A Unified Description of Quark and Lepton Mass Matrices in a Universal Seesaw Model

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In a framework of the universal seesaw model which intends to give a unified description of the quark and lepton mass matrices by assuming a seesaw mechanism not only for the neutrinos but also for the charged leptons and quarks, a possible form of the neutrino mass matrix is investigated. In the mass matrices $\overline{f}_L m_L F_R + \overline{F}_L m_R f_R + \overline{F}_L M_F F_R$ (f: quarks and leptons; F: hypothetical heavy fermions), m_L and m_R are universal for up- and down-fermions, and M_F have only fermion-sector-dependent structures. The observed top quark mass enhancement is understood by $\det M_F = 0$ ($F = U$). The quark and lepton masses and mixings (including the neutrino sector) are well-satisfactorily described by only one complex parameter b_f in M_F which is invariant under a permutation symmetry S_3, when m_L and m_R are fixed by using the observed charged lepton mass values as the inputs. However, so far, the universal seesaw model has not been able to give bimaximal mixing for the neutrino sector under the parameters which well describe the observed masses and mixings of the quarks and charged leptons, although it can give either maximal mixing of ν_e-ν_μ or ν_μ-ν_τ. In the present paper, we consider that m_L and m_R are universal for up- and down-fermions, but those in the lepton sectors are different from those in the quark sectors by a rotation. The new universal seesaw model can reasonably give the observed nearly bimaximal mixing.

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I. INTRODUCTION

A. What is the universal seesaw model?

Stimulated by the recent progress of neutrino experiments, there has been considerable interest in a unified description of the quark and lepton mass matrices. As one of such unified models, a non-standard model, the so-called "universal seesaw model" (USM) [1], is well known. The model describes not only the neutrino mass matrix M_ν but also the quark mass matrices M_u and M_d and charged lepton mass matrix M_e by seesaw-type matrices universally: The model has hypothetical fermions F_i ($F = U, D, N, E$; $i = 1, 2, 3$) in addition to the conventional quarks and leptons f_i ($f = u, d, \nu, e$; $i = 1, 2, 3$), and these fermions are assigned to $f_L = (2, 1)$, $f_R = (1, 2)$, $F_L = (1, 1)$ and $F_R = (1, 1)$ of $SU(2)_L \times SU(2)_R$. The 6×6 mass matrix which is sandwiched between the fields $(\overline{f}_L, \overline{F}_L)$ and (f_R, F_R) is given by

$$M^{6\times 6} = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} , \tag{1.1}$$

where m_L and m_R are universal for all fermion sectors ($f = u, d, \nu, e$) and only M_F have structures dependent on the fermion sectors $F = U, D, N, E$. For $\Lambda_L < \Lambda_R \ll \Lambda_S$, where $\Lambda_L = O(m_L)$, $\Lambda_R = O(m_R)$ and $\Lambda_S = O(M_F)$, the 3×3 mass matrix M_f for the fermions f is given by the well-known seesaw expression

$$M_f \simeq -m_L M_F^{-1} m_R . \tag{1.2}$$

Thus, the model answers the question why the masses of quarks (except for top quark) and charged leptons are so small compared with the electroweak scale Λ_L ($\sim 10^2$ GeV).

Recently, in order to understand the observed fact $m_t \sim \Lambda_L$ (m_t is the top quark mass), the authors have proposed a universal seesaw mass matrix model with an ansatz [2–4]

$$\det M_F = 0 , \tag{1.3}$$

for the up-quark sector ($F = U$). In the model, one of the fermion masses $m(U_i)$ is zero [say, $m(U_3) = 0$], so that the seesaw mechanism does not work for the third family, i.e., the fermions (u_{3L}, U_{3R}) and (U_{3L}, u_{3R}) acquire masses of

$$m(u_{3L}, U_{3R}) \sim O(m_L) , \quad m(U_{3L}, u_{3R}) \sim O(m_R) , \tag{1.4}$$

respectively. We identify (u_{3L}, U_{3_R}) as the top quark (t_L, t_R). Thus, we can understand the question why only the top quark has a mass of the order of Λ_L.

More explicitly speaking, in the USM, the effective Hamiltonian is given by

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$$H_{int} = Y^u_{Lij}\overline{q}_{Li}\widetilde{\phi}_L U_{Rj} + Y^d_{Lij}\overline{q}_{Li}\phi_L D_{Rj} + Y^\nu_{Lij}\overline{\ell}_{Li}\widetilde{\phi}_L N_{Rj} + Y^e_{Lij}\overline{\ell}_{Li}\phi_L E_{Rj}$$
$$+ Y^u_{Rij}\overline{q}_{Ri}\widetilde{\phi}_R U_{Lj} + Y^d_{Rij}\overline{q}_{Ri}\phi_R D_{Lj} + Y^\nu_{Rij}\overline{\ell}_{Ri}\widetilde{\phi}_R N_{Lj} + Y^e_{Rij}\overline{\ell}_{Ri}\phi_R E_{Lj} \qquad (1.5)$$
$$+ Y^u_{Sij}\overline{U}_{Li}\Phi U_{Rj} + Y^d_{Sij}\overline{D}_{Li}\Phi^\dagger D_{Rj} + Y^\nu_{Sij}\overline{N}_{Li}\Phi N_{Rj} + Y^e_{Sij}\overline{E}_{Li}\Phi^\dagger E_{Rj} + h.c. \ ,$$

where

$$q_{L/R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L/R} , \quad \ell_{L/R} = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_{L/R} ,$$
$$\phi_{L/R} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_{L/R} , \quad \widetilde{\phi}_{L/R} = \begin{pmatrix} \overline{\phi}^0 \\ -\phi^- \end{pmatrix}_{L/R} . \qquad (1.6)$$

Here, the $SU(2)_L \times SU(2)_R$ singlet Higgs scalar Φ has been introduced in order to give a heavy mass M_F and it has a vacuum expectation value (VEV) $\langle \Phi \rangle \sim \Lambda_S$ at an energy scale $\mu = \Lambda_S$.

Note that the quantum number of the fermion N_L is identical with that of the fermion N^c_R [$\equiv (N_R)^c \equiv C\overline{N}^T_R$]. Therefore, the neutral fermions N_L and N_R can acquire the following Majorana mass terms at $\mu = \Lambda_S$:

$$H_{Majorana} = \left(Y^L_{Sij}\overline{N}_{Li} N^c_{Lj} + Y^R_{Sij}\overline{N}^c_{Ri} N_{Rj} \right) \Phi + h.c. \ . \qquad (1.7)$$

Then, the neutrino mass matrix is given as follows

$$\left(\overline{\nu}_L \ \overline{\nu}^c_R \ \overline{N}_L \ \overline{N}^c_R \right) \begin{pmatrix} 0 & 0 & 0 & m_L \\ 0 & 0 & m^T_R & 0 \\ 0 & m_R & M_L & M_D \\ m^T_L & 0 & M^T_D & M_R \end{pmatrix} \begin{pmatrix} \nu^c_L \\ \nu_R \\ N^c_L \\ N_R \end{pmatrix} , \qquad (1.8)$$

where $M_D = Y^\nu_S \langle \Phi \rangle$, $M_L = Y^L_S \langle \Phi \rangle$ and $M_R = Y^R_S \langle \Phi \rangle$. Since $O(M_D) \sim O(M_L) \sim O(M_R) \gg O(m_R) \gg O(m_L)$, we obtain the mass matrix M_ν for the active neutrinos ν_L

$$M_\nu \simeq -m_L M^{-1}_R m^T_L \ . \qquad (1.9)$$

If we take the ratio $O(m_L)/O(m_R)$ suitably small, we can understand the smallness of the observed neutrino masses reasonably.

For an embedding of the model into a grand unification scenario, for example, see Ref. [5], where a possibility of $SO(10) \times SO(10)$ has been discussed.

B. What is the democratic universal seesaw model?

As an extended version of the USM, the "democratic" USM [2,3] is also well known. The model has successfully given the quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) [6] matrix parameters in terms of the charged lepton masses. The outline of the model is as follows:

(i) The mass matrices m_L and m_R have the same structure except for their phase factors

$$m^f_L = m^f_R/\kappa = m_0 Z^f \ , \qquad (1.10)$$

where κ is a constant with $\kappa \gg 1$ and Z^f are given by

$$Z^f = P(\delta^f) Z \ , \qquad (1.11)$$

$$P(\delta^f) = \mathrm{diag}(e^{i\delta^f_1}, e^{i\delta^f_2}, e^{i\delta^f_3}) \ , \qquad (1.12)$$

$$Z = \mathrm{diag}\,(z_1, z_2, z_3) \ , \qquad (1.13)$$

with $z_1^2 + z_2^2 + z_3^2 = 1$. (The fermion masses m^f_i are independent of the parameters δ^f_i. Only the values of the CKM matrix parameters $|V_{ij}|$ depend on the parameters δ^f_i).

(ii) In the basis on which the matrices m^f_L and m^f_R are diagonal, the mass matrices M_F are given by the form

$$M_F = m_0 \lambda (\mathbf{1} + 3b_f X), \qquad (1.14)$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} . \qquad (1.15)$$

(iii) The parameter b_f for the charged lepton sector is given by $b_e = 0$, so that in the limit of $\kappa/\lambda \ll 1$, the parameters z_i are given by

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}} \ . \qquad (1.16)$$

Then, the up- and down-quark masses are successfully given by the choice of $b_u = -1/3$ and $b_d = -e^{i\beta_d}$ ($\beta_d = 18^\circ$), respectively. Here, note that the choice $b_u = -1/3$ gives $\mathrm{Tr} M_U = 0$, so that the case give $m_t \sim O(m_L)$. Another motivation for the choice $b_u = -1/3$ is that the model with $b_e = 0$ and $b_u = -1/3$ has led to the successful relation [7,2]

$$\frac{m_u}{m_c} \simeq \frac{3}{4}\frac{m_e}{m_\mu}, \tag{1.17}$$

which is almost independent of the value of the seesaw suppression factor κ/λ. For the choice of $b_u = -1/3$ and $b_d = -e^{i\beta_d}$ ($\beta_d = 18^\circ$), the CKM matrix parameters are successfully given [2,3] by taking

$$\delta_1^u - \delta_1^d = \delta_2^u - \delta_2^d = 0 \ , \quad \delta_3^u - \delta_3^d \simeq \pi \ . \tag{1.18}$$

C. What is the problem?

It seems that the present model is almost successful from the phenomenological point of view, so that the future task is only to give more reliable theoretical base to the model. However, the democratic USM has a serious problem in the neutrino phenomenology.

In the present model, the parameters z_i are fixed by the observed charged lepton masses as shown in (1.16), and the adjustable parameter is only the parameter b_ν defined by (1.14). If we take $b_\nu \simeq -1/2$, we can obtain the mixing matrix [8]

$$U \simeq \left(\begin{array}{ccc} 1 & \sqrt{m_e/2m_\mu} & \sqrt{m_e/2m_\mu} \\ -\sqrt{m_e/m_\mu} & 1/\sqrt{2} & \mp 1/\sqrt{2} \\ -\sqrt{m_e/m_\tau} & \pm 1/\sqrt{2} & 1/\sqrt{2} \end{array} \right) , \tag{1.19}$$

so that we can explain a large mixing between ν_μ and ν_τ which is suggested in the atmospheric neutrino data [9]. On the other hand, if we take $b_\nu \simeq -1$, we obtain the mixing matrix [8]

$$U \simeq \left(\begin{array}{ccc} 1/\sqrt{2} & \mp 1/\sqrt{2} & -\sqrt{m_e/m_\tau} \\ \pm 1/\sqrt{2} & 1/\sqrt{2} & -\sqrt{m_\mu/m_\tau} \\ \sqrt{m_\mu/2m_\tau} & \sqrt{m_\mu/2m_\tau} & 1 \end{array} \right) , \tag{1.20}$$

so that we can explain a large mixing $\nu_e \leftrightarrow \nu_\mu$ which is suggested in the solar neutrino data [10]. Thus, there is no solution which gives the observed nearly bimaximal mixing under the condition $\Delta m^2_{solar}/\Delta m^2_{atm} \sim 10^{-2}$.

This suggests that the present model is too tight. therefore, in the present paper, we loosen the constraint (1.11) as

$$Z^f = P(\delta^f)\widehat{Z}^f , \tag{1.21}$$

and we assume

$$\widehat{Z}^\ell = R_X^T \widehat{Z}^q R_X \ , \tag{1.22}$$

$$\widehat{Z}^q \equiv Z = \mathrm{diag}(z_1, z_2, z_3), \tag{1.23}$$

where $\widehat{Z}^\ell \equiv \widehat{Z}^e = \widehat{Z}^\nu$ and $\widehat{Z}^q \equiv \widehat{Z}^d = \widehat{Z}^u$. Since we take the same assumption $b_e = 0$ as before, the rotation (1.22) in the charged lepton sector does not affect to the masses, so that the relation (1.16) is still satisfied. For the quark sectors, we also take the same assumption $Z^d = P(\delta^d)Z$ and $Z^u = P(\delta^u)Z$ as before, so that the phenomenological success in the quark sectors is not lost.

However, generally speaking, it is not so notable that we can give the nearly bimaximal mixing in the neutrino sector, because we have three additional parameters in the rotation matrix R_X. The problem is whether the rotation R_X has a physical meaning or not. In the next section, we will investigate a rotation matrix R_X which leads to the observed nearly bimaximal mixing and suggests an interesting relation between quarks and leptons.

II. S$_2$ SYMMETRY VERSUS S$_3$ SYMMETRY

A. Basic assumption

For the quark sectors, the model is unchanged from the previous model, i.e., the mass terms are given by

$$m_0 \sum_{f=u,d} \left[\overline{f}_L Z P(\delta^f) F_R + \kappa \overline{F}_L P^\dagger(\delta^f) Z f_R \right.$$
$$\left. + \lambda \overline{F}_L (\mathbf{1} + 3b_f X) F_R \right] + h.c. \ . \tag{2.1}$$

On the basis on which the mass matrices m_L^f and m_R^f are diagonal, the mass matrix M_F is invariant under the permutation symmetry S$_3$. As investigated in Ref. [2,3], in order to give reasonable values of the CKM matrix parameters, it was required to choose

$$P(\delta^u)P^\dagger(\delta^d) = P(\delta^u - \delta^d) = \mathrm{diag}(1, 1, -1) \ , \tag{2.2}$$

although the origin of such a phase inversion is still open question. In this paper, we assume

$$P(\delta^u) = \mathrm{diag}(1, 1, -1) \ , \quad P(\delta^d) = \mathrm{diag}(1, 1, 1) \ . \tag{2.3}$$

For the lepton sectors, we assume

$$m_0 \sum_{f=e,\nu} \left[\overline{f}_L \widehat{Z}^\ell P(\delta^f) F_R + \kappa \overline{F}_L P^\dagger(\delta^f) \widehat{Z}^\ell f_R \right.$$
$$\left. + \lambda \overline{F}_L (\mathbf{1} + 3b_f X) F_R \right] + h.c. \ , \tag{2.4}$$

where, for convenience, we have dropped the Majorana mass terms (1.7) from the expression (2.4), since it is clear that we assume that the Majorana mass terms have the same structure as the Dirac mass term. When we define

$$f' = R_X f \ , \quad F' = R_X F \ , \tag{2.5}$$

the mass terms (2.4) can be rewritten as

$$m_0 \sum_{f=e,\nu} \left[\overline{f}'_L Z R_X P(\delta^f) R_X^T F'_R + \kappa \overline{F}'_L R_X P^\dagger(\delta^f) R_X^T Z f'_R \right.$$

$$\left. + \lambda \overline{F}'_L (\mathbf{1} + 3b_f X^\ell) F'_R \right] + h.c. \ , \qquad (2.6)$$

where

$$X^\ell = R_X X R_X^T \ . \qquad (2.7)$$

We take $P(\delta^e) = \mathbf{1}$ corresponding to (2.3). The fields e' and E' are fields on the basis on which m'_L and m'_R are diagonal. However, differently from the quark sectors, the heavy fermion mass matrix M'_E is not S_3-invariant because X^ℓ is not democratic. Also note that since we consider

$$P(\delta^\nu) = \mathrm{diag}(1, 1, -1) \ , \qquad (2.8)$$

corresponding to (2.3), the mass term $\overline{\nu}'_L m'_L N'_R$ is not diagonal. It is not that the fields ν'_L is defined as the mass matrix $m_L^{\nu\prime}$ is diagonal, but that the field ν'_L is defined as the partner of e'_L whose mass matrix $m_L^{e\prime}$ is diagonal. The effective charged lepton and neutrino mass matrices are given by

$$M_e \simeq -m_0 \frac{\kappa}{\lambda} R_X^T Z R_X (\mathbf{1} + 3a_e X) R_X^T Z R_X \ , \qquad (2.9)$$

$$M_\nu \simeq -m_0 \frac{1}{\lambda} R_X^T Z R_X P(\delta^\nu)(\mathbf{1} + 3a_\nu X) P(\delta^\nu) R_X^T Z R_X \ , \qquad (2.10)$$

where we have used

$$(\mathbf{1} + 3b_f X)^{-1} = \mathbf{1} + 3a_f X \ , \qquad (2.11)$$

$$a_f = -b_f/(1 + 3b_f) \ . \qquad (2.12)$$

Since we assume the same condition $a_e = 0$ as before, M_e is diagonalized by the rotation R_X as $R_X M_e R_X^T = D_e \equiv \mathrm{diag}(m_e, m_\mu, m_\tau)$, so that the neutrino mixing matrix U_ν is obtained from

$$U_\nu^T M'_\nu U_\nu = D_\nu \equiv \mathrm{diag}(m_1^\nu, m_2^\nu, m_3^\nu) \ , \qquad (2.13)$$

where

$$M'_\nu = -m_0 \frac{1}{\lambda} Z(\mathbf{1} + 3a_\nu X^\nu) Z \ , \qquad (2.14)$$

$$X^\nu = R_X P(\delta^\nu) X P(\delta^\nu) R_X^T \ . \qquad (2.15)$$

The rotation R_X means a rotation between the basis in the quark sectors and that in the lepton sectors. Our interests are as follows: What rotation R_X can give reasonable neutrino masses and mixings? What relation does it suggest between quarks and leptons?

B. A special form of R_X

We consider the following special form of R_X:

$$R_X = \begin{pmatrix} x_3 & x_2 & x_1 \\ \sqrt{\frac{2}{3}} - x_3 & \sqrt{\frac{2}{3}} - x_2 & \sqrt{\frac{2}{3}} - x_1 \\ \sqrt{\frac{2}{3}}(x_1 - x_2) & \sqrt{\frac{2}{3}}(x_3 - x_1) & \sqrt{\frac{2}{3}}(x_2 - x_3) \end{pmatrix} \ , \qquad (2.16)$$

where x_i satisfy the relations

$$x_1^2 + x_2^2 + x_3^2 = 1 \ , \qquad (2.17)$$

$$x_1 + x_2 + x_3 = \sqrt{\frac{3}{2}} \ . \qquad (2.18)$$

The explicit expressions of x_i are given by

$$x_1 = \frac{1}{\sqrt{6}} - \frac{2s}{2\sqrt{3}} \ ,$$

$$x_2 = \frac{1}{\sqrt{6}} + \frac{s}{2\sqrt{3}} - \frac{c}{2} \ , \qquad (2.19)$$

$$x_3 = \frac{1}{\sqrt{6}} + \frac{s}{2\sqrt{3}} + \frac{c}{2} \ ,$$

where

$$s = \sin\left(\frac{\pi}{4} - \varepsilon\right) \ , \quad c = \cos\left(\frac{\pi}{4} - \varepsilon\right) \ . \qquad (2.20)$$

Independently of the parameter ε in the explicit expression (2.19), the rotation R_X transforms the 3×3 democratic matrix to a 2×2 democratic matrix as

$$R_X X_3 R_X^T = X_2 \ , \qquad (2.21)$$

where

$$X_3 \equiv X = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \ , \quad X_2 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ . \qquad (2.22)$$

The relation (2.21) shows that the heavy lepton mass matrices M'_F for the lepton fields with the prime, i.e., in the expression (2.6), are invariant under a permutation symmetry S_2, although the mass matrices M_F for the lepton fields without the prime, i.e., in the expression (2.4), have the S_3 invariant forms. In other words, the relation (2.21) tell us that on the bases on which the mass matrices m_L^f for the down-fermions $f = d, e$ are diagonal, the corresponding heavy quark mass matrices M_F $(F = U, D)$ are S_3-invariant, while the heavy lepton mass matrices M'_F $(F = E, N)$ are S_2-invariant.

In the next section, we will give numerical study of the neutrino mass matrix (2.14) for a case with the rotation (2.16).

III. NUMERICAL STUDY OF THE MASS MATRIX

When we take $P(\delta^\nu) = \mathrm{diag}(1,1,-1)$, (2.8), the matrix X^ν defined by (2.15) is expressed as

$$X^\nu = \begin{pmatrix} y_1^2 & y_1 y_2 & y_1 y_3 \\ y_1 y_2 & y_2^2 & y_2 y_3 \\ y_1 y_3 & y_2 y_3 & y_3^2 \end{pmatrix} , \tag{3.1}$$

where

$$y_1 = \frac{1}{\sqrt{3}}(x_3 + x_2 - x_1) ,$$

$$y_2 = -\frac{1}{3\sqrt{3}}(x_3 + x_2 - 5x_1) , \tag{3.2}$$

$$y_3 = \frac{2\sqrt{2}}{3}(x_3 - x_2) ,$$

and $y_1^2 + y_2^2 + y_3^2 = 1$.

Note that the parameters x_i satisfy the relation

$$x_1^2 + x_2^2 + x_3^2 = \frac{2}{3}(x_1 + x_2 + x_3)^2 , \tag{3.3}$$

and, on the other hand, it is well known that the observed charged lepton masses satisfy the relation [12]

$$m_e + m_\mu + m_\tau = \frac{2}{3}\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2 , \tag{3.4}$$

i.e.,

$$z_1^2 + z_2^2 + z_3^2 = \frac{2}{3}(z_1 + z_2 + z_3)^2 . \tag{3.5}$$

Therefore, the choice $x_i = z_i$ is interesting. The observed charged lepton masses give the values

$$z_1 = 0.016473 , \quad z_2 = 0.23687 , \quad z_3 = 0.97140 , \tag{3.6}$$

which corresponds to $\varepsilon = 2.268^\circ$ in the expression (2.19). Then, we obtain

$$X_\ell = \begin{pmatrix} 0.47346 & -0.14909 & 0.47651 \\ -0.14909 & 0.04695 & -0.15006 \\ 0.47651 & -0.15006 & 0.47959 \end{pmatrix} , \tag{3.7}$$

which gives

$$(M_\nu)_{11} = z_1^2(1 + 1.4204 a_\nu) . \tag{3.8}$$

Therefore, if we take $a_\nu = -2/3$ (i.e., $b_\nu = -2/3$), we obtain $(M_\nu)_{11} \simeq 0$. When we take $x_i = z_i$ and $b_\nu = -2/3$, the numerical result of M_ν is

$$M_\nu = \frac{m_0}{\lambda}\begin{pmatrix} 1.44 \times 10^{-5} & 0.001164 & -0.015250 \\ 0.001164 & 0.050839 & 0.069054 \\ -0.015250 & 0.069054 & 0.038526 \end{pmatrix} , \tag{3.9}$$

so that we obtain

$$\begin{aligned} m_1 &= 0.004223\, m_0/\lambda , \\ m_2 &= -0.029637\, m_0/\lambda , \\ m_3 &= 0.114793\, m_0/\lambda , \end{aligned} \tag{3.10}$$

$$U_\nu = \begin{pmatrix} 0.920097 & -0.38289 & 0.08267 \\ 0.31730 & 0.60476 & -0.73047 \\ -0.22970 & -0.69833 & -0.67792 \end{pmatrix} , \tag{3.11}$$

$$\sin^2 2\theta_{solar} = 0.496 , \tag{3.12}$$

$$\sin^2 2\theta_{atm} = 0.981 , \tag{3.13}$$

$$R = \frac{\Delta m_{21}^2}{\Delta m_{31}^2} = \frac{0.000861}{0.01230} = 0.0700 , \tag{3.14}$$

The predicted values (3.11) - (3.13) are not so in disagreement with the observed values, although the value of $\sin^2 2\theta_{solar}$, (3.11), is somewhat small compared with the recent observed value [11] $\sin^2 2\theta_{solar} \simeq 0.76$, and the value of R, (3.13), is somewhat large compared with the observed value [9,11]

$$R_{obs} \simeq \frac{5.0 \times 10^{-5}\mathrm{eV}^2}{2.5 \times 10^{-3}\mathrm{eV}^2} = 2.0 \times 10^{-2} . \tag{3.15}$$

If we adjust the parameter a_ν slightly, we can obtain the following numerical results at $a_\nu = -0.650$ ($b_\nu = -0.684$):

$$R = 0.0197 , \tag{3.16}$$

$$\sin^2 2\theta_{solar} = 0.797 , \tag{3.17}$$

$$\sin^2 2\theta_{atm} = 0.979 . \tag{3.18}$$

In conclusion, the model with $x_i = z_i$ and $b_\nu \simeq -2/3$ can give reasonable numerical results for the neutrino masses and mixings.

IV. CONCLUDING REMARKS

We have proposed a revised model of the democratic universal seesaw model in order to give a unified description of the quark and lepton mass matrices. In the original model, the mass matrices m_L and m_R were given by a universal structure Z independently of the fermion sectors $f = u, d, e, \nu$. The constraint is too tight, so that the model could not give the observed nearly bimaximal neutrino mixing. In the revised model, the mass matrices m_L^f (also m_R^f) are still $m_L^u = m_L^d \equiv m_L^q$ and

$m_L^e = m_L^\nu \equiv m_L^\ell$, while m_L^ℓ is related to m_L^q by a rotation R_X as $m_L^\ell = R_X^T m_L^q R_X$. If we take a special rotation R_X which transform the 3×3 democratic matrix X_3 to the 2×2 democratic matrix X_2 as (2.21) and we take the parameters x_i as $x_i = z_i \propto \sqrt{m_i^e}$ and $b_\nu \simeq -2/3$, we can obtain reasonable values of neutrino masses and mixings.

For the quark and charged lepton sectors, in the original democratic universal seesaw model [2,3], we have already obtained reasonable values of the masses and mixings by taking $b_e = 0$, $b_u = -1/3$, and $b_d \simeq -1$. Those values of b_f are unchanged in the present revised model, and moreover, in order to explain the observed nearly bimaximal neutrino mixing, the value $b_\nu \simeq -2/3$ was newly required. What do these parameter values

$$b_e = 0 \ , \quad b_u = -1/3 \ , \quad b_\nu \simeq -2/3 \ , \quad b_d \simeq -1 \ , \tag{4.1}$$

mean? This is a future task to us.

What does the rotation R_X with $R_X X_3 R_X^T = X_2$ mean? Especially, in order to obtain reasonable values of neutrino masses and mixings, it was required to take $x_i = z_i$. This R_X gives

$$R_X \begin{pmatrix} \sqrt{m_\tau} \\ \sqrt{m_\mu} \\ \sqrt{m_e} \end{pmatrix} = \sqrt{m_e + m_\mu + m_\tau} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ . \tag{4.2}$$

This suggests that the rotation R_X is highly related to the charged lepton mass spectrum. This is also our future task.

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