

# Radiative Neutrino Masses in a SUSY GUT Model

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## Abstract

Radiatively-induced neutrino mass matrix is investigated within the framework of an SU(5) SUSY GUT model. The model has matter fields of three families  $\bar{5}_{L(+);i} + 5_{L(+);i}$  in addition to the ordinary matter fields  $\bar{5}_{L(-);i} + 10_{L(+);i}$  and Higgs fields  $H_{(+)} + \bar{H}_{(0)}$ , where  $(+, 0, -)$  denote the transformation properties  $(\omega^{+1}, \omega^0, \omega^{-1})$  ( $\omega^3 = -1$ ) under a discrete symmetry  $Z_3$ .  $R$ -parity violating terms are given by  $\bar{5}_{L(+)}\bar{5}_{L(+)}10_{L(+)}$ , while the Yukawa interactions are given by  $\bar{H}_{(0)}\bar{5}_{L(-)}10_{L(+)}$ , i.e. the  $\bar{5}$ -fields in both are different from each other. The  $Z_3$  symmetry is only broken by the terms  $\bar{5}_{L(+);i}5_{L(+);i}$  softly, so that the  $\bar{5}_{L(+);i} \leftrightarrow \bar{5}_{L(-);i}$  mixings appear at  $\mu < M_X$ . Of the  $R$ -parity violating terms  $\bar{5}_{L(+)}\bar{5}_{L(+)}10_{L(+)}$ , only the terms  $(e_L\nu_{-}\nu_L e)_R^c$  sizably appear at  $\mu < M_X$ .

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## 1 Introduction

The idea of the radiative neutrino mass [1] is an antithesis to the idea of the neutrino seesaw mechanism [2]: in the latter model, the neutrinos acquire Dirac masses of the same order as quark and charged lepton masses and the smallness of the observed neutrino masses is explained by the seesaw mechanism due to large Majorana masses of the right-handed neutrinos  $\nu_R$ , while, in the former model, there are no right-handed neutrinos, so that there are no Dirac mass terms, and small Majorana neutrino masses are generated radiatively. Currently, the latter idea is influential, because it is hard to embed the former model into a grand unification theory (GUT). A supersymmetric (SUSY) model with  $R$ -parity violation can provide neutrino masses [3], but the model cannot be embedded into GUT, because the  $R$ -parity violating terms induce proton decay inevitably [4].

Recently, Sato and the author [5] have proposed a model with  $R$ -parity violation within the framework of an SU(5) SUSY GUT. In the model, there are no  $R$ -parity violating terms  $\bar{5}_L\bar{5}_L10_L$  ( $\bar{5}_L$  and  $10_L$  denote  $\bar{5}_L$ -plet and  $10$ -plet matter fields in SU(5) SUSY GUT), which are forbidden by a discrete symmetry  $Z_2$ . At  $\mu < M_X$  ( $M_X$  is a unification scale of the SU(5) GUT), the  $Z_2$  symmetry is softly broken, and  $\bar{H}_d \leftrightarrow \bar{5}_L$  mixing is induced, so that the  $R$ -parity violation terms  $\bar{5}_L\bar{5}_L10_L$  are effectively induced from the Yukawa interactions  $\bar{H}_d\bar{5}_L10_L$ .

In contrast to the above scenario, in the present paper, we propose another model with  $R$ -parity violation within the framework of an SU(5) SUSY GUT: we have quark and lepton fields  $\bar{5}_L + 10_L$ , which contribute to the Yukawa interactions as  $H_u10_L10_L$  and  $\bar{H}_d\bar{5}_L10_L$ ; we also have additional matter fields  $\bar{5}'_L + 5'_L$  which contribute to the  $R$ -parity violating terms  $\bar{5}'_L\bar{5}'_L10_L$ . Since the two  $\bar{5}_L$  and  $\bar{5}'_L$  are different from each other, the  $R$ -parity violating interactions are usually invisible. The  $R$ -parity violating effects become visible only through  $\bar{5}_L \leftrightarrow \bar{5}'_L$  mixings in low energy phenomena.

In order to make such a scenario, i.e. in order to allow the interactions  $\bar{5}'_L \bar{5}'_L 10_L$ , but to forbid  $\bar{5}_L \bar{5}_L 10_L$  and  $\bar{5}_L \bar{5}'_L 10_L$ , we introduce a discrete symmetry  $Z_3$ . (We cannot build such a model by using  $Z_2$  symmetry.) We denote fields with the transformation properties  $\Psi \rightarrow \omega^{+1}\Psi$ ,  $\Psi \rightarrow \omega^0\Psi$  and  $\Psi \rightarrow \omega^{-1}\Psi$  ( $\omega^3 = -1$ ) as  $\Psi_{(+)}$ ,  $\Psi_{(0)}$  and  $\Psi_{(-)}$ , respectively. We consider matter fields  $\bar{5}_{L(-)i} + 10_{L(+)i}$  ( $i = 1, 2, 3$ : family indices) which contribute the Yukawa interactions as

$$W_Y = (Y_u)_{ij} H_{(+)} 10_{L(+)i} 10_{L(+)j} + (Y_d)_{ij} \bar{H}_{(0)} \bar{5}_{L(-)i} 10_{L(+)j} , \quad (1.1)$$

and additional matter fields  $\bar{5}_{L(+)i} + 5_{L(+)i}$  which contribute the  $R$ -parity interactions as

$$W_{\mathcal{H}} = \lambda_{ijk} \bar{5}_{L(+)i} \bar{5}_{L(+)j} 10_{L((+)k)} . \quad (1.2)$$

The  $R$ -parity violating interactions  $\bar{5}_{L(-)} \bar{5}_{L(-)} 10_{L((+)}$  and  $\bar{5}_{L(-)} \bar{5}_{L(+) } 10_{L((+)}$  are forbidden by the  $Z_3$  symmetry.

In order to give  $\bar{5}_{L(-)} \leftrightarrow \bar{5}_{L(+)}$  mixings,

$$\begin{aligned} \bar{5}_{L(-)i} &= c_i \bar{5}_{Li} + s_i \bar{5}'_{Li} , \\ \bar{5}_{L(+)i} &= -s_i \bar{5}_{Li} + c_i \bar{5}'_{Li} , \end{aligned} \quad (1.3)$$

where  $s_i = \sin \theta_i$  and  $c_i = \cos \theta_i$ , we consider a superpotential

$$W_5 = [\bar{5}_{L(-)i} (M_5 - g_5 \Phi_{(0)}) + M_i^{SB} \bar{5}_{L(+)i}] 5_{L(+)i} , \quad (1.4)$$

where  $\Phi_{(0)}$  is a 24-plet Higgs field with the vacuum expectation value (VEV)  $\langle \Phi_{(0)} \rangle = v_{24} \text{diag}(2, 2, 2, -3, -3)$ , which gives doublet-triplet splitting in the mass terms  $\bar{5}_{L(-)i} 5_{L(+)i}$  at  $\mu < M_X$ , i.e.

$$M^{(2)} = M_5 + 3g_5 v_{24} , \quad M^{(3)} = M_5 - 2g_5 v_{24} . \quad (1.5)$$

The discrete symmetry  $Z_3$  is softly broken by the  $M_i^{SB}$ -terms in (1.4). Then, we obtain

$$W_5 = \sum_{a=2,3} \sqrt{(M^{(a)})^2 + (M_i^{SB})^2} \bar{5}_{Li}^{(a)} 5_{L(+)i}^{(a)} , \quad (1.6)$$

where the index  $(a)$  denotes that the field  $\Psi^{(a)}$  with  $a = 2$  ( $a = 3$ ) is a doublet (triplet) component of  $SU(5) \rightarrow SU(2) \times SU(3)$ , and

$$s_i^{(a)} = \frac{M^{(a)}}{\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}} , \quad c_i^{(a)} = \frac{M_i^{SB}}{\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}} . \quad (1.7)$$

The field  $\bar{5}_{Li}^{(a)}$  has a mass  $\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}$ , while  $\bar{5}_{Li}^{(a)}$  are massless. We regard  $\bar{5}_{Li} + 10_{L(+)i}$  as the observed quarks and leptons at low energy scale ( $\mu < M_X$ ). Then, the effective  $R$ -parity violating terms at  $\mu < M_X$  are given by

$$W_{\mathcal{H}}^{eff} = s_i^{(a)} s_j^{(b)} \lambda_{ijk} \bar{5}_{Li}^{(a)} \bar{5}_{Lj}^{(b)} 10_{L(+)k} . \quad (1.8)$$

In order to suppress the unwelcome term  $d_R^c d_R^c u_R^c$  in the effective  $R$ -parity violating terms (1.8), we assume a fine tuning

$$M^{(2)} \sim M_X , \quad M^{(3)} \sim m_{SUSY} , \quad M_i^{SB} \sim M_X \times 10^{-1} , \quad (1.9)$$

so that

$$s_i^{(2)} \simeq 1, \quad c_i^{(2)} \simeq \frac{M_i^{SM}}{M^{(2)}} \sim 10^{-1}; \quad s_i^{(3)} \simeq \frac{M^{(3)}}{M_i^{SM}} \sim 10^{-12}, \quad c_i^{(3)} \simeq 1. \quad (1.10)$$

Therefore, the  $R$ -parity violating terms  $d_R^c d_R^c u_R^c$  and  $d_R^c (e_L u_L - \nu_L d_L)$  are suppressed by  $s^{(3)} s^{(3)} \sim 10^{-24}$  and  $s^{(3)} s^{(2)} \sim 10^{-12}$ , respectively. Thus, proton decay caused by terms  $d_R^c d_R^c u_R^c$  and  $d_R^c (e_L u_L - \nu_L d_L)$  is suppressed by a factor  $(s^{(3)})^3 s^{(2)} \sim 10^{-36}$ . On the other hand, radiative neutrino masses are generated by the  $R$ -parity violating term  $(e_L \nu_L - \nu_L e_L) e_R^c$  with a factor  $s^{(2)} s^{(2)} \simeq 1$ . The numerical choice (1.9) gives

$$\begin{aligned} m(\bar{5}_{Li}^{(2)}) &\simeq M^{(2)} \sim M_X, \\ m(\bar{5}_{Li}^{(3)}) &\simeq M_i^{SB} \sim M_X \times 10^{-1}. \end{aligned} \quad (1.11)$$

Since  $m(\bar{5}_{Li}^{(3)}) < M_X$ , the triplet fields  $\bar{5}_{Li}^{(3)}$  can basically contribute to the renormalization group equation (RGE) effects at  $\mu < M_X$ . However, since we consider  $M_i^{SM} \sim M_X \times 10^{-1}$ , the numerical effect does almost not spoil the gauge-coupling-constant unification at  $\mu = M_X \sim 10^{16}$  GeV.

The up-quark masses are generated by the Yukawa interactions (1.1), so that we obtain the up-quark mass matrix  $M_u$

$$(M_u)_{ij} = (Y_u)_{ij} v_u, \quad (1.12)$$

where  $v_u = \langle H_{(+)}^0 \rangle$ . From the Yukawa interaction (1.1), we also obtain the down-quark mass matrix  $M_d$  and charged lepton mass matrix  $M_e$  as

$$(M_d^\dagger)_{ij} = c_i^{(3)} (Y_d)_{ij} v_d, \quad (M_e^*)_{ij} = c_i^{(2)} (Y_d)_{ij} v_d, \quad (1.13)$$

i.e.

$$(M_d^\dagger)_{ij} = (c_i^{(3)} / c_i^{(2)}) (M_e^*)_{ij}, \quad (1.14)$$

where  $v_d = \langle \bar{H}_{(0)}^0 \rangle$ . Note that  $M_d^T$  has a structure different from  $M_e$ , because the values of  $c_i^{(2)}$  can be different from each other. (The idea  $M_d^T \neq M_e$  based on a mixing between two  $\bar{5}_L$  has been discussed, for example, by Bando and Kugo [6] in the context of an  $E_6$  model.)

In order to give doublet-triplet splitting for the Higgs fields  $H_{(+)}$  and  $\bar{H}_{(0)}$ , we assume the ‘‘missing partner mechanism’’ [7]: for example, we consider

$$W_H = \lambda H_{(+)} \bar{H}_{50(-)} \langle H_{75(0)} \rangle + \bar{\lambda} \bar{H}_{(0)} H_{50(0)} \langle H_{75(0)} \rangle, \quad (1.15)$$

which gives mass to the triplets in  $H_{(+)} + \bar{H}_{(0)}$ , but not to the doublets, where  $H_{50(0)}$  ( $\bar{H}_{50(-)}$ ) and  $H_{75(0)}$  are 50-plet and 75-plet Higgs fields, respectively.

## 2 Radiative neutrino mass matrix

In this section, we investigate a possible form of the radiatively-induced neutrino mass matrix  $M_{rad}$ . Contribution from non-zero VEVs of sneutrinos  $\langle \tilde{\nu} \rangle \neq 0$  to the neutrino mass matrix will be discussed in the next section.

In the present model, since we do not have a term which induces  $\widehat{e}_R^+ \leftrightarrow \overline{H}_{(0)}^+$  mixing, there is no Zee-type diagram [1], which is proportional to the Yukawa vertex  $(Y_d)_{ij}$  and R-parity violating vertex  $\lambda_{ijk}$ . (The  $\widehat{e}_R^+ \leftrightarrow \overline{H}_{(0)}^+$  mixing can come from interactions of a type  $\overline{H} \overline{H} 10_{L(+)}$ . However, in the present model,  $\overline{5}$ -plet Higgs fields are only on type  $\overline{H}_{(0)}$ . Therefore, the combination  $\overline{H}_{(0)} \overline{H}_{(0)} 10_{L(+)}$  is forbidden because of the antisymmetric property of SU(5) 10-plet fields  $10_{L(+)}$ . Even after the SU(5) is broken,  $\overline{H}_{(0)}^{(2)} \overline{H}_{(0)}^{(2)}$  cannot couple to the SU(2) singlet  $\widehat{e}_R^+$  because SU(2) singlet composed of  $2 \times 2$  must be antisymmetric. Therefore, we cannot bring the  $\overline{H}_{(0)}^{(2)} \overline{H}_{(0)}^{(2)} \widehat{e}_R^+$  term even as a soft supersymmetry breaking term.)

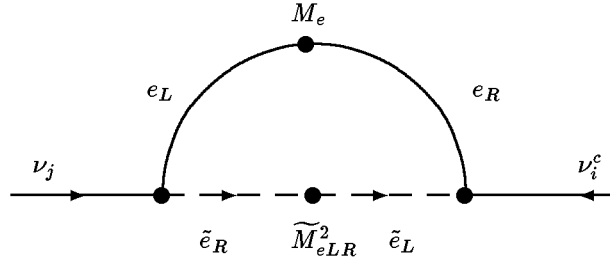


Figure 1: Radiative generation of neutrino Majorana mass

Only the radiative neutrino masses in the present scenario come from a charged-lepton loop diagram: the radiative diagram with  $(\nu_L)_j \rightarrow (e_R)_l + (\tilde{e}_L^c)_n$  and  $(e_L)_k + (\tilde{e}_L^c)_m \rightarrow (\nu_L^c)_i$ . The contributions  $(M_{rad})_{ij}$  from the charged lepton loop are given, except for the common factors, as follows:

$$(M_{rad})_{ij} = s_i s_j s_k s_n \lambda_{ikm} \lambda_{jnl} (M_e)_{kl} (\widetilde{M}_{eLR}^{2T})_{mn} + (i \leftrightarrow j), \quad (2.1)$$

where  $s_i = s_i^{(2)}$ ,  $m_i = m(e_i) = (m_e, m_\mu, m_\tau)$  and  $M_e$  and  $\widetilde{M}_{eLR}^2$  are charged-lepton and charged-slepton-LR mass matrices, respectively. Since  $\widetilde{M}_{eLR}^2$  is proportional to  $M_e$ , i.e.,  $\widetilde{M}_{eLR}^2 = A M_e$ , we obtain

$$(M_{rad})_{ij} = 2 A s_i s_j s_k s_n \lambda_{ikm} \lambda_{jnl} (M_e)_{kl} (M_e)_{nm}. \quad (2.2)$$

Therefore, the mass matrix  $M_{rad}$  on the basis with  $M_e = D_e \equiv \text{diag}(m_e, m_\mu, m_\tau)$  is given by

$$(M_{rad})_{ij} = m_0 s_i s_j s_k s_l \lambda_{ikl} \lambda_{jlk} \frac{m_k m_l}{m_3^2}, \quad (2.3)$$

where

$$m_0 = \frac{2}{16\pi^2} A m_3^2 F(m_{\tilde{e}_R}^2, m_{\tilde{e}_L}^2), \quad (2.4)$$

$$F(m_a^2, m_b^2) = \frac{1}{m_a^2 - m_b^2} \ln \frac{m_a^2}{m_b^2}. \quad (2.5)$$

Since the coefficient  $\lambda_{ijk}$  is antisymmetric in the permutation  $i \leftrightarrow j$ , it is useful to define

$$\lambda_{ijk} = \varepsilon_{ijl} h_{lk} , \quad (2.6)$$

and

$$H_{ij} = h_{ij} m_j s_j . \quad (2.7)$$

Then, we can rewrite (2.4) as

$$(M_{rad})_{ij} = \frac{m_0}{m_3^2} s_i s_j \varepsilon_{ikm} \varepsilon_{jln} H_{ml} H_{nk} . \quad (2.8)$$

The expression (2.8) is explicitly given as follows:

$$M_{11} = s_1^2 [H_{23}^2 + H_{32}^2 - 2H_{22}H_{33}] , \quad (2.9)$$

$$M_{22} = s_2^2 [H_{31}^2 + H_{13}^2 - 2H_{33}H_{11}] , \quad (2.10)$$

$$M_{33} = s_3^2 [H_{12}^2 + H_{21}^2 - 2H_{11}H_{22}] , \quad (2.11)$$

$$M_{12} = M_{21} = s_1 s_2 [(H_{12} + H_{21})H_{33} - H_{23}H_{13} - H_{32}H_{31}] , \quad (2.12)$$

$$M_{13} = M_{31} = s_1 s_3 [(H_{13} + H_{31})H_{22} - H_{23}H_{21} - H_{32}H_{12}] , \quad (2.13)$$

$$M_{23} = M_{32} = s_2 s_3 [(H_{23} + H_{32})H_{11} - H_{31}H_{21} - H_{13}H_{12}] , \quad (2.14)$$

where  $M_{ij} \equiv (M_{rad})_{ij}$  and we have dropped a common factor  $m_0/m_3^2$ . As discussed in (1.10), in a phenomenological investigation in the next section, we will take  $s_1 = s_2 = s_3 = 1$  for simplicity.

### 3 Phenomenology

In general, the sneutrinos  $\tilde{\nu}_i$  can have VEVs  $v_i \equiv \langle \tilde{\nu}_i \rangle \neq 0$  [8]. Since the mass matrix for  $(\nu_1, \nu_2, \nu_3, \widetilde{W}^0)$  (except for the radiative masses) is given by

$$\begin{pmatrix} 0 & 0 & 0 & \frac{1}{2}g v_1 \\ 0 & 0 & 0 & \frac{1}{2}g v_2 \\ 0 & 0 & 0 & \frac{1}{2}g v_3 \\ \frac{1}{2}g v_1 & \frac{1}{2}g v_2 & \frac{1}{2}g v_3 & M_{\widetilde{W}} \end{pmatrix} , \quad (3.1)$$

where, for simplicity, we have dropped the elements for  $\widetilde{B}^0$ , the contribution  $M_{\tilde{\nu}}$  from  $\langle \tilde{\nu}_i \rangle \neq 0$  to the neutrino masses is expressed by

$$M_{\tilde{\nu}} \simeq -\frac{g^2}{4} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} (M_{\widetilde{W}})^{-1} (v_1 \ v_2 \ v_3) = -\frac{g^2}{4M_{\widetilde{W}}} \begin{pmatrix} v_1^2 & v_1 v_2 & v_1 v_3 \\ v_1 v_2 & v_2^2 & v_2 v_3 \\ v_1 v_3 & v_2 v_3 & v_3^2 \end{pmatrix} , \quad (3.2)$$

under the seesaw approximation. Note that the matrix  $M_{\tilde{\nu}}$  is a rank-1 matrix. Therefore, in the present model, the neutrino mass matrix  $M_\nu$  is given by

$$M_\nu = M_{rad} + M_{\tilde{\nu}} . \quad (3.3)$$

We have estimated the absolute magnitudes of the radiative masses in (2.3)–(2.5). On the other hand, it is hard to estimate the absolute values of  $\langle \tilde{\nu}_i \rangle$ , because, in the present model, there is neither a term corresponding to the so-called “ $\mu$ -term”  $\mu \bar{H}_d H_u$  nor  $\bar{5}_{L(-)i} \leftrightarrow \bar{H}_{(0)}$  mixing terms, so that the sneutrinos  $\tilde{\nu}_i$  cannot have the VEVs  $\langle \tilde{\nu}_i \rangle$  at the tree level. The non-zero VEVs appears only through the renormalization group equation (RGE) effect[9]. The contribution highly depends on an explicit model of the SUSY breaking. Therefore, in the present paper, we will deal with the relative ratio of the contributions  $M_{\tilde{\nu}}$  to  $M_{rad}$  as a free parameter.

The recent neutrino data [10, 11, 12] have indicated that  $\sin^2 2\theta_{atm} \simeq 1$  and  $\tan^2 \theta_{solar} \simeq 0.5$ . In response to these observations, He and Zee have found a phenomenological neutrino mass matrix [13]

$$M_\nu = m_0 \begin{pmatrix} 2+x & 0 & 0 \\ 0 & 1-y+x & 1+y \\ 0 & 1+y & 1-y+x \end{pmatrix} + m_0 \varepsilon \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \quad (3.4)$$

which leads to a nearly bimaximal mixing

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \quad (3.5)$$

i.e.

$$\sin^2 2\theta_{atm} = 1 , \quad (3.6)$$

$$\tan^2 \theta_{solar} = \frac{1}{2} . \quad (3.7)$$

(Although He and Zee gave the mass matrix (3.4) with  $x = 0$  in Ref. [13], since a term which is proportional to a unit matrix does not affect the mixing matrix form, the most general form of the He–Zee mass matrix is given by (3.4).) The mass matrix (3.4) gives the following mass eigenvalues:

$$\begin{aligned} m_{\nu 1} &= m_0(2+x) , \\ m_{\nu 2} &= m_0(2+x+3\varepsilon) , \\ m_{\nu 3} &= m_0(x-2y) , \end{aligned} \quad (3.8)$$

and

$$\Delta m_{21}^2 = m_{\nu 2}^2 - m_{\nu 1}^2 = 12\varepsilon \left( 1 + \frac{1}{2}x + \frac{3}{4}\varepsilon \right) m_0^2 , \quad (3.9)$$

$$\Delta m_{32}^2 = m_{\nu_3}^2 - m_{\nu_2}^2 = -4 \left( 1 + x - y + \frac{2}{3}\varepsilon \right) \left( 1 + y + \frac{3}{2}\varepsilon \right) m_0^2, \quad (3.10)$$

$$R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = -\frac{3(2+x+3\varepsilon/2)\varepsilon}{2(1+x-y)(1+y)}. \quad (3.11)$$

(Therefore, the parameter  $y$  has to be  $y \neq -1$  and  $y \neq 1+x$ .)

It is likely that the VEVs  $\langle \tilde{\nu}_i \rangle$  are democratic on the basis on which the charged lepton mass matrix is diagonal, i.e.

$$\langle \tilde{\nu}_1 \rangle = \langle \tilde{\nu}_2 \rangle = \langle \tilde{\nu}_3 \rangle, \quad (3.12)$$

so that we can regard the second term in the He–Zee matrix (3.4) as  $M_{\tilde{\nu}}$  which originates in the sneutrino VEVs. Then, it is interesting whether our radiative mass matrix (2.8) can give the first term in the He–Zee mass matrix (3.4) or not.

Corresponding to the assumption (3.12), we may also suppose that the coefficients  $h_{ij}$  are invariant under the permutation among  $\ell_{Li} = (\nu_{Li}, e_{Li})$  which belong to  $\bar{5}_{Li}$  (not among  $e_{Ri}^c$  which belong to  $10_{Li}$ ). The most simple case will be

$$h = \lambda \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (3.13)$$

Then, we obtain the radiative neutrino mass matrix

$$M_{rad} = m_0 \lambda^2 \frac{m_1^2}{m_3^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad (3.14)$$

which corresponds the first term in the He–Zee mass matrix (3.4) with  $x = -2$  and  $y = -2$ , so that

$$R \simeq \frac{9}{4}\varepsilon^2. \quad (3.15)$$

From the best fit values of  $\Delta m_{ij}^2$  [10, 11, 12],

$$R_{obs} = \frac{6.9 \times 10^{-5} \text{ eV}^2}{2.5 \times 10^{-3} \text{ eV}^2} = 2.76 \times 10^{-2}, \quad (3.16)$$

we obtain

$$\varepsilon = 0.111, \quad (3.17)$$

and

$$m_{\nu 1} = 0, \quad m_{\nu 2} = 0.0083 \text{ eV}, \quad m_{\nu 3} = 0.050 \text{ eV}, \quad (3.18)$$

where we have used the best fit values [10, 11, 12]  $\Delta m_{atm}^2 = 2.5 \times 10^{-3} \text{ eV}^2$  and  $\Delta m_{solar}^2 = 6.9 \times 10^{-5} \text{ eV}^2$ . In the present model (3.14), the absolute magnitude of  $m_{\nu 3}$ , which is radiatively generated, is given by

$$m_{\nu 3} = \frac{1}{4\pi} AF\lambda^2 m_e^2 = 1.9 \times 10^{-2} \hat{A}\hat{F}\lambda^2 \text{ eV}, \quad (3.19)$$

where  $\hat{A}$  and  $\hat{F}$  are numerical values of the parameters  $A$  and  $F$  in unit of TeV, which are defined by  $\hat{M}_{eLR}^2 = AM_e$  and the equation (2.5), respectively. If we, for example, take  $A \simeq 1 \text{ TeV}$  and  $\hat{m}_{eL}^2 \simeq \hat{m}_{eR}^2 \simeq 0.5 \text{ TeV}$ , we obtain  $m_{\nu 3} \simeq 0.075\lambda^2 \text{ eV}$ . Thus, roughly speaking, the choice  $m_{SUSY} \sim 1 \text{ TeV}$  and  $\lambda \sim 1$  can give a reasonable magnitude of  $m_{\nu 3}$ .

## 4 Summary

In conclusion, we have proposed a model with  $R$ -parity violation within the framework of an  $SU(5)$  SUSY GUT. In the model, we have matter fields  $10_{L(+)} + \bar{5}_{L(-)} + 5_{L(+)} + \bar{5}_{L(+)}$  and Higgs fields  $H_{(+)}$  and  $\bar{H}_{(0)}$ , where  $(+, 0, -)$  denote their transformation properties  $(\omega^{+1}, \omega^0, \omega^{-1})$  under a discrete symmetry  $Z_3$ , respectively. Although  $\bar{5}_{L(-)}5_{L(+)}$  acquires a heavy mass  $M_5$  at  $\mu = M_X$ , the effective

masses of the triplet and doublet components  $\bar{5}_{L(-)}^{(3)}5_{L(+)}^{(3)}$  and  $\bar{5}_{L(-)}^{(2)}\bar{5}_{L(+)}^{(2)}$ ,  $M^{(3)}$  and  $M^{(2)}$ , are given by  $M^{(3)} \sim M_W$  and  $M^{(2)} \sim M_X$ , respectively, because we consider a fine tuning term  $g_5 \bar{5}_{L(-)} \Phi_{(0)} 5_{L(+)}$  with VEVs  $\langle \Phi_{(0)} \rangle = v_{24}(2, 2, 2, -3, -3)$ . At an intermediate energy scale  $\mu = M_I \sim 10^{15} \text{ GeV}$ , the

$Z_3$  symmetry is broken by the term  $M^{SB} \bar{5}_{L(+)} 5_{L(+)}$ , so that masses of  $\bar{5}_{L(-)}^{(3)}$  and  $\bar{5}_{L(-)}^{(2)}$  are given by

$m(\bar{5}_{L(+)}^{(3)}) \simeq M^{SB} \sim M_I$  and  $m(\bar{5}_{L(-)}^{(2)}) \simeq M^{(2)} \sim M_X$ . In other words, at a low energy scale, the massless

matter fields are  $\bar{5}_{L(-)}^{(3)} + \bar{5}_{L(+)}^{(2)} + 10_{L(+)}$ . Therefore, the  $R$ -parity violating interactions  $\bar{5}_{L(+)} \bar{5}_{L(+)} 10_{L(+)}$  are invisible in the triplet sector, while those are visible in the doublet sector. Since we take the fine tuning parameters  $M^{(3)}$ ,  $M^{(2)}$  and  $M^{SB}$  as  $M^{(3)} \sim m_{SUSY}$ ,  $M^{(2)} \sim M_X$  and  $M^{SB} \sim M_X \times 10^{-1}$ , the mixing angles  $\theta_i^{(a)}$  between  $\bar{5}_{L(+)}^{(a)}$  and  $\bar{5}_{L(-)}^{(a)}$  (the observed quarks and leptons  $\bar{5}_L$  are defined as

$\bar{5}_L i = c_i \bar{5}_{L(-)} i - s_i \bar{5}_{L(+)} i$ ) are given by  $s_i^{(3)} \simeq M^{(3)}/M_i^{SB} \sim 10^{-12}$  and  $c_i^{(2)} \simeq M_i^{SB}/M^{(2)} \sim 10^{-1}$ , i.e. the triplet components in the effective  $R$ -parity violating interactions  $\bar{5}_L \bar{5}_L 10_{L(+)}$  are highly suppressed by the factors  $s_i^{(3)} \sim 10^{-12}$ , while the doublet components are visible because of  $s_i^{(2)} \sim 1$ .

In the present model, the radiative neutrino masses are generated only through the charged lepton loop. The general radiative mass matrix form  $M_{rad}$  is given by the expression (2.8) [(2.9)–(2.14)]. If there are contributions  $M_{\tilde{\nu}}$  from VEVs of the sneutrinos  $\langle \tilde{\nu} \rangle \neq 0$  to the neutrino mass matrix  $M_\nu$  with suitable magnitudes relative to  $M_{rad}$ , especially, with a democratic form (3.12), we can obtain the He-Zee neutrino mass matrix form (3.4), which leads to a nearly bimaximal mixing with  $\sin^2 2\theta_{atm} = 1$  and  $\tan^2 \theta_{solar} = 1/2$ . Of course, this is only an example of the explicit mass matrix form. It is likely that



more plausible ansatz for a flavor symmetry can give more elegant mass matrix form which gives beautiful explanations for the observed neutrino and lepton-flavor-violation phenomena. Search for such a flavor symmetry is one of our future tasks.

In the present paper, we did not discuss the quark and charged lepton mass matrices. In the present model, the down-quark mass matrix  $M_d$  is related to the charged lepton mass matrix  $M_e$  as  $M_d^T = CM_e$  with  $C \neq \mathbf{1}$ . Investigation of a possible structure of  $C$  is also a future task in the model.

It is interesting to extend the model to a further large unification group. In the present SU(5) model, we have two types of the matter fields with the transformation properties  $\omega^{+1}$  and  $\omega^{-1}$  under the discrete symmetry  $Z_3$ , i.e.  $\bar{5}_{L(+)} + 10_{L(+)}$  and  $\bar{5}_{L(-)} + 5_{L(+)}$ . For example, if we suppose an SU(10) model, we can regard  $\bar{5}_{L(+)} + 10_{L(+)}$  [ $+1_{L(+)}$ ] and  $\bar{5}_{L(\pm)} + 5_{L(\pm)}$  as  $16_{(+)}$  and  $10_{(\pm)}$  of SO(10), respectively. We are also interested in a 27-plet representation of  $E_6$ , which is decomposed into  $16 + 10 + 1$  of SO(10). Thus, the present model has a possibility of a further extension.

In conclusion, the present model will bring fruitful results not only in phenomenology, but also in a theoretical extension.

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