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Neutrino Masses Without Seesaw Mechanism in a SUSY SU(5) Model with Additional $\bar{5}'_L + 5'_L$

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Abstract

Within the framework of an SU(5) SUSY GUT model, a possible general form of the neutrino mass matrix induced by R-parity violation is investigated. The model has matter fields $\overline{5}'_L + 5'_L$ in addition to the ordinary matter fields $\overline{5}_L + 10_L$ and Higgs fields $H_u + \overline{H}_d$. The R-parity violating terms are given by $\overline{5}_L\overline{5}_L 10_L$, while the Yukawa interactions are given by $\overline{H}_d\overline{5}'_L 10_L$. Since the matter fields $\overline{5}'_L$ and $\overline{5}_L$ are different from each other at the unification scale, the R-parity violation effects at a low energy scale appear only through the $\overline{5}'_L \leftrightarrow \overline{5}_L$ mixings. In order to make this R-parity violation effect harmless for proton decay, a discrete symmetry Z_3 and a triplet-doublet splitting mechanism analogous to that in the 5-plet Higgs fields are assumed.

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1 Introduction

As an origin of the neutrino masses, the idea of the radiative neutrino mass [1] is very interesting as well as the idea of the neutrino seesaw mechanism [2]. However, currently, the latter idea is influential, because it is hard to embed the former model into a grand unification theory (GUT). For example, a supersymmetric (SUSY) model with R-parity violation can provide radiative neutrino masses [3], but the model cannot be embedded into GUT, because the R-parity violating terms induce proton decay inevitably [4].

Recently, the author [5] has proposed a model with R-parity violation within the framework of an SU(5) SUSY GUT: we have quark and lepton fields $\overline{5}_L + 10_L$, which contribute to the Yukawa interactions as $H_u 10_L 10_L$ and $\overline{H}_d \overline{5}_L 10_L$; we also have additional matter fields $\overline{5}'_L + 5'_L$ which contribute to the R-parity violating terms $\overline{5}'_L\overline{5}'_L 10_L$. Since the two $\overline{5}_L$ and $\overline{5}'_L$ are different from each other, the R-parity violating interactions are usually invisible. The R-parity violating effects become visible only through $\overline{5}_L \leftrightarrow \overline{5}'_L$ mixings in low energy phenomena.

In the previous model [5], a discrete symmetry Z_3 has been assumed, and their quantum numbers have been assigned as $\overline{5}_{L(-)} + 10_{L(+)} + \overline{5}'_{L(+)} + 5'_{L(+)}$ and $\overline{H}_{d(0)} + H_{u(+)}$, where we have denoted fields with the transformation properties $\Psi \to \omega^{+1}\Psi$, $\Psi \to \omega^{0}\Psi$ and $\Psi \to \omega^{-1}\Psi$ ($\omega = e^{i2\pi/3}$) as $\Psi_{(+)}$, $\Psi_{(0)}$ and $\Psi_{(-)}$, respectively. Therefore, in the set $\overline{5}_L + 10_L$, the fields $\overline{5}_{L(-)}$ and $10_{L(+)}$ have different transformation properties each other. In contrast to the previous model, in the present paper, we will propose a model with alternative assignments

$$(\overline{5}_L + 10_L)_{(+)} + (\overline{5}'_L + 5'_L)_{(0)} + \overline{H}_{d(-)} + H_{u(+)} \ . \tag{1.1}$$

Although the mechanism of the harmless R-parity violation is the same as the previous model, since the Z_3 quantum number assignment is different from the previous one, the structure of the model is completely different from the previous one.

In the present paper, we will investigate not only the radiatively-induced neutrino masses, but also the contributions from the vacuum expectation values (VEV) of the sneutrinos, $\langle\widetilde{\nu}\rangle$, although in the previous paper the estimate of $\langle\widetilde{\nu}\rangle$ was merely based on an optimistic speculation.

2 Harmless R-parity violation mechanism

Under the Z_3 quantum number assignment (1.1), the Z_3 invariant tri-linear terms in the superpotential are only the following three terms:

$$W_{tri} = (Y_u)_{ij} H_{u(+)} 10_{L(+)i} 10_{L(+)j} + (Y_d)_{ij} \overline{H}_{d(-)} \overline{5}'_{L(0)i} 10_{L(+)j} + \lambda_{ijk} \overline{5}_{L(+)i} \overline{5}_{L(+)j} 10_{L(+)k} \ . \tag{2.1}$$

Similarly, the Z_3 invariant bi-linear terms are only two: $\overline{H}_{d(-)} H_{u(+)}$ and $\overline{5}_{L(0)} H_{L(0)}$. In order to give doublet-triplet splitting, we assume the following "effective" bi-linear terms

$$W_{bi} = \overline{H}_{d(-)} (\mu + g_H \langle\Phi_{(0)}\rangle) H_{u(+)} + \overline{5}'_{L(0)i} (M_5 - g_5 \langle\Phi_{(0)}\rangle) 5'_{L(0)i} + M_i^{SB} \overline{5}_{L(+)i} 5'_{L(0)i} \ , \tag{2.2}$$

where $\Phi_{(0)}$ is a 24-plet Higgs field with the VEV $\langle\Phi_{(0)}\rangle = v_{24}\mathrm{diag}(2,2,2,-3,-3)$, so that, for example, the effective masses $M^{(a)}$ in the term $\overline{5}'^{(a)}_{L(0)} 5'^{(a)}_{L(0)}$ ($5_L^{(2)}$ and $5_L^{(3)}$ denote doublet and triplet components of the fields 5_L, respectively) are given by

$$M^{(2)} = M_5 + 3 g_5 v_{24} \ , \quad M^{(3)} = M_5 - 2 g_5 v_{24} \ . \tag{2.3}$$

The last term in Eq. (2.2) has been added in order to break the Z_3 symmetry softly. We define the $\overline{5}_L \leftrightarrow \overline{5}'_L$ mixing as follows:

$$\begin{aligned} \overline{5}'_{L(0)i} &= c_i \overline{5}^{q\ell}_{Li} + s_i \overline{5}^{heavy}_{Li} \ , \\ \overline{5}_{L(+)i} &= -s_i \overline{5}^{q\ell}_{Li} + c_i \overline{5}^{heavy}_{Li} \ , \end{aligned} \tag{2.4}$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. Then, we can rewrite the second and third terms in Eq. (2.2) as

$$\sum_{a=2,3} \sqrt{(M^{(a)})^2 + (M_i^{SB})^2} \left(\overline{5}^{heavy}_{Li}\right)^{(a)} \left(5^{heavy}_{Li}\right)^{(a)} \ , \tag{2.5}$$

where $5^{heavy}_L = 5'_{L(0)}$ and

$$s_i^{(a)} = \frac{M^{(a)}}{\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}} \ , \quad c_i^{(a)} = \frac{M_i^{SB}}{\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}} \ . \tag{2.6}$$

The fields $\overline{5}^{heavy(a)}_{Li}$ have masses $\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}$, while $\overline{5}^{q\ell(a)}_{Li}$ are massless. We regard $\overline{5}^{q\ell}_{Li} + 10_{L(+)i}$ as the observed quarks and leptons at low energy scale ($\mu < M_{GUT}$). (Hereafter, we will simply denote $\overline{5}^{q\ell}_{Li}$ and $10_{L(+)i}$ as $\overline{5}_{Li}$ and 10_{Li}, respectively.)

Then, the effective R-parity violating terms at $\mu < M_{GUT}$ are given by

$$W_{R\!\!\!/}^{eff} = s_i^{(a)} s_j^{(b)} \lambda_{ijk} \overline{5}_{Li}^{(a)} \overline{5}_{Lj}^{(b)} 10_{Lk} \ . \tag{2.7}$$

In order to suppress the unwelcome term $d_R^c d_R^c u_R^c$ in the effective R-parity violating terms (2.7), we assume a fine tuning

$$M^{(2)} \sim M_{GUT}, \quad M^{(3)} \sim m_{SUSY}, \quad M_i^{SB} \sim M_{GUT} \times 10^{-1}, \tag{2.8}$$

where m_{SUSY} denotes a SUSY breaking scale ($m_{SUSY} \sim 1$ TeV), so that

$$s_i^{(2)} = 1 - O(10^{-2}) \ , \quad c_i^{(2)} \simeq \frac{M_i^{SB}}{M^{(2)}} \sim 10^{-1} \ ; \qquad s_i^{(3)} \simeq \frac{M^{(3)}}{M_i^{SB}} \sim 10^{-12} \ , \quad c_i^{(3)} = 1 - O(10^{-24}) \ . \tag{2.9}$$

Note that in the present model the observed down-quarks $d_{Ri}^c = (\overline{5}_{Li}^{q\ell})^{(3)}$ are given by $(\overline{5}_{Li}^{q\ell})^{(3)} \simeq (\overline{5}'_{L(0)i})^{(3)}$,

while the observed lepton doublets $(\nu_{Li}, e_{Li}) = (\overline{5}_{Li}^{q\ell})^{(2)}$ are given by $(\overline{5}_{Li}^{q\ell})^{(2)} \simeq -(\overline{5}_{L(+)i})^{(2)}$.

From Eq. (2.9), the R-parity violating terms $d_R^c d_R^c u_R^c$ and $d_R^c(e_L u_L - \nu_L d_L)$ are suppressed by $s^{(3)} s^{(3)} \sim 10^{-24}$ and $s^{(3)} s^{(2)} \sim 10^{-12}$, respectively. Thus, proton decay caused by terms $d_R^c d_R^c u_R^c$ and $d_R^c(e_L u_L - \nu_L d_L)$ is suppressed by a factor $(s^{(3)})^3 s^{(2)} \sim 10^{-36}$. On the other hand, radiative neutrino masses are generated by the R-parity violating term $(e_L \nu_L - \nu_L e_L) e_R^c$ with a factor $s^{(2)} s^{(2)} \simeq 1$.

The up-quark masses are generated by the Yukawa interactions (2.1), so that we obtain the up-quark mass matrix M_u as $(M_u)_{ij} = (Y_u)_{ij} v_u$, where $v_u = \langle H_{u(+)}^0 \rangle$. We also obtain the down-quark mass matrix M_d and charged lepton mass matrix M_e as

$$M_d^\dagger = C^{(3)} Y_d v_d \quad M_e^* = C^{(2)} Y_d v_d \ , \tag{2.10}$$

where

$$C^{(a)} = \mathrm{diag}(c_1^{(a)}, c_2^{(a)}, c_3^{(a)}) \ , \tag{2.11}$$

so that

$$M_d^T = \left(C^{(3)} C^{(2)-1} \right)^* M_e \ , \tag{2.12}$$

where $v_d = \langle \overline{H}_{d(-)}^0 \rangle$. Note that M_d^T has a structure different from M_e, because the values of $c_i^{(2)}$ ($i = 1, 2, 3$) can be different from each other. (The idea $M_d^T \neq M_e$ based on a mixing between two $\overline{5}_L$ has been discussed, for example, by Bando and Kugo [6] in the context of an E_6 model.)

3 General form of the neutrino mass matrix

First, we investigate a possible form of the radiatively-induced neutrino mass matrix M_{rad}. In the present model, since we do not have a term which induces $\widehat{e}_R^+ \leftrightarrow \overline{H}_{d(-)}^+$ mixing, there is no Zee-type diagram [1], which is proportional to the Yukawa vertex $(Y_d)_{ij}$ and R-parity violating vertex λ_{ijk} .

Only the radiative neutrino masses in the present scenario come from a charged-lepton loop diagram: the radiative diagram with $(\nu_L)_j \to (e_R)_l + (\widetilde{e}^c_L)_n$ and $(e_L)_k + (\widetilde{e}^c_L)_m \to (\nu^c_L)_i$. The contributions $(M_{rad})_{ij}$ from the charged lepton loop are given, except for the common factors, as follows:

$$(M_{rad})_{ij} = s_i s_j s_k s_n \lambda^*_{ikm} \lambda^*_{jnl} (M_e)^*_{kl} (\widetilde{M}^{2T}_{eLR})^*_{mn} + (i \leftrightarrow j) \ , \tag{3.1}$$

where $s_i = s_i^{(2)}$, and M_e and $\widetilde{M}^2_{eLR}$ are charged-lepton and charged-slepton-LR mass matrices, respectively. (In the present paper, we define the charged lepton mass matrix M_e and the neutrino mass matrix M_ν as $\overline{e}_L M_e e_R$ and $\overline{\nu}_L M_\nu \nu^c_L$, respectively, so that the complex conjugate quantities λ^*_{ijk} and so on have appeared in the expression (3.1).) Since $\widetilde{M}^2_{eLR}$ is proportional to M_e, i.e. $\widetilde{M}^2_{eLR} = (A + \mu^{(2)} \tan\beta) M_e$ ($\mu^{(2)} = \mu - 3 g_H v_{24}$, and A is the coefficient of the soft SUSY breaking terms $(Y_d)_{ij} (\tilde{\nu}, \tilde{e})^T_{Li} \tilde{e}^c_{Lj} \overline{H}_d$ with $A \sim 1$ TeV), we obtain

$$(M_{rad})_{ij} = 2(A + \mu^{(2)} \tan\beta) s_i s_j s_k s_n \lambda^*_{ikm} \lambda^*_{jnl} (M_e)^*_{kl} (M_e)^*_{nm} \ . \tag{3.2}$$

Since the coefficient λ_{ijk} is antisymmetric in the permutation $i \leftrightarrow j$, it is useful to define

$$\lambda_{ijk} = \varepsilon_{ijl} L_{lk} \ , \tag{3.3}$$

and

$$K = (S M_e L^T)^* \ . \tag{3.4}$$

Then, the radiative neutrino mass matrix is given by

$$(M_{rad})_{ij} = m_0^{-1} s_i s_j \varepsilon_{ikm} \varepsilon_{jln} K_{ml} K_{nk} \ . \tag{3.5}$$

The coefficient m_0^{-1} is calculated from one-loop diagram (Fig.1) as

$$m_0^{-1} = \frac{2}{16\pi^2} (A + \mu^{(2)} \tan\beta) F(m^2_{\tilde{e}_R}, m^2_{\tilde{e}_L}) \ , \tag{3.6}$$

where

$$F(m_a^2, m_b^2) = \frac{1}{m_a^2 - m_b^2} \ln \frac{m_a^2}{m_b^2} \ . \tag{3.7}$$

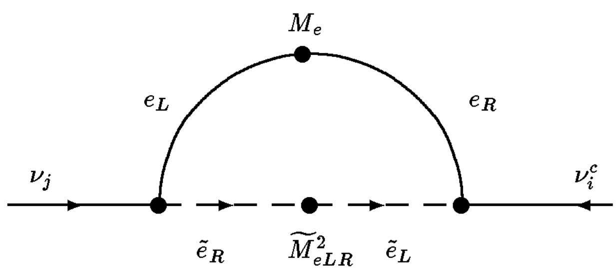

Figure 1: Radiative generation of neutrino Majorana mass

Next, let us investigate the contributions from the VEVs of sneutrinos $\langle\widetilde{\nu}_i\rangle$. In general, the sneutrinos $\widetilde{\nu}_i$ can have VEVs $v_i \equiv \langle\widetilde{\nu}_i\rangle \neq 0$ [7], if there are one or more of the following terms: $\mu_i \overline{5}_{Li} H_u$ in superpotential W, and $B_i \overline{5}_{Li} H_u + m^2_{HLi} \overline{5}_{Li} \bar{H}_d^\dagger$ in the bilinear soft SUSY breaking terms V_{soft}. In the present model, there is no such a term at tree level, because these terms are forbidden by the Z_3 symmetry. However, only an effective m^2_{HLi}-term can appear via the loop diagram $\overline{H}_d \to (\overline{5}^{ql}_L)^c + (10_L)^c \to \overline{5}^{ql}_L$ (Fig. 2). The contribution m^2_{HLi} is proportional to

$$s_i s_j \lambda_{ijk} (M_e)_{jk} = s_i \varepsilon_{ijk} K^*_{jk} \ . \tag{3.8}$$

On the other hand, the contribution M_{VEV} from $\langle\widetilde{\nu}_i\rangle \neq 0$ to the neutrino mass matrix is proportional to

$$\begin{pmatrix} v_1^2 & v_1 v_2 & v_1 v_3 \\ v_1 v_2 & v_2^2 & v_2 v_3 \\ v_1 v_3 & v_2 v_3 & v_3^2 \end{pmatrix} , \tag{3.9}$$

and $v_i \equiv \langle\widetilde{\nu}_i\rangle$ are proportional to the values $(m^2_{HLi})^*$, so that the mass matrix M_{VEV} is given by

$$(M_{VEV})_{ij} = \xi m_0^{-1} s_i s_j \varepsilon_{ikl} \varepsilon_{jmn} K_{kl} K_{mn} \ , \tag{3.10}$$

where ξ is a relative ratio of M_{VEV} to M_{rad}.

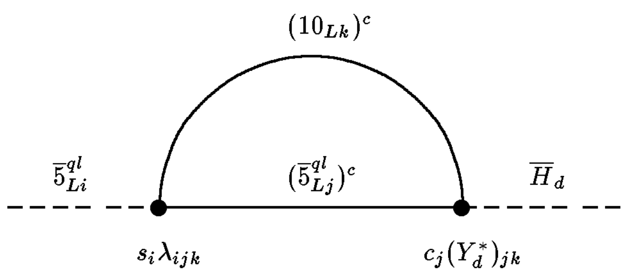

Figure 2: Effective $\overline{5}^{ql}_L \overline{H}_d^\dagger$ term

In conclusion, the neutrino mass matrix M_ν in the present model is given by the form

$$(M_\nu)_{ij} = m_0^{-1} s_i s_j \varepsilon_{ikl} \varepsilon_{jmn} \left(K_{kn} K_{ml} + \xi K_{kl} K_{mn} \right) \ , \tag{3.11}$$

i.e.

$$\begin{aligned} M_\nu \ &= \ m_0^{-1} S \left\{ \left[(K - K^T)(K - K^T) - \mathbf{1}\mathrm{Tr}(KK - KK^T) \right] (1 + \xi) \right. \\ &\quad \left. + \left[(K + K^T) - \mathbf{1}\mathrm{Tr}K \right] \mathrm{Tr}K - (KK + K^T K^T) + \mathbf{1}\mathrm{Tr}(KK) \right\} S \ , \end{aligned} \tag{3.12}$$

where $\mathbf{1}$ is a 3×3 unit matrix.

4 General features of the neutrino mass matrix

In the present model, if the charged lepton mass matrix M_e and the structure of λ_{ijk} (i.e. L_{ij}) are given, then we can obtain $K = (S M_e L^T)^*$, so that we can predict neutrino masses and mixings. However, at present, we have many unknown parameters, so that in order to give explicit predictions of the neutrino masses and mixings, we must put a further assumption on the parameters K_{ij}. In the present section, we investigate general features of the neutrino mass matrix (3.11) [or (3.12)] without making any explicit assumptions about flavor symmetries.

So far, the expression of M_ν, (3.12), has been given in the initial flavor basis, where $\overline{5}_{L(+)} \leftrightarrow \overline{5}'_{L(0)}$ mixings have been taken place a diagonal form

$$S^{(a)} = \mathrm{diag}(s_1^{(a)}, s_2^{(a)}, s_3^{(a)}) \ , \quad C^{(a)} = \mathrm{diag}(c_1^{(a)}, c_2^{(a)}, c_3^{(a)}) \ , \tag{4.1}$$

and the matrix K has been defined by Eq. (3.4), $K = (SM_eL^T)^*$. Since S, M_e and L are transformed as

$$\begin{array}{rcl} M_e & \to & M'_e = U_5^\dagger M_e U_{10}^* \ , \\ L & \to & L' = U_5^\dagger L U_{10} \ , \\ S & \to & S' = U_5^\dagger S U_5 \ , \end{array} \tag{4.2}$$

under a rotation of the flavor basis

$$10_L \to 10'_L = U_{10}^\dagger 10_L \ , \quad \overline{5}_L^{ql} \to (\overline{5}_L^{q\ell})' = U_5^\dagger \overline{5}_L^{q\ell} \ , \tag{4.3}$$

the matrix K transforms as

$$K \to K' = U_5^T \ K \ U_5 \ . \tag{4.4}$$

We have a great interest in the form of M'_ν in the flavor basis with $M'_e = D_e \equiv \mathrm{diag}(m_e, m_\mu, m_\tau)$. Hereafter, we denote the quantities M'_ν, K', and so on in the $M'_e = D_e$ basis as $\widehat{M}_e$, $\widehat{K}$ and so on, respectively. The matrix $\widehat{K}$ is expressed as

$$\widehat{K} = \widehat{S} D_e \widehat{L}^\dagger \simeq D_e (U_R^e)^\dagger L^\dagger (U_L^e) \ , \tag{4.5}$$

where $U_5 = U_L^e$ and $U_{10} = U_R^e$, and we have put $\widehat{S} \simeq \mathbf{1}$ because of $S \simeq \mathbf{1}$ as we have assumed in Eq. (2.9).

Here, let us summarize general features of the present neutrino mass matrix (3.12).

(i) If the matrix K defined by Eq. (3.4) satisfies $K^T = K$ in the initial basis, the matrix K' in the arbitrary basis also satisfies $K'^T = K'$, so that the present model gives $\langle \widetilde{\nu}'_i \rangle = 0$ in the arbitrary basis. For such a case, the neutrino mass matrix is simply given by

$$M_\nu = -m_0^{-1} S \left[2KK - 2K\mathrm{Tr}K - \mathrm{Tr}(KK) + (\mathrm{Tr}K)^2 \right] S \ . \tag{4.6}$$

(ii) When K is symmetric under the flavor $2 \leftrightarrow 3$ permutation, the neutrino mass matrix M_ν is also symmetric under the $2 \leftrightarrow 3$ permutation. It is well-known [8] that when the neutrino mass matrix $\widehat{M_\nu}$ is symmetric under the $2 \leftrightarrow 3$ permutation, the mass matrix $\widehat{M_\nu}$ gives a nearly bimaximal mixing, i.e. $\sin^2 2\theta_{23} = 1$ and $|U_{13}|^2 = 0$, which are favorable to the observed atmospheric [9], K2K [10] and CHOOZ [11] data. In the present model, the $2 \leftrightarrow 3$ symmetry of $\widehat{M_\nu}$ means that the parameters

$$\widehat{K}_{ij} = K_{kl} (U_L^e)_{ki} (U_L^e)_{lj} \ , \tag{4.7}$$

are symmetric under the $2 \leftrightarrow 3$ permutation. In other words, the $2 \leftrightarrow 3$ symmetry of $\widehat{M_\nu}$ is due to special structures of U_L^e and K. For example, when K and U_L^e are given by the textures

$$K = \begin{pmatrix} K_{11} & 0 & 0 \\ 0 & K_{22} & K_{23} \\ 0 & K_{32} & K_{33} \end{pmatrix} \ , \tag{4.8}$$

$$U_L^e = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -s & \frac{1}{\sqrt{2}}c & \frac{1}{\sqrt{2}}c \\ c & \frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}s \end{pmatrix} , \tag{4.9}$$

the matrix $\widehat{K}$ is $2 \leftrightarrow 3$ symmetric:

$$\widehat{K} = \begin{pmatrix} f & a & a \\ a' & g & b \\ a' & b & g \end{pmatrix} , \tag{4.10}$$

so that the neutrino mass matrix $\widehat{M}_\nu$ is also $2 \leftrightarrow 3$ symmetric:

$$\begin{aligned} (\widehat{M}_\nu)_{11} &= -2(g^2 - b^2) m_0^{-1} , \\ (\widehat{M}_\nu)_{12} &= (\widehat{M}_\nu)_{13} = (\widehat{M}_\nu)_{21} = (\widehat{M}_\nu)_{31} = (a + a')(g^2 - b^2) m_0^{-1} , \\ (M_\nu)_{22} &= (\widehat{M}_\nu)_{33} = \left[(a - a')^2 (1 + \xi) + 2(aa' - fg) \right] m_0^{-1} , \\ (\widehat{M}_\nu)_{23} &= (\widehat{M}_\nu)_{32} = -\left[(a - a')^2 (1 + \xi) + 2(aa' - fb) \right] m_0^{-1} , \end{aligned} \tag{4.11}$$

and K in the initial basis is given by

$$\begin{aligned} K_{11} &= g - b , \\ K_{22} &= (g + b)c^2 - \sqrt{2}(a + a')cs + fs^2 , \\ K_{33} &= (g + b)s^2 + \sqrt{2}(a + a')cs + fc^2 , \\ K_{23} &= \sqrt{2}(-as^2 + a'c^2) + (g + b - f)cs , \\ K_{32} &= \sqrt{2}(ac^2 - a's^2) + (g + b - d)cs . \end{aligned} \tag{4.12}$$

Finally, let us show a simple example which is suggested by above comments (i) and (ii). We assume that $M_e M_e^\dagger$ on the initial basis is $2 \leftrightarrow 3$ symmetric:

$$M_e M_e^\dagger = \begin{pmatrix} F & A & A \\ A & G & B \\ A & B & G \end{pmatrix} , \tag{4.13}$$

so that U_L^e has a form of a nearly bimaximal mixing. For simplicity, we assume that U_L^e is given by the full bimaximal mixing form

$$U_L^e = (U_L^e)^T = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} , \tag{4.14}$$

which demands the constraint $F = B + G$ on the matrix (4.13). Then, the eigenvalues $D_e^2 = \mathrm{diag}(m_e^2,\ m_\mu^2,\ m_\tau^2)$

are given by

$$
\begin{aligned}
m_e^2 &= G - B \ , \\
m_\mu^2 &= G + B - \sqrt{2}A \ , \\
m_\tau^2 &= G + B + \sqrt{2}A \ ,
\end{aligned}
\tag{4.15}
$$

On the other hand, we assume that K in the initial basis is given by the form (4.8) with $K_{23} = K_{32}$, so that we obtain $a = a'$ and

$$
\widehat{M}_\nu = 2m_0^{-1} \begin{pmatrix} -(b^2-g^2) & a(g-b) & a(g-b) \\ a(g-b) & a^2 - fg & -(a^2 - fb) \\ a(g-b) & -(a^2-fb) & a^2 - fg \end{pmatrix} \ . \tag{4.16}
$$

Note that the mass matrix (4.16) does not include the contributions (ξ-terms) from nonvanishing sneutrino VEVs because of $K^T = K$. The mass matrix (4.16) gives the following eigenvalues and mixings:

$$
\begin{aligned}
m_{\nu 1} &= (g-b)\left[\sqrt{9(g+b)^2 + 2f(g+b) + f^2} - (g+b+f)\right] m_0^{-1} \ , \\
-m_{\nu 2} &= -(g-b)\left[\sqrt{9(g+b)^2 + 2f(g+b) + f^2} + g+b+f\right] m_0^{-1} \ , \\
m_{\nu 3} &= -2\left[2a^2 - (g+b)f\right] m_0^{-1} \ ,
\end{aligned}
\tag{4.17}
$$

$$
\widehat{U}_\nu = \begin{pmatrix} c_\nu & s_\nu & 0 \\ \frac{1}{\sqrt{2}}s_\nu & -\frac{1}{\sqrt{2}}c_\nu & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s_\nu & -\frac{1}{\sqrt{2}}c_\nu & -\frac{1}{\sqrt{2}} \end{pmatrix} \ , \tag{4.18}
$$

$$
s_\nu = \sqrt{\frac{m_{\nu 1}}{m_{\nu 1} + m_{\nu 2}}} \ , \quad c_\nu = \sqrt{\frac{m_{\nu 2}}{m_{\nu 1} + m_{\nu 2}}} \ , \tag{4.19}
$$

so that we obtain

$$
\tan^2 \theta_{solar} = \frac{m_{\nu 1}}{m_{\nu 2}} \ , \tag{4.20}
$$

together with $\sin^2 2\theta_{atm} = 1$ and $|U_{13}|^2 = 1$. For a further simple case with $f = 0$, which demands

$$
K_{23} = K_{32} = \frac{1}{2}(K_{33} + K_{22}) \ , \tag{4.21}
$$

we obtain $m_{\nu 1} = m_{\nu 2}/2 = 2(g^2 - b^2)m_0^{-1}$, so that

$$
\tan^2 \theta_{solar} = \frac{1}{2} \ , \tag{4.22}
$$

$$R \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{3}{4}\frac{(g^2-b^2)^2}{a^4-(g^2-b^2)^2} , \tag{4.23}$$

where we have considered

$$a^2 = \frac{1}{8}(K_{33}-K_{22})^2 \gg g^2 - b^2 = K_{11}^2(K_{33}+K_{22})^2 . \tag{4.24}$$

The result (4.22) is favorable to the recent solar [12] and KamLAND data [13]. Of course, this is only an example, and the result (4.22) is not a prediction which is inevitably driven from the general form of M_ν.

5 Summary

In conclusion, within the framework of a SUSY GUT model, we have proposed an R-parity violation mechanism which is harmless for proton decay and investigated a general form of the neutrino mass matrix M_ν. As we have given in Eq. (3.12), the form of M_ν is described in terms of the matrix K defined in Eq. (3.4). (i) If $K^T = K$, the VEVs of sneutrinos are exactly zero, $\langle\widetilde{\nu}_i\rangle = 0$, in the arbitrary basis, so that M_ν is given only by the radiative contributions. (ii) If $\widehat{K}$ is $2 \leftrightarrow 3$ symmetric, then $\widehat{M_\nu}$ is also $2 \leftrightarrow 3$ symmetric, so that $\widehat{M_\nu}$ can predict $\sin^2 2\theta_{atm} = 1$ and $|U_{13}|^2 = 0$.

In order to demonstrate that the general form indeed has a phenomenologically favorable parameter range, we have given a simple example of K and $M_e M_e^\dagger$ in the last part of the section 4. Although such a simple form of K, (4.8), with the constraint (4.23) is likely, the investigation of the origin of the possible form K will be our next task. The purpose of the present paper is not to give a special model for neutrino phenomenology, and it is to demonstrate that it is indeed possible to build a neutrino mass matrix model with R-parity violation, i.e. without a seesaw mechanism, even if the model is within a framework of GUT.

The present model has assigned Z_3 quantum numbers to the superfields differently from those in the previous model [5] with $\overline{5}_L \leftrightarrow \overline{5}'_L$ mixing: we have been able to assign the same Z_3 quantum number to the matter fields $\overline{5}_L$ and 10_L (and also to $\overline{5}'_L$ and $5'_L$). This re-assignment will give fruitful potentiality for a further extension of the present model.

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