H_e = y_e \left(\bar{\ell}_{L1} \phi^d_{L1} E_{R1} + \bar{\ell}_{L2} \phi^d_{L2} E_{R2} + \bar{\ell}_{L3} \phi^d_{L3} E_{R3} \right). \tag{2.11}$$

The form (2.11) corresponds to the case

$$y_0 = y_e, \quad y_1 = 0, \quad y_2 = \frac{1}{\sqrt{3}} y_e, \quad y_3 = y_4 = \sqrt{\frac{2}{3}} y_e, \tag{2.12}$$

in the general form (2.10).

The general study under the S_3 symmetry will be found in a recent paper[12] by the author. Here, the details are skiped.

2.4 A simple example of the S_3-invariant neutrino interaction

Now, let us speculate an S_3-invariant neutrino interaction with a concise form[9]

$$H_\nu = y_\nu \left(\frac{\bar{\ell}_\pi N_\pi + \bar{\ell}_\eta N_\eta + \bar{\ell}_\sigma N_\sigma}{\sqrt{3}} \phi^u_\sigma + \frac{\bar{\ell}_\pi N_\eta + \bar{\ell}_\eta N_\pi}{\sqrt{2}} \phi^u_\pi + \frac{\bar{\ell}_\pi N_\pi - \bar{\ell}_\eta N_\eta}{\sqrt{2}} \phi^u_\eta \right). \tag{2.13}$$

Here, we have assumed the universality of the coupling constants to the ϕ_σ, ϕ_π and ϕ_η terms. Then, we obtain a simple mass spectrum

$$
\begin{aligned}
m_{\nu 1} &= \left(\tfrac{1}{\sqrt{6}} - \tfrac{1}{2}\right)^2 m_0^\nu, \\
m_{\nu 2} &= \tfrac{1}{6} m_0^\nu, \\
m_{\nu 3} &= \left(\tfrac{1}{\sqrt{6}} + \tfrac{1}{2}\right)^2 m_0^\nu,
\end{aligned}
\tag{2.14}
$$

where we have used the relation

$$v_\pi^2 + v_\eta^2 = v_\sigma^2, \tag{2.15}$$

from the S_3 Higgs potential model. By putting $m_{\nu 3} = \sqrt{\Delta m^2_{atm}}$ from the atmospheric neutrino oscillation data[13], we obtain

$$
\begin{aligned}
m_{\nu 1} &= (5.3^{+0.4}_{-0.3}) \times 10^{-4} \text{ eV}, \\
m_{\nu 2} &= (1.05^{+0.07}_{-0.05}) \times 10^{-2} \text{ eV}, \\
m_{\nu 3} &= (5.22^{+0.35}_{-0.25}) \times 10^{-2} \text{ eV}.
\end{aligned}
\tag{2.16}
$$

The present case predicts

$$R = \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{4\sqrt{6} - 9}{4\sqrt{6} + 9} = 0.0423. \tag{2.17}$$

The predicted value is somewhat larger than the observed value[14, 13]

$$R_{obs} \equiv \frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} = \frac{(7.9^{+0.6}_{-0.5}) \times 10^{-5}\text{eV}^2}{(2.72^{+0.38}_{-0.25}) \times 10^{-3}\text{eV}^2}. = (2.9 \pm 0.5) \times 10^{-2} \tag{2.18}$$

However, the case cannot, at present, be ruled out within three sigma.

If $v_\pi/v_\eta \neq 0$, m_L^ν cannot be diagonal on the basis $(\nu_\pi, \nu_\eta, \nu_\sigma)$. The mixing angle $\theta_{\pi\eta}$ of the further rotation between ν_π-ν_η is given by

$$\tan\theta_{\pi\eta} = v_\pi/v_\eta. \tag{2.19}$$

It is well known that the $2 \leftrightarrow 3$ symmetry[15] is promising for neutrino mass matrix description. We also assume the $2 \leftrightarrow 3$ symmetry for $\langle\phi_i^u\rangle$, i.e. $v_2^u = v_3^u$, which leads to

$$\langle\phi_\pi^u\rangle = 0. \tag{2.20}$$

Therefore, the present model gives the exact tribimaximal mixing:

$$\sin^2 2\theta_{23} = 1, \quad \tan^2\theta_{12} = 1/2, \quad \theta_{13} = 0. \tag{2.21}$$

Note that if we require the $2 \leftrightarrow 3$ symmetry for the fields $\ell_{Li} = (\nu_{Li}, e_{Li})$, the symmetry will affect the charged lepton sector, too. Here, we have assumed the $2 \leftrightarrow 3$ symmetry only for $\langle\phi_i^u\rangle$, not for $\langle\phi_i^d\rangle$, so that the symmetry does not affect the charged lepton mass matrix.

2.5 Summery of the S_3 model

In conclusion, in the S_3 model, the following assumptions have been done:
(i) We have assumed a universal seesaw model.
(ii) The S_3 symmetry in the Yukawa interactions is strictly unbroken. The symmetry S_3 is broken only though the VEV of Higgs scalars ϕ_i.
(iii) We have required the universality of the coupling constants on the basis (e_1, e_2, e_3) for the charged lepton sector, while we have assumed that on the basis $(\nu_\pi, \nu_\eta, \nu_\sigma)$ for the neutrino sector.

As a result of those assumptions, we have obtainde the tribimaximal mixing (2.1) and the charged lepton mass formula (1.5).

3 Prospect of the Charged Lepton Mass Formula

3.1 Brannen's speculations

Recently, Brannen has speculated a neutrino mass relation[16] similar to the charged lepton mass relation (1.5):

$$m_{\nu 1} + m_{\nu 2} + m_{\nu 3} = \frac{2}{3}(-\sqrt{m_{\nu 1}} + \sqrt{m_{\nu 2}} + \sqrt{m_{\nu 3}})^2 . \tag{3.1}$$

Of course, we cannot extract the values of the neutrino mass ratios $m_{\nu 1}/m_{\nu 2}$ and $m_{\nu 2}/m_{\nu 3}$ from the neutrino oscillation data Δm^2_{solar} and Δm^2_{atm} unless we have more information on the neutrino masses, so that we cannot judge whether the observed neutrino masses satisfy the relation (3.1) or not.

Generally, the masses which satisfy the relations (1.5) and (3.1) can be expressed as a bilinear form

$$m_{fi} = (z_{fi})^2 m_{f0}, \tag{3.2}$$

where

$$\begin{array}{l} z_{f1} = \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \sin \xi_f , \\ z_{f2} = \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \sin(\xi_f + \frac{2}{3}\pi), \\ z_{f3} = \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \sin(\xi_f + \frac{4}{3}\pi), \end{array} \tag{3.3}$$

$$(z_{f1})^2 + (z_{f2})^2 + (z_{f3})^2 = 1. \tag{3.4}$$

Then, Brannen has also speculated the relation[16]

$$\xi_\nu = \xi_e + \frac{\pi}{12}. \tag{3.5}$$

From the observed charged lepton mass values, we obtain

$$\xi_e = \frac{\pi}{4} - \varepsilon = 42.7324^\circ \quad (\varepsilon = 2.2676^\circ). \tag{3.6}$$

Therefore, the Brannen relation (3.5) gives

$$\xi_\nu = 57.7324^\circ, \tag{3.7}$$

which predicts

$$R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = 0.0318. \tag{3.8}$$

The value (3.8) is in good agreement with the observed value 0.029 ± 0.005, (2.17). Therefore, the speculations by Brannen are favorable to the observed neutrino data.

We can understand Brannen's first relation (3.1) by assuming two different scalars i.e. $\phi^u \neq \phi^d$. However, Brannen's second relation (3.5) is hard to be derived from the conventional symmetries. it is an open question that the relation (3.5) is accidental or not.[12]

Besides, Brannen[16] and Rosen[17] have speculated that the observed value $\xi_e = 42.7324^\circ$ is given by the relation

$$\xi_e = \frac{\pi}{6} + \frac{2}{9}. \tag{3.9}$$

Since the value 2/9 means

$$\frac{2}{9}\,\text{rad} = 12.732395^\circ, \tag{3.10}$$

the speculation (3.9) is in excellent agreement with the observed value $\xi_e = 42.7324^\circ$ from the charged lepton masses. This is an amazing coincidence.

However, at present, there is no reason for the relation (3.9). Too adhering to this coincidence will again push the formula (1.5) into a mysterious world, so that it is not recommended that we take the relation (3.9) seriously at present.

3.2 From Seesaw to Frogatt-Nielsen

The seesaw model with the three $\mathrm{SU(2)}_L$ doublet scalars ϕ_i causes the flavor changing neutral current (FCNC) problem. Therefore, the seesaw model may, for example, be translated into a Frogatt-Nielsen-type model:

$$H_{eff} = y_e \bar{l}_L H_L^d \frac{\phi^d}{\Lambda_d}\frac{\phi^d}{\Lambda_d} e_R + y_\nu \bar{l}_L H_L^u \frac{\phi^u}{\Lambda_u}\nu_R + y_R \overline{\nu}_R \Phi \nu_R^*. \tag{3.11}$$

The argument about flavor structure is essentially unchanged under the present model-changing. However, the scenario for the symmetry breaking energy scale will be considerably changed: The energy scale of $v_i = \langle \phi_i \rangle$ in the seesaw model is of the order of 10^2 GeV, while that in the Frogatt-Nielsen[18] type model is of the order of 10^{19} GeV. In the Frogatt-Nielsen-type model, the S_3-broken structure of the effective Yukawa coupling constants is given at the Planck mass scale. However, this is no problem, because the formula (1.1) is not so sensitive to the renormalization group equation (RGE) effects as far as the lepton sector is concerned.[19]

3.3 From S_3 to A_4

Recently, Ma[20] has explained the observed tribimaximal mixing on the basis of A_4. This suggests that the A_4 symmetry is also promising as a symmetry of leptons.

3.3.1 Irreducible representations of A_4

When we denote **3** of A_4 as

$$\overline{\psi} = \begin{pmatrix} \overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3 \end{pmatrix} \sim \mathbf{3}, \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \sim \mathbf{3}, \tag{3.12}$$

we can make **1**, **1**', **1**" from 3×3 as follows:

$$\begin{aligned} (\overline{\psi}\psi)_{\mathbf{1}} &= \tfrac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2 + \overline{\psi}_3\psi_3), \\ (\overline{\psi}\psi)_{\mathbf{1}'} &= \tfrac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2\omega + \overline{\psi}_3\psi_3\omega^2), \\ (\overline{\psi}\psi)_{\mathbf{1}"} &= \tfrac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2\omega^2 + \overline{\psi}_3\psi_3\omega), \end{aligned} \tag{3.13}$$

where

$$\omega = e^{i\frac{2}{3}\pi} = \frac{-1 + i\sqrt{3}}{2}. \tag{3.14}$$

When we define

$$\begin{aligned} (\overline{\psi}\psi)_{\sigma} &= \tfrac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2 + \overline{\psi}_3\psi_3), \\ (\overline{\psi}\psi)_{\eta} &= \tfrac{1}{\sqrt{6}}(2\overline{\psi}_1\psi_1 - \overline{\psi}_2\psi_2 - \overline{\psi}_3\psi_3), \\ (\overline{\psi}\psi)_{\pi} &= \tfrac{1}{\sqrt{2}}(\overline{\psi}_3\psi_3 - \overline{\psi}_2\psi_2), \end{aligned} \tag{3.15}$$

we can express (3.13) as

$$\begin{aligned} (\overline{\psi}\psi)_{\mathbf{1}} &= (\overline{\psi}\psi)_{\sigma}, \\ (\overline{\psi}\psi)_{\mathbf{1}'} &= \tfrac{1}{\sqrt{2}}\left[(\overline{\psi}\psi)_{\eta} - i(\overline{\psi}\psi)_{\pi}\right], \\ (\overline{\psi}\psi)_{\mathbf{1}"} &= \tfrac{1}{\sqrt{2}}\left[(\overline{\psi}\psi)_{\eta} + i(\overline{\psi}\psi)_{\pi}\right]. \end{aligned} \tag{3.16}$$

Therefore, if we define $(\phi_\pi, \phi_\eta, \phi_\sigma)$ as

$$\begin{aligned} \phi_{\mathbf{1}} &= \phi_\sigma, \\ \phi_{\mathbf{1}'} &= \tfrac{1}{\sqrt{2}}(\phi_\eta - i\phi_\pi), \\ \phi_{\mathbf{1}"} &= \tfrac{1}{\sqrt{2}}(\phi_\eta + i\phi_\pi), \end{aligned} \tag{3.17}$$

we can write an A_4 invariant Yukawa interaction as follows:

$$\begin{aligned} &(\overline{\psi}\psi)_{\mathbf{1}}\phi_{\mathbf{1}} + (\overline{\psi}\psi)_{\mathbf{1}'}\phi_{\mathbf{1}"} + (\overline{\psi}\psi)_{\mathbf{1}"}\phi_{\mathbf{1}'} \\ &= (\overline{\psi}\psi)_{\sigma}\phi_\sigma + (\overline{\psi}\psi)_{\eta}\phi_\eta + (\overline{\psi}\psi)_{\pi}\phi_\pi. \end{aligned} \tag{3.18}$$

Thus, we can translate the relations in S_3 into those in A_4. The A_4 symmetry will be also promising in the symmetry of the leptons[21].

3.3.2 Higgs potential which gives $v_\pi^2 + v_\eta^2 = v_\sigma^2$

The existence of the λ_2-term, $\phi_\sigma^2(\phi_\pi^2 + \phi_\eta^2)$, in the S_3 Higgs potential model (2.9) was essential for the derivation of the relation (2.7). In an A_4 model, the λ_2-term can be composed as

$$\phi_{\mathbf{1}}\phi_{\mathbf{1}}\left(\phi_{\mathbf{1}'}\phi_{\mathbf{1}"} + \phi_{\mathbf{1}"}\phi_{\mathbf{1}'}\right) = \phi_\sigma^2\left(\phi_\pi^2 + \phi_\eta^2\right). \tag{3.19}$$

Thus, the charged lepton mass relation (1.5) can also be derived from the A_4 model.

3.4 From non-SUSY to SUSY

Since the formula (1.5) is so exact, we need to find a condition which is protected against large corrections. Ma[22] has recently proposed a supersymmetric S_3-invariant Higgs potential, because supersymmetry is unbroken even after the superfields acquire VEVs:

$$W = \frac{1}{2}m\left(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2\right) + \frac{1}{3}\lambda\phi_\sigma^3 + \lambda\phi_\sigma\left(\phi_\pi^2 + \phi_\eta^2\right), \tag{3.20}$$

which also leads to the relation $v_\pi^2 + v_\eta^2 = v_\sigma^2$, (2.7).

By extending this idea, and by applying a symmetry $\Sigma(81)$ to the model, Ma[23] has proposed a new model which leads to the charged lepton mass relation (1.5) :

$$\begin{aligned} W = &\tfrac{1}{2}m_0\chi_0^2 + \tfrac{1}{2}m_1\chi_1^2 + \tfrac{1}{2}m_2\chi_2^2 \\ &+ m_3\chi_1\chi_2 + \tfrac{1}{3}\lambda(\chi_0^3 + \chi_1^3 + \chi_2^3 + 6\chi_0\chi_1\chi_2). \end{aligned} \tag{3.21}$$

This will throw new light on the formula (1.5).

4 Summary

The charged lepton mass formula is not supernatural beings, so that the investigation should be done along the conventional mass matrix approach based on the field theory.

A promising possibility has come up: The charged lepton mass formula (1.5) and the tribimaximal mixing (2.1) are related each other and both can be understood from S_3 or A_4 .

Our next step is to attack the quark masses and mixing by applying the present approach.

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