Universal Texture of Quark and Lepton Mass Matrices and a Discrete Symmetry Z_3

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Abstract

Recent neutrino data have been favourable to a nearly bimaximal mixing, which suggests a simple form of the neutrino mass matrix. Stimulated by this matrix form, a possibility that all the mass matrices of quarks and leptons have the same form as in the neutrinos is investigated. The mass matrix form is constrained by a discrete symmetry Z_3 and a permutation symmetry S_2. The model, of course, leads to a nearly bimaximal mixing for the lepton sectors, while, for the quark sectors, it can lead to reasonable values of the CKM mixing matrix and masses.

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I. INTRODUCTION

Recent neutrino oscillation experiments [1] have highly suggested a nearly bimaximal mixing ($\sin^2 2\theta_{12} \sim 1$, $\sin^2 2\theta_{23} \simeq 1$) together with a small ratio $R \equiv \Delta m_{12}^2/\Delta m_{23}^2 \sim 10^{-2}$. This can be explained by assuming a neutrino mass matrix form [2]– [7] with a permutation symmetry between second and third generations. We think that quarks and leptons should be unified. It is therefore interesting to investigate a possibility that all the mass matrices of the quarks and leptons have the same matrix form, which leads to a nearly bimaximal mixing and $U_{13} = 0$ in the neutrino sector, against the conventional picture that the mass matrix forms in the quark sectors will take somewhat different structures from those in the lepton sectors. In the present paper, we will assume that the mass matrix form is invariant under a discrete symmetry Z_3 and a permutation symmetry S_2.

Phenomenologically, our mass matrices M_u, M_d, M_ν and M_e (mass matrices of up quarks (u, c, t), down quarks (d, s, b), neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ and charged leptons (e, μ, τ), respectively) are given as follows:

$$M_f = P_{Lf}^\dagger \widehat{M}_f P_{Rf}, \tag{1.1}$$

with

$$\widehat{M}_f = \begin{pmatrix} 0 & A_f & A_f \\ A_f & B_f & C_f \\ A_f & C_f & B_f \end{pmatrix} \quad (f = u, d, \nu, e)\,, \tag{1.2}$$

where P_{Lf} and P_{Rf} are the diagonal phase matrices and A_f, B_f, and C_f are real parameters. Namely the components are different in $\widehat{M}_f$, but their mutual relations are the same. This structure of mass matrix was previously suggested and used for the neutrino mass matrix in Refs [2]– [7], using the basis where the charged-lepton mass matrix is diagonal, motivated by the experimental finding of maximal ν_μ–ν_τ mixing [1]. In this paper, we consider that this structure is fundamental for both quarks and leptons, although it was speculated from the neutrino sector. Therefore, we assume that all the mass matrices have this structure.

Let us look at the universal characters of the model. Hereafter, for brevity, we will omit the flavour index. The eigen-masses m_i of Eq. (1.2) are given by

$$-m_1 = \frac{1}{2}\left(B + C - \sqrt{8A^2 + (B+C)^2}\right), \tag{1.3}$$

$$m_2 = \frac{1}{2}\left(B + C + \sqrt{8A^2 + (B+C)^2}\right), \tag{1.4}$$

$$m_3 = B - C. \tag{1.5}$$

The texture's components of $\widehat{M}$ are expressed in terms of eigen-masses m_i as

$$\begin{aligned} A &= \sqrt{\frac{m_2 m_1}{2}}, \\ B &= \frac{1}{2} m_3 \left(1 + \frac{m_2 - m_1}{m_3}\right), \\ C &= -\frac{1}{2} m_3 \left(1 - \frac{m_2 - m_1}{m_3}\right). \end{aligned} \tag{1.6}$$

That is, $\widehat{M}$ is diagonalized by an orthogonal matrix O as

$$O^T \widehat{M} O = \begin{pmatrix} -m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{1.7}$$

with

$$O \equiv \begin{pmatrix} c & s & 0 \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1.8}$$

Here c and s are defined by

$$c = \sqrt{\frac{m_2}{m_2 + m_1}}, \quad s = \sqrt{\frac{m_1}{m_2 + m_1}}. \tag{1.9}$$

It should be noted that the elements of O are independent of m_3 because of the above structure of $\widehat{M}$.

The zeros in this mass matrix are constrained by the discrete symmetry that is discussed in the next section, defined at a unification scale (the scale does not always mean "grand unification scale"). This discrete symmetry is broken below $\mu = M_R$, at which the right-handed neutrinos acquire heavy Majorana masses, as we discuss in Sec. IV. Therefore, the matrix form (1.1) will, in general, be changed by renormalization group equation (RGE) effects. Nevertheless, we would like to emphasize that we can use the expression (1.1) with (1.2) for the predictions of the physical quantities in the low-energy region. This will be discussed in the appendix.

This article is organized as follows. In Sec. II we discuss the symmetry property of our model. Our model is realized when we consider two Higgs doublets in each up-type and down-type quark (lepton) mass matrices. The quark mixing matrix in the present model is argued in Sec. III. In Sec. IV, the lepton mixing matrix is analyzed. Sec. V is devoted to a summary.

II. Z_3 SYMMETRY AND MASS MATRIX FORM

We assume a permutation symmetry between second and third generations, except for the phase factors. However, the condition $(\widehat{M}_f)_{11} = 0$ cannot be derived from such a symmetry. Therefore, in addition to the $2 \leftrightarrow 3$ symmetry, we assume a discrete symmetry Z_3, under which symmetry the quark and lepton fields ψ_L, which belong to 10_L, $\overline{5}_L$ and 1_L of SU(5) ($1_L = \nu_R^c$), are transformed as

$$\begin{aligned} \psi_{1L} &\to \psi_{1L}, \\ \psi_{2L} &\to \omega\psi_{2L}, \\ \psi_{3L} &\to \omega\psi_{3L}, \end{aligned} \tag{2.1}$$

where $\omega^3 = +1$. (Although we use a terminology of SU(5), at present, we do not consider the SU(5) grand unification.) Then, the bilinear terms $\overline{q}_{Li}u_{Rj}$, $\overline{q}_{Li}d_{Rj}$, $\overline{\ell}_{Li}\nu_{Rj}$, $\overline{\ell}_{Li}e_{Rj}$ and $\overline{\nu}^c_{Ri}\nu_{Rj}$ $[\nu_R^c = (\nu_R)^c = C\overline{\nu_R}^T$ and $\overline{\nu}_R^c = \overline{(\nu_R^c)}]$ are transformed as follows:

$$\begin{pmatrix} 1 & \omega^2 & \omega^2 \\ \omega^2 & \omega & \omega \\ \omega^2 & \omega & \omega \end{pmatrix}, \tag{2.2}$$

where

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \ , \quad \ell_L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix} \ . \tag{2.3}$$

Therefore, if we assume two SU(2) doublet Higgs scalars H_1 and H_2, which are transformed as

$$H_1 \to \omega H_1, \quad H_2 \to \omega^2 H_2, \tag{2.4}$$

the Yukawa interactions are given as follows

$$\begin{aligned} H_{\rm int} = & \sum_{A=1,2} \left(Y^u_{(A)ij} \overline{q}_{Li} \widetilde{H}_A u_{Rj} + Y^d_{(A)ij} \overline{q}_{Li} H_A d_{Rj} \right) \\ & + \sum_{A=1,2} \left(Y^\nu_{(A)ij} \overline{\ell}_{Li} \widetilde{H}_A \nu_{Rj} + Y^e_{(A)ij} \overline{\ell}_{Li} H_A e_{Rj} \right) \\ & + \left(Y^R_{(1)ij} \overline{\nu}^c_{Ri} \widetilde{\Phi}^0 \nu_{Rj} + Y^R_{(2)ij} \overline{\nu}^c_{Ri} \Phi^0 \nu_{Rj} \right) + \text{h.c.} \ , \end{aligned} \tag{2.5}$$

where

$$H_A = \begin{pmatrix} H_A^+ \\ H_A^0 \end{pmatrix} \ , \quad \widetilde{H}_A = \begin{pmatrix} \overline{H}_A^0 \\ -H_A^- \end{pmatrix} \ , \tag{2.6}$$

so that

$$Y^u_{(1)},\ Y^d_{(2)},\ Y^\nu_{(1)},\ Y^e_{(2)},\ Y^R_{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix} \ , \quad Y^u_{(2)},\ Y^d_{(1)},\ Y^\nu_{(2)},\ Y^e_{(1)},\ Y^R_{(1)} = \begin{pmatrix} 0 & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix} . \tag{2.7}$$

In (2.7), the symbol $*$ denotes non-zero quantities. Here, in order to give heavy Majorana masses of the right-handed neutrinos ν_R, we have assumed an SU(2) singlet Higgs scalar Φ^0, which is transformed as H_1.

In the present model, the phase difference $\arg(Y^f_{(1)} + Y^f_{(2)})_{21} - \arg(Y^f_{(1)} + Y^f_{(2)})_{31}$ plays an essential role. Therefore, for the permutation symmetry S_2, we put the following assumption: the permutation symmetry can be applied to only the special basis that the all Yukawa coupling constants are real. (Of course, for the Z_3 symmetry, such an assumption is not required.) We consider that the phase factors are caused by an additional mechanism after the requirement of the permutation symmetry S_2 (after the manifestation of the linear combination $Y_{(1)}+Y_{(2)}$). In the present paper, we consider that although the Z_3 symmetry is rigorously defined for the fields by (2.1), the permutation symmetry S_2 is rather phenomenological one (i.e. Ansatz) for the mass matrix shape. Then, under such the S_2 symmetry, the general forms of $Y_f \equiv Y^f_{(1)} + Y^f_{(2)}$ are given by

$$Y_f = P^\dagger_{Lf} \widehat{Y}_f P_{Rf} = \begin{pmatrix} 0 & ae^{-i(\delta^f_{L1}-\delta^f_{R2})} & ae^{-i(\delta^f_{L1}-\delta^f_{R3})} \\ ae^{-i(\delta^f_{L2}-\delta^f_{R1})} & be^{-i(\delta^f_{L2}-\delta^f_{R2})} & ce^{-i(\delta^f_{L2}-\delta^f_{R3})} \\ ae^{-i(\delta^f_{L3}-\delta^f_{R1})} & ce^{-i(\delta^f_{L3}-\delta^f_{R2})} & be^{-i(\delta^f_{L3}-\delta^f_{R3})} \end{pmatrix} . \tag{2.8}$$

We have already assumed that $\psi_L = (\nu_, e_L, d^c_R; u_L, d_L, u^c_R, e^c_R; \nu^c_R)$ have the same transformation (2.1) under the discrete symmetry Z_3, so that $\overline{\psi}_{Li}\psi^c_{Lj}$ are transformed as (2.2). From this analogy, we assume that the phase matrices P_{Lf} and P_{Rf} come from the replacement $\psi_L \to P_f \psi_L$, i.e.

$$\overline{\psi}_L Y_f \psi^c_L \to \overline{\psi}_L P^\dagger_f \widehat{Y}_f P^\dagger_f \psi^c_L \ . \tag{2.9}$$

However, differently from the transformation (2.1), we do not assume in (2.9) that all the phase matrices P_f are identical, but we assume that they are flavour dependent. This explains the assumption

$$\delta^f_{Li} = -\delta^f_{Ri} \equiv \delta^f_i \ , \tag{2.10}$$

in the expression (2.8). (However, this assumption (2.10) is not essential for the numerical predictions in the present paper, because the predictions of the physical quantities depend on only the phases δ^f_{Li}.)

Since the present model has two Higgs doublets horizontally, in general, flavour-changing neutral currents (FCNCs) are caused by the exchange of Higgs scalars. However, this FCNC problem is a common subject to be overcome not only in the present model but also in most models with two Higgs doublets. The conventional mass matrix models based on a

GUT scenario cannot give realistic mass matrices without assuming more than two Higgs scalars [8]. Besides, if we admit that two such scalars remain until the low energy scale, the well-known beautiful coincidence of the gauge coupling constants at $\mu \sim 10^{16}$ GeV will be spoiled. Although the present model is not based on a GUT scenario, as are the conventional mass matrix models, for the FCNC problem, we optimistically consider [9] that only one component of the linear combinations among those Higgs scalars survives at the low energy scale $\mu = m_Z$, while the other component is decoupled at $\mu < M_X$. The study of the RGE effects given in the appendix will be based on such an "effective" one-Higgs scalar scenario.

III. QUARK MIXING MATRIX

The quark mass matrices

$$M_f = P_f^\dagger \widehat{M}_f P_f^\dagger \quad (f = u, d), \tag{3.1}$$

are diagonalized by the bi-unitary transformation

$$D_f = U_{Lf}^\dagger M_f U_{Rf} \ , \tag{3.2}$$

where $U_{Lf} \equiv P_f^\dagger O_f$, $U_{Rf} \equiv P_f O_f$, and O_d (O_u) is given by Eq. (1.8). Then, the Cabibbo–Kobayashi–Maskawa (CKM) [10] quark mixing matrix V is given by

$$\begin{aligned} V &= U_{Lu}^\dagger U_{Ld} = O_u^T P_u P_d^\dagger O_d \\ &= \begin{pmatrix} c_u c_d + \rho s_u s_d & c_u s_d - \rho s_u c_d & -\sigma s_u \\ s_u c_d - \rho c_u s_d & s_u s_d + \rho c_u c_d & \sigma c_u \\ -\sigma s_d & \sigma c_d & \rho \end{pmatrix}, \end{aligned} \tag{3.3}$$

where ρ and σ are defined by

$$\rho = \frac{1}{2}(e^{i\delta_3} + e^{i\delta_2}) = \cos\frac{\delta_3 - \delta_2}{2} \exp i\left(\frac{\delta_3 + \delta_2}{2}\right) \ , \tag{3.4}$$

$$\sigma = \frac{1}{2}(e^{i\delta_3} - e^{i\delta_2}) = \sin\frac{\delta_3 - \delta_2}{2} \exp i\left(\frac{\delta_3 + \delta_2}{2} + \frac{\pi}{2}\right) \ . \tag{3.5}$$

Here we have put $P \equiv P_u P_d^\dagger \equiv \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3})$, and we have taken $\delta_1 = 0$ without loss of generality.

Then, the explicit magnitudes of the components of V are expressed as

$$|V_{cb}| = |\sigma|\, c_u = \frac{\sin\frac{\delta_3-\delta_2}{2}}{\sqrt{1+m_u/m_c}}, \tag{3.6}$$

$$|V_{ub}| = |\sigma|\, s_u = \frac{\sin\frac{\delta_3-\delta_2}{2}}{\sqrt{1+m_u/m_c}}\sqrt{\frac{m_u}{m_c}}, \tag{3.7}$$

$$|V_{ts}| = |\sigma|\, c_d = \frac{\sin\frac{\delta_3-\delta_2}{2}}{\sqrt{1+m_d/m_s}}, \tag{3.8}$$

$$|V_{td}| = |\sigma|\, s_d = \frac{\sin\frac{\delta_3-\delta_2}{2}}{\sqrt{1+m_d/m_s}}\sqrt{\frac{m_d}{m_s}}, \tag{3.9}$$

$$\begin{aligned} |V_{us}| &= c_u s_d \left|1 - \rho \frac{s_u}{c_u}\frac{c_d}{s_d}\right| = \sqrt{\frac{m_c}{m_c+m_u}}\sqrt{\frac{m_d}{m_s+m_d}} \\ &\times \left[1 - 2\cos\frac{\delta_3-\delta_2}{2}\cos\frac{\delta_3+\delta_2}{2}\sqrt{\frac{m_u m_s}{m_c m_d}} + \cos^2\frac{\delta_3-\delta_2}{2}\left(\frac{m_u m_s}{m_c m_d}\right)\right]^{\frac{1}{2}}, \end{aligned} \tag{3.10}$$

$$\begin{aligned} |V_{cd}| &= c_u s_d \left|\rho - \frac{s_u}{c_u}\frac{c_d}{s_d}\right| = \sqrt{\frac{m_c}{m_c+m_u}}\sqrt{\frac{m_d}{m_s+m_d}} \\ &\times \left[\cos^2\frac{\delta_3-\delta_2}{2} - 2\cos\frac{\delta_3-\delta_2}{2}\cos\frac{\delta_3+\delta_2}{2}\sqrt{\frac{m_u m_s}{m_c m_d}} + \left(\frac{m_u m_s}{m_c m_d}\right)\right]^{\frac{1}{2}}. \end{aligned} \tag{3.11}$$

It should be noted that the elements of V are independent of m_t and m_b. The independent parameters in the expression $|V_{ij}|$ are $\theta_u = \tan^{-1}(m_u/m_c)$, $\theta_d = \tan^{-1}(m_d/m_s)$, δ_3, and δ_2. Among them, the two parameters θ_u and θ_d are already fixed by the quark masses of the first and second generations. Therefore, the present model has two adjustable parameters δ_3 and δ_2 to reproduce the observed CKM matrix parameters [11]:

$$\begin{aligned} &|V_{us}|_{\rm exp} = 0.2196 \pm 0.0026, \quad |V_{cb}|_{\rm exp} = 0.0412 \pm 0.0020, \\ &|V_{ub}|_{\rm exp} = (3.6 \pm 0.7) \times 10^{-3}, \end{aligned} \tag{3.12}$$

It should be noted that the predictions

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{s_u}{c_u} = \sqrt{\frac{m_u}{m_c}} = \sqrt{\frac{2.33}{677}} = 0.0586 \pm 0.0064\ , \tag{3.13}$$

$$\frac{|V_{td}|}{|V_{ts}|} = \frac{s_d}{c_d} = \sqrt{\frac{m_d}{m_s}} = \sqrt{\frac{4.69}{93.4}} = 0.224 \pm 0.014 \tag{3.14}$$

are almost independent of the RGE effects, because they do not contain the phase difference, $(\delta_3-\delta_2)$, which is highly dependent on the energy scale as we discuss in the appendix [see (A.9)] and we know that the ratios m_u/m_c and m_d/m_s are almost independent of the RGE effects. In the numerical results of (3.13) and (3.14), we have used the running quark mass at $\mu=m_Z$ [12]:

$$\begin{aligned} m_u(m_Z) &= 2.33^{+0.42}_{-0.45}\,\mathrm{MeV},\ \ m_c(m_Z)=677^{+56}_{-61}\,\mathrm{MeV}, \\ m_d(m_Z) &= 4.69^{+0.60}_{-0.66}\,\mathrm{MeV},\ \ m_s(m_Z)=93.4^{+11.8}_{-13.0}\,\mathrm{MeV}. \end{aligned} \tag{3.15}$$

The predicted value (3.13) is somewhat small with respect to the present experimental value $|V_{ub}|/|V_{cb}|=0.08\pm0.02$, but it is within the error.

The heavy-quark-mass-independent predictions (3.13) and (3.14) have first been derived from a special ansatz for quark mixings by Branco and Lavoura [13], and later, a similar formulation has also be given by Fritzsch and Xing [14]. For example, the CKM matrix V is given by the form $V=R_{12}(\theta_u)R_{23}(\theta_Q,\phi_Q)R_{12}^T(\theta_d)$ in the Fritzsch–Xing ansatz, and their rotation $R_{23}(\theta_Q,\phi_Q)$ with a phase ϕ_Q corresponds to $R_{23}(-\pi/4)P_uP_d^\dagger R_{23}^T(-\pi/4)$ in the present model, because the present rotation given in (1.8) is expressed as $O_f=R_{23}(-\pi/4)R_{12}(\theta_f)$. However, we would like to emphasize that the $2\leftrightarrow 3$ mixing in V comes from only the relative phase difference $(\delta_2-\delta_3)$, and it is independent of the forms of the up- and down-mixing matrices (1.8). The present mass matrix texture is completely different from theirs. The rederivation of (3.13) and (3.14) in the present model will illuminate the farsighted instates by Branco and Lavoura.

Next let us fix the parameters δ_3 and δ_2. When we use the expressions (3.6)–(3.11) at $\mu=m_Z$, the parameters δ_2 and δ_3 do not mean the phases that are evolved from those at $\mu=M_X$. Hereafter, we use the parameters δ_2 and δ_3 as phenomenological parameters that approximately satisfy the relations (3.6)–(3.11) at $\mu=m_Z$. In order to fix the value of $\delta_3-\delta_2$, we use the relation (3.6), which leads to

$$\sin\frac{\delta_3-\delta_2}{2}=\sqrt{1+\frac{m_u}{m_c}}\,|V_{cb}|_{\rm exp}=0.0401\pm0.0018\ , \tag{3.16}$$

$$\delta_3-\delta_2=4.59^\circ\pm0.21^\circ. \tag{3.17}$$

Then, we obtain

$$|V_{ub}| = \sqrt{\frac{m_u}{m_c}}\,|V_{cb}|_{\rm exp} = 0.00234 \pm 0.00028, \tag{3.18}$$

$$|V_{ts}| = \sqrt{\frac{1+\frac{m_u}{m_c}}{1+\frac{m_d}{m_s}}}\,|V_{cb}|_{\rm exp} = 0.0391 \pm 0.0018, \tag{3.19}$$

$$|V_{td}| = \sqrt{\frac{1+\frac{m_u}{m_c}}{1+\frac{m_d}{m_s}}}\sqrt{\frac{m_d}{m_s}}\,|V_{cb}|_{\rm exp} = 0.00880 \pm 0.00094, \tag{3.20}$$

which are consistent with the present experimental data. Therefore, the value (3.17) is acceptable as reasonable. Then, by using the value (3.17) and the expression (3.10), we can obtain the remaining parameter $(\delta_3 + \delta_2)$:

$$\delta_3 + \delta_2 = 93^\circ \pm 22^\circ \quad \text{or} \quad -80^\circ \pm 22^\circ \ . \tag{3.21}$$

Since $\sin(\delta_3 - \delta_2)/2 \simeq 0.04$ and $\cos(\delta_3 + \delta_2)/2 \simeq 0.2$, the present model also predicts the following approximated relations

$$|V_{us}| = c_u s_d \left|1 - \rho \frac{s_u}{c_u}\frac{c_d}{s_d}\right| \simeq \sqrt{\frac{m_d}{m_s}}, \tag{3.22}$$

$$|V_{cd}| = c_u s_d \left|\rho - \frac{s_u}{c_u}\frac{c_d}{s_d}\right| \simeq \sqrt{\frac{m_d}{m_s}}, \tag{3.23}$$

$$|V_{td}| = |\sigma|\, s_d = \sqrt{|V_{cb}|^2 + |V_{ub}|^2}\sqrt{\frac{m_d}{m_s + m_d}} \simeq |V_{cb}| \cdot |V_{us}| \,. \tag{3.24}$$

Using the rephasing of the up-type and down-type quarks, Eq. (3.3) is changed to the standard representation of the CKM quark mixing matrix

$$\begin{aligned} V_{\rm std} &= {\rm diag}(e^{\alpha_1^u}, e^{\alpha_2^u}, e^{\alpha_2^u})\ V\ {\rm diag}(e^{\alpha_1^d}, e^{\alpha_2^d}, e^{\alpha_2^d}) \\ &= \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} . \end{aligned} \tag{3.25}$$

Here, α_i^q comes from the rephasing in the quark fields to make the choice of phase convention. The CP-violating phase δ in the representation (3.25) is expressed with the expression V in Eq. (3.3) by

$$\delta = \arg\left[\left(\frac{V_{12}V_{22}^*}{V_{13}V_{23}^*}\right) + \frac{|V_{12}|^2}{1-|V_{13}|^2}\right] , \tag{3.26}$$

so that we obtain

$$\delta = \pm(80^\circ \pm 22^\circ). \tag{3.27}$$

It is interesting that nearly maximal $|\sin\delta|$ is realized in the present model.

The rephasing invariant Jarlskog parameter J [15] is defined by $J = \mathrm{Im}(V_{us}V_{cs}^*V_{ub}^*V_{cb})$. In the present model with (3.6)–(3.11), the parameter J is given by

$$\begin{aligned} J &= |\sigma|^2|\rho|c_u s_u c_d s_d \sin\frac{\delta_3+\delta_2}{2} \\ &= \frac{|V_{ub}|}{|V_{cb}|}\frac{|V_{td}||V_{ts}||V_{tb}|}{1+|V_{ub}/V_{cb}|^2}\sin\frac{\delta_3+\delta_2}{2}. \end{aligned} \tag{3.28}$$

Using the relation $|V_{td}| \simeq |V_{cb}||V_{us}|$ in (3.24), and the experimental findings $|V_{us}|^2 \gg |V_{cb}|^2 \gg |V_{ub}|^2$, $|V_{ts}| \simeq |V_{cb}|$, and $|V_{tb}| \simeq 1$, we obtain

$$J \simeq |V_{ub}||V_{cb}||V_{us}|\sin\frac{\delta_3+\delta_2}{2}. \tag{3.29}$$

On the other hand, in the standard expression of V, (3.25), J is given by

$$\begin{aligned} J &= c_{13}^2 s_{13} c_{12} s_{12} c_{23} s_{23}\sin\delta \\ &= \frac{|V_{ud}||V_{us}||V_{ub}||V_{cb}||V_{tb}|}{1-|V_{ub}|^2}\sin\delta \simeq |V_{us}||V_{ub}||V_{cb}|\sin\delta. \end{aligned} \tag{3.30}$$

Comparing Eq. (3.29) with Eq. (3.30), we obtain

$$\sin\delta \simeq \sin\frac{\delta_3+\delta_2}{2}. \tag{3.31}$$

By using the numerical results (3.17)–(3.21), we obtain

$$|J| = (1.91 \pm 0.38) \times 10^{-5}. \tag{3.32}$$

IV. LEPTON MIXING MATRIX

Let us discuss the lepton sectors. We assume that the neutrino masses are generated via the seesaw mechanism [16]:

$$M_\nu = -M_D M_R^{-1} M_D^T \ . \tag{4.1}$$

Here M_D and M_R are the Dirac neutrino and the right-handed Majorana neutrino mass matrices, which are defined by $\overline{\nu}_L M_D \nu_R$ and $\overline{\nu}_R^c M_R \nu_R$, respectively. Since $M_D = P_\nu^\dagger \widehat{M}_D P_\nu^\dagger$ and $M_R = P_\nu^\dagger \widehat{M}_R P_\nu^\dagger$ according to the assumption (2.9), we obtain

$$\begin{aligned} M_\nu &= -P_\nu^\dagger \widehat{M}_D \widehat{M}_R^{-1} \widehat{M}_D^T P_\nu^\dagger \\ &= P_\nu^\dagger \begin{pmatrix} 0 & \sqrt{\frac{m_2 m_1}{2}} & \sqrt{\frac{m_2 m_1}{2}} \\ \sqrt{\frac{m_2 m_1}{2}} & \frac{1}{2} m_3 \left(1 + \frac{m_2 - m_1}{m_3}\right) & -\frac{1}{2} m_3 \left(1 - \frac{m_2 - m_1}{m_3}\right) \\ \sqrt{\frac{m_2 m_1}{2}} & -\frac{1}{2} m_3 \left(1 - \frac{m_2 - m_1}{m_3}\right) & \frac{1}{2} m_3 \left(1 + \frac{m_2 - m_1}{m_3}\right) \end{pmatrix} P_\nu^\dagger \ . \end{aligned} \tag{4.2}$$

Here and hereafter, m_1, m_2 and m_3 denote neutrino masses unless they are specifically mentioned. In the last expression, we have used the fact[1] that the product of $AB^{-1}A$ of the matrices A and B with the texture (1.1) with (1.2) again becomes a matrix with the texture (1.1) with (1.2).

On the other hand, the charged lepton mass matrix M_e is given by

$$M_e = P_e^\dagger \begin{pmatrix} 0 & \sqrt{\frac{m_\mu m_e}{2}} & \sqrt{\frac{m_\mu m_e}{2}} \\ \sqrt{\frac{m_\mu m_e}{2}} & \frac{1}{2} m_\tau \left(1 + \frac{m_\mu - m_e}{m_\tau}\right) & -\frac{1}{2} m_\tau \left(1 - \frac{m_\mu - m_e}{m_\tau}\right) \\ \sqrt{\frac{m_\mu m_e}{2}} & -\frac{1}{2} m_\tau \left(1 - \frac{m_\mu - m_e}{m_\tau}\right) & \frac{1}{2} m_\tau \left(1 + \frac{m_\mu - m_e}{m_\tau}\right) \end{pmatrix} P_e^\dagger \ , \tag{4.3}$$

where m_e, m_μ and m_τ are charged lepton masses.

Those mass matrices M_e and M_ν are diagonalized as $(P_e^\dagger O_e)^\dagger M_e (P_e O_e) = D_e$ and $(P_\nu^\dagger O_\nu)^\dagger M_\nu (P_\nu O_\nu) = D_\nu$, respectively, where

[1]The seesaw invariant texture form was discussed systematically in [17].

$$O_e = \begin{pmatrix} c_e & s_e & 0 \\ -\frac{s_e}{\sqrt{2}} & \frac{c_e}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_e}{\sqrt{2}} & \frac{c_e}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad O_\nu = \begin{pmatrix} c_\nu & s_\nu & 0 \\ -\frac{s_\nu}{\sqrt{2}} & \frac{c_\nu}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_\nu}{\sqrt{2}} & \frac{c_\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{4.4}$$

Here c_e and s_e are obtained from Eq. (1.9) by replacing m_1 and m_2 in it by m_e and m_μ; c_ν and s_ν are also obtained by taking the neutrino masses m_i. Therefore, the Maki–Nakagawa–Sakata–Pontecorv (MNSP) lepton mixing matrix [18] U can be written as

$$\begin{aligned} U &= O_e^T P O_\nu \\ &= \begin{pmatrix} c_e c_\nu + \rho_\nu s_e s_\nu & c_e s_\nu - \rho_\nu s_e c_\nu & -\sigma_\nu s_e \\ s_e c_\nu - \rho_\nu c_e s_\nu & s_e s_\nu + \rho_\nu c_e c_\nu & \sigma_\nu c_e \\ -\sigma_\nu s_\nu & \sigma_\nu c_\nu & \rho_\nu \end{pmatrix}, \end{aligned} \tag{4.5}$$

where $P \equiv P_e P_\nu^\dagger \equiv \mathrm{diag}(e^{i\delta_{\nu 1}}, e^{i\delta_{\nu 2}}, e^{i\delta_{\nu 3}})$. Hereafter we take $\delta_{\nu 1} = 0$ without loss of generality.

The explicit forms of absolute magnitudes of the components of U are given by expressions similar to (3.4)–(3.12), where $|V_{ij}|$, (m_u, m_c, m_t), and (m_d, m_s, m_b) are replaced by $|U_{ij}|$, (m_1, m_2, m_3), and (m_e, m_μ, m_τ), respectively. It should again be noted that the elements of U are independent of m_τ and m_3. The independent parameters of the unitary matrix U are $\theta_e = \tan^{-1}(m_e/m_\mu)$, $\theta_\nu = \tan^{-1}(m_1/m_2)$, $\delta_{\nu 3}$, and $\delta_{\nu 2}$. Among them, θ_e is given by charged-lepton masses of the first and second generations. Therefore, the model has the three adjustable parameters $\delta_{\nu 3}$, $\delta_{\nu 2}$, and m_1/m_2 to reproduce the experimental values [11].

Let us estimate the values θ_ν, $\delta_{\nu 3}$ and $\delta_{\nu 2}$ by fitting the experimental data. In the following discussions we consider the normal mass hierarchy $\Delta m^2_{23} = m_3^2 - m_2^2 > 0$ for the neutrino mass. The case of the inverse mass hierarchy $\Delta m^2_{23} < 0$ is quite similar to it. It follows from the CHOOZ [19], solar [20], and atmospheric neutrino experiments [1] that

$$|U_{13}|^2_{\rm exp} < 0.03 \ . \tag{4.6}$$

From the global analysis of the SNO solar neutrino experiment [20],

$$\Delta m_{12}^2 = m_2^2 - m_1^2 = \Delta m_{\rm sol}^2 = 5.0 \times 10^{-5}\,{\rm eV}^2, \tag{4.7}$$

$$\tan^2\theta_{12} = \tan^2\theta_{\rm sol} = 0.34, \tag{4.8}$$

with $\chi^2_{min}/{\rm dof} = 57.0/72$, for the large mixing angle (LMA) MSW solution. From the atmospheric neutrino experiment [1], we also have

$$\Delta m_{23}^2 = m_3^2 - m_2^2 \simeq \Delta m_{\rm atm}^2 = 2.5 \times 10^{-3}\,{\rm eV}^2, \tag{4.9}$$

$$\sin^2 2\theta_{23} \simeq \sin^2 2\theta_{\rm atm} = 1.0, \tag{4.10}$$

with $\chi^2_{min}/{\rm dof} = 163.2/170$.

Independently of the parameters $\delta_{\nu 3}$ and $\delta_{\nu 2}$, the model predicts the following two ratios:

$$\frac{|U_{13}|}{|U_{23}|} = \frac{s_e}{c_e} = \sqrt{\frac{m_e}{m_\mu}} = \sqrt{\frac{0.487}{103}} = 0.0688, \tag{4.11}$$

$$\frac{|U_{31}|}{|U_{32}|} = \frac{s_\nu}{c_\nu} = \sqrt{\frac{m_1}{m_2}}\ . \tag{4.12}$$

Here we have used the running charged-lepton mass at $\mu = m_Z$ [12]: $m_e(m_Z) = 0.48684727 \pm 0.00000014$ MeV, and $m_\mu(m_Z) = 102.75138 \pm 0.00033$ MeV. The neutrino mixing angle $\theta_{\rm atm}$ under the constraint $|\Delta m_{23}^2| \gg |\Delta m_{12}^2|$ is given by

$$\begin{aligned} \sin^2 2\theta_{\rm atm} &\equiv 4\,|U_{23}|^2\,|U_{33}|^2 \\ &= 4\,|\rho_\nu|^2\,|\sigma_\nu|^2\,c_e^2 = \sin^2(\delta_{\nu 3} - \delta_{\nu 2})\sqrt{\frac{m_\mu}{m_\mu + m_e}}\ . \end{aligned} \tag{4.13}$$

The observed fact $\sin^2 2\theta_{\rm atm} \simeq 1.0$ highly suggests $\delta_{\nu 3} - \delta_{\nu 2} \simeq \pi/2$. Hereafter, for simplicity, we take

$$\delta_{\nu 3} - \delta_{\nu 2} = \frac{\pi}{2}\ . \tag{4.14}$$

Under the constraint (4.13), the model predicts

$$|U_{13}|^2 = \frac{1}{2}\frac{m_e}{m_\mu + m_e} = 0.00236\ ,\ \ {\rm or}\ \ \sin^2 2\theta_{13} = 0.00942\ . \tag{4.15}$$

This value is consistent with the present experimental constraints (4.6) and can be checked in neutrino factories [21], which have sensitivity to $\sin^2 2\theta_{13}$ for

$$\sin^2 2\theta_{13} \geq 10^{-5}. \tag{4.16}$$

The mixing angle $\theta_{\rm sol}$ in the present model is given by

$$\begin{aligned}
\sin^2 2\theta_{\rm sol} &\equiv 4\left|U_{11}\right|^2 \left|U_{12}\right|^2 \\
&\simeq \frac{4m_2 m_1}{(m_2+m_1)^2}\left[1-\sqrt{2}\cos\frac{\delta_{\nu 3}+\delta_{\nu 2}}{2}\sqrt{\frac{m_e m_2}{m_\mu m_1}}+\frac{1}{2}\left(\frac{m_e m_2}{m_\mu m_1}\right)\right] \\
&\simeq \frac{4m_1/m_2}{(1+m_1/m_2)^2} \ ,
\end{aligned} \tag{4.17}$$

which leads to

$$\frac{m_1}{m_2} \simeq \tan^2\theta_{\rm sol} = 0.34 \ , \tag{4.18}$$

where we have used the best fit value (4.8). This value (4.18) guarantees the validity of the approximation (4.17), because of $\sqrt{(m_e/m_\mu)/(m_1/m_2)} \simeq 0.12$. Then, we can obtain the neutrino masses

$$\begin{aligned}
m_1 &= 0.0026\,{\rm eV} \ , \\
m_2 &= 0.0075\,{\rm eV} \ , \\
m_3 &= 0.050\,{\rm eV} \ ,
\end{aligned} \tag{4.19}$$

where we have used the observed best fit values of $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm atm}$, (4.7) and (4.9), respectively.

Next let us discuss the CP violation phases in the lepton mixing matrix. The Majorana neutrino fields do not have the freedom of rephasing invariance, so that we can use only the rephasing freedom of M_e to transform Eq. (4.5) to the standard form

$$U_{\rm std} \equiv \begin{pmatrix}
c_{\nu 13}c_{\nu 12} & c_{\nu 13}s_{\nu 12}e^{i\beta} & s_{\nu 13}e^{i(\gamma-\delta_\nu)} \\
\left(-c_{\nu 23}s_{\nu 12}-s_{\nu 23}c_{\nu 23}s_{\nu 13}e^{i\delta_\nu}\right)e^{-i\beta} & c_{\nu 23}c_{\nu 12}-s_{\nu 23}s_{\nu 12}s_{\nu 13}e^{i\delta_\nu} & s_{\nu 23}c_{\nu 13}e^{i(\gamma-\beta)} \\
\left(s_{\nu 23}s_{\nu 12}-c_{\nu 23}c_{\nu 12}s_{\nu 13}e^{i\delta_\nu}\right)e^{-i\gamma} & \left(-s_{\nu 23}c_{\nu 12}-c_{\nu 23}s_{\nu 12}s_{\nu 13}e^{i\delta_\nu}\right)e^{-i(\gamma-\beta)} & c_{\nu 23}c_{\nu 13}
\end{pmatrix} , \tag{4.20}$$

as

$$U_{\rm std} = {\rm diag}(e^{i\alpha_1^e}, e^{i\alpha_2^e}, e^{i\alpha_3^e})\ U\ {\rm diag}(e^{\pm i\pi/2}, 1, 1)\ . \tag{4.21}$$

Here, α_i^e comes from the rephasing in the charged lepton fields to make the choice of phase convention, and the specific phase $\pm\pi/2$ is added on the right-hand side of U in order to change the neutrino eigen-mass m_1 to a positive quantity. Similarly to the quark sector, the CP-violating phase δ_ν in the representation (4.20) is expressed as

$$\delta_\nu = \arg\left[\frac{U_{12}U_{22}^*}{U_{13}U_{23}^*} + \frac{|U_{12}|^2}{1-|U_{13}|^2}\right] \simeq \arg\left(\frac{U_{12}U_{22}^*}{U_{13}U_{23}^*}\right) \simeq \arg\rho_\nu^* + \pi = -\frac{\delta_{\nu 3}+\delta_{\nu 2}}{2} + \pi\ . \tag{4.22}$$

Though the lepton mixing matrix includes the additional Majorana phase factors β and γ [22,23], the number of parameters which will become experimentally available in the near future is practically four, as in the Dirac case. The additional phase parameters are determined as

$$\beta = \arg\left(\frac{U_{\rm std\ 12}}{U_{\rm std\ 11}}\right) = \arg\left(\frac{U_{12}}{U_{11}e^{\pm i\pi}}\right) \simeq 0 \mp \frac{\pi}{2}\ , \tag{4.23}$$

and

$$\gamma = \arg\left(\frac{U_{\rm std\ 13}}{U_{\rm std\ 11}}e^{i\delta_\nu}\right) = \arg\left(\frac{U_{13}}{U_{11}e^{\pm i\pi}}e^{i\delta_\nu}\right) \simeq \arg(-\sigma_\nu) + \delta_\nu \mp \frac{\pi}{2} \simeq \frac{\pi}{2} \mp \frac{\pi}{2}\ , \tag{4.24}$$

by using the relations $m_e \ll m_\mu$ and $(\delta_{\nu 3} - \delta_{\nu 2})/2 \simeq \pi/4$. Hence, we can also predict the averaged neutrino mass $\langle m_\nu \rangle$ [23], which appears in the neutrinoless double beta decay, as follows:

$$\begin{aligned}\langle m_\nu \rangle &\equiv \left| -m_1 U_{11}^2 + m_2 U_{12}^2 + m_3 U_{13}^2 \right| \\ &= \left| -2\rho_\nu c_e s_e \sqrt{m_1 m_2} + \rho_\nu^2 s_e^2 (m_2 - m_1) + m_3 s_e^2 \right|\ .\end{aligned} \tag{4.25}$$

The value of (4.25) is highly sensitive to the value of $(\delta_{\nu 3} + \delta_{\nu 2})/2$, which is unknown at present, because the values $s_e/c_e = \sqrt{m_e/m_\mu} \simeq 0.070$ and $\sqrt{m_1 m_2}/m_3 \simeq 0.088$ are in the same order. For $(\delta_{\nu 3} + \delta_{\nu 2})/2 = 0$, $\pi/2$ and π, we obtain the numerical results $\langle m_\nu \rangle =$ 0.00018 eV, 0.00049 eV and 0.00069 eV, respectively. However, these values should not be taken strictly because the value m_1/m_2 is also sensitive to the observed value of $\tan^2\theta_{sol}$. In any cases, the predicted value of $\langle m_\nu \rangle$ will be less than the order of 10^{-3} eV.

The rephasing-invariant parameter J in the lepton sector is defined by $J = \mathrm{Im}(U_{12}U_{22}^*U_{13}^*U_{23})$, which is explicitly given by

$$\begin{aligned} J &= |\sigma_\nu|^2|\rho_\nu|c_\nu s_\nu c_e s_e \sin\frac{\delta_{\nu 3}+\delta_{\nu 2}}{2} \\ &= \frac{|U_{13}|}{|U_{23}|}\frac{|U_{31}||U_{32}||U_{33}|}{1+|U_{13}/U_{23}|^2}\sin\frac{\delta_{\nu 3}+\delta_{\nu 2}}{2} \leq \frac{|U_{13}|}{|U_{23}|}\frac{|U_{31}||U_{32}||U_{33}|}{1+|U_{13}/U_{23}|^2} \ . \end{aligned} \tag{4.26}$$

The upper bound is described in terms of the ratio m_1/m_2, so that we obtain

$$J \leq 0.019. \tag{4.27}$$

It should be noted that if we again assume the maximal CP violation in the lepton sector, the magnitude of the rephasing invariant $|J|$ can be considerably larger than in the quark sector, $|J_{\rm quark}| \simeq 2\times 10^{-5}$.

V. CONCLUSION

In conclusion, stimulated by recent neutrino data, which suggest a nearly bimaximal mixing, we have investigated a possibility that all the mass matrices of quarks and leptons have the same texture as the neutrino mass matrix. We have assumed that the mass matrix form is constrained by a discrete symmetry Z_3 and a permutation symmetry S_2, i.e. that the texture is given by the form (1.1) with (1.2). The most important feature of the present model is that the textures (1.1)–(1.2) are practically applicable to the predictions at the low energy scale (the electroweak scale), although we assume that the textures are exactly given at a unification scale.

It is well known that the matrix form (1.1) leads to a bimaximal mixing in the neutrino sector. In the present model, the mixing angle θ_{12}^f between the first and second generations is given by

$$\tan\theta_{12}^f = \sqrt{m_1^f/m_2^f} \ , \tag{5.1}$$

where m_1^f and m_2^f are the first and second generation fermion masses. This leads to a large mixing in the lepton mixing matrix (MNSP matrix) U with $m_1 \sim m_2$ (neglecting

$\tan\theta^e_{12} = \sqrt{m_e/m_\mu}$ in the charge lepton sector), and it also leads to the famous formula [24] $|V_{us}| \simeq \sqrt{m_d/m_s}$ in the quark mixing matrix (CKM matrix) V (neglecting $\tan\theta^u_{12} = \sqrt{m_u/m_c}$ in the up-quark sector). In the present model the mixing angle θ^f_{23} between the second and third generation is fixed as $\theta^f_{23} = \pi/4$. However, the (2,3) component of the quark mixing matrix V (and also the lepton mixing matrix U) is highly dependent on the phase difference $\delta_3 - \delta_2$, as follows

$$V_{23} = \frac{1}{\sqrt{1 + m^u_1/m^u_2}} \sin\frac{\delta_3 - \delta_2}{2} \ , \tag{5.2}$$

where $\delta_i = \delta^u_i - \delta^d_i$. Replacing the arguments by their leptonic counterparts, we have the same form for U_{23}. We have understood the observed values V_{23} and U_{23} by taking $(\delta_3 - \delta_2)/2$ as a small value for the quark sectors and as $\pi/2$ for the lepton sectors, respectively. As predictions, which are independent of such phase parameters, there are two relations

$$\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{m_u}{m_c}} \ , \quad \frac{|V_{td}|}{|V_{ts}|} = \sqrt{\frac{m_d}{m_s}} \ , \tag{5.3}$$

(and the similar relations for U). The relations (5.3) are in good agreement with experiments. The relation $|U_{13}/U_{23}| = \sqrt{m_e/m_\mu}$ in the lepton sectors leads to $|U_{13}|^2 \simeq m_e/2m_\mu = 0.0024$ if we accept $\sin^2 2\theta_{\rm atm} = 1.0$. This value will be testable in the near future.

Since, in the present model, each mass matrix M_f (i.e. the Yukawa coupling Y_f) takes different values of A_f, B_f, and so on, the present model cannot be embedded into a GUT scenario. In spite of such a demerit, however, it is worth while noting that it can give a unified description of quark and lepton mass matrices with the same texture.

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APPENDIX

The mass matrix texture (1.1) with (1.2), which is defined at the unification energy scale $\mu = M_X$, is applicable to the phenomenology at the electroweak scale $\mu = m_Z$. In the present appendix, we demonstrate this for the quark mass matrices M_u and M_d.

It is well known [25] that the energy scale dependences $R(A) = A(\mu)/A(M_X)$ for observable quantities A approximately satisfy the relations $R(|V_{ub}|) \simeq R(|V_{cb}|) \simeq R(|V_{td}|) \simeq R(|V_{ts}|) \simeq R(m_d/m_b) \simeq R(m_s/m_b)$, and that the ratios $R(|V_{us}|)$, $R(|V_{cd}|)$, $R(m_d/m_s)$ and $R(m_u/m_c)$ are approximately constant. This is caused by the fact that the Yukawa coupling constant y_t of the top quark is extremely large with respect to other coupling constants. The above relations on R are well explained by the approximation $y_t^2 \gg y_b^2, y_c^2, \cdots$. Therefore, we will also use approximation below.

The one-loop RGE for the Yukawa coupling constants Y_f $(f = u, d)$ has the form

$$\frac{dY_f}{dt} = \frac{1}{16\pi^2}\left(C_f \mathbf{1} + C_{ff} Y_f Y_f^\dagger + C_{ff'} Y_{f'} Y_{f'}^\dagger\right) Y_f \ , \tag{A.1}$$

where $f' = d$ $(f' = u)$ for $f = u$ $(f = d)$, and the coefficients C_f, C_{ff} and $C_{ff'}$ are energy scale dependent factors which are calculated from the one-loop Feynman diagrams. We start from the Yukawa coupling constants $Y_f(M_X)$, corresponding to the mass matrix form (1.1) [with (1.2)].

Since the matrix $Y_u Y_u^\dagger$ is approximately given by

$$Y_u(M_X) Y_u^\dagger(M_X) \simeq \frac{m_t^2}{v_u^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -e^{+i\delta_u} \\ 0 & -e^{-i\delta_u} & 1 \end{pmatrix} , \tag{A.2}$$

where $\delta_u = \delta_3^u - \delta_2^u$, $v_u/\sqrt{2} = \langle H_u^0 \rangle$, and we have used the relations (1.6) and the approximation $y_t^2 \gg y_b^2, y_c^2, \cdots$, the up-quark Yukawa coupling constant $Y_u(\mu)$ in the neighbourhood of $\mu = M_X$ is given by the form

$$Y_u(\mu) \simeq r_u(\mu)\left[\mathbf{1} + \varepsilon_u(\mu)\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -e^{+i\delta_u} \\ 0 & -e^{-i\delta_u} & 1 \end{pmatrix}\right] Y_u(M_X) \simeq \frac{r_u(\mu)}{v_u/\sqrt{2}}$$

$$\begin{pmatrix} 0 & A_u e^{-i\delta_2^u} & A_u e^{-i\delta_3^u} \\ A_u(1+\varepsilon_u-\varepsilon_u)e^{-i\delta_2^u} & B_u(1+\varepsilon_u-\varepsilon_u C_u/B_u)e^{-2i\delta_2^u} & C_u(1+\varepsilon_u-\varepsilon_u B_u/C_u)e^{-i(\delta_2^u+\delta_3^u)} \\ A_u(1+\varepsilon_u-\varepsilon_u)e^{-i\delta_3^u} & C_u(1+\varepsilon_u-\varepsilon_u B_u/C_u)e^{-i(\delta_2^u+\delta_3^u)} & B_u(1+\varepsilon_u-\varepsilon_u C_u/B_u)e^{-2i\delta_3^u} \end{pmatrix} . \tag{A.3}$$

Although this form is one in $\mu \simeq M_X$, but, since the texture keeps the same form under the small change of energy scale, as a result, the texture of $Y_u(\mu)$ given by (A.3) holds at any energy scale μ. Therefore, we can obtain the expression (1.1) at an arbitrary energy scale μ. (The demonstration (A.3) has been done for the case $P_R = P_L^\dagger$ mentioned in (2.10). However, the conclusion does not depend on this choice.)

On the other hand, the evolution of the down-quark Yukawa coupling constant $Y_d(\mu)$ is somewhat complicated. By a way similar to (A.3), we obtain

$$Y_d(\mu) \simeq r_d(\mu) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1+\varepsilon_d & -\varepsilon_d e^{+i\delta_u} \\ 0 & -\varepsilon_d e^{-i\delta_u} & 1+\varepsilon_d \end{pmatrix} Y_d(M_X) \simeq \frac{r_d(\mu)}{v_d/\sqrt{2}}$$

$$P_d^\dagger \begin{pmatrix} 0 & A_d & A_d \\ A_d(1+\varepsilon_d-\varepsilon_d e^{+i(\delta_u-\delta_d)}) & B_d(1+\varepsilon_d-\varepsilon_d e^{+i(\delta_u-\delta_d)}C_d/B_d) & C_d(1+\varepsilon_d-\varepsilon_d e^{+i(\delta_u-\delta_d)}B_d/C_d) \\ A_d(1+\varepsilon_d-\varepsilon_d e^{-i(\delta_u-\delta_d)}) & C_d(1+\varepsilon_d-\varepsilon_d e^{-i(\delta_u-\delta_d)}B_d/C_d) & B_d(1+\varepsilon_d-\varepsilon_d e^{-i(\delta_u-\delta_d)}C_d/B_d) \end{pmatrix} P_d^\dagger \tag{A.4}$$

where $\delta_u - \delta_d = (\delta_3^u - \delta_2^u) - (\delta_3^d - \delta_2^d) = \delta_3 - \delta_2$. Note that the part that is sandwiched between $P_d^\dagger$ and $P_d^\dagger$ includes imaginary parts and those phase factors cannot be removed by an additional phase matrix $P_d(\mu)$ into the form $P_d^\dagger(\mu)\widehat{Y}_d(\mu)P_d^\dagger(\mu)$. However, the quantity that has the physical meaning is $Y_d Y_d^\dagger$. When we define

$$\xi e^{-i\alpha} = 1 + \varepsilon_d(1 - e^{-i(\delta_u-\delta_d)}) \ , \quad \eta e^{+i\beta} = 1 + \varepsilon_d(1 + e^{+i(\delta_u-\delta_d)}) \ , \tag{A.5}$$

we obtain

$$Y_d(\mu)Y_d^\dagger(\mu) \simeq \frac{r_d^2(\mu)}{v_d^2/2} P_d^\dagger P_\beta \begin{pmatrix} A_d^2 & A_d(B_d+C_d) & A_d(B_d+C_d) \\ A_d(B_d+C_d)\eta & A_d^2\xi^2 + (B_d^2+C_d^2)\eta^2 & A_d^2\xi^2 e^{2i(\alpha-\beta)} + 2B_dC_d\eta^2 \\ A_d(B_d+C_d)\eta & A_d^2\xi^2 e^{2i(\alpha+\beta)} + 2B_dC_d\eta^2 & A_d^2\xi^2 + (B_d^2+C_d^2)\eta^2 \end{pmatrix} P_\beta P_d^\dagger \tag{A.6}$$

where

$$P_\beta = \mathrm{diag}(1, e^{i\beta}, e^{-i\beta}) \ , \tag{A.7}$$

so that we can obtain a real matrix for the part which is sandwiched by the phase matrix $P_d P_\beta$ under the approximation $A_d^2/|B_dC_d| \simeq 0$. This means that we can practically write

$$\widehat{Y}_d(\mu) \simeq \frac{r_d(\mu)}{v_d/\sqrt{2}} \begin{pmatrix} 0 & A_d & A_d \\ A_d\xi & B_d\eta & C_d\eta \\ A_d\xi & C_d\eta & B_d\eta \end{pmatrix} \ , \tag{A.8}$$

with

$$P_d^\dagger(\mu) = \mathrm{diag}(1, e^{-i(\delta_2^d-\beta)}, e^{-i(\delta_3^d+\beta)}) \ , \tag{A.9}$$

at an arbitrary energy scale μ. It should be noted that the changes of the phases $\delta_2^d \to \delta_2^d - \beta$ and $\delta_3^d \to \delta_3^d + \beta$ do not come from the evolution of the phases $\delta_2^d(\mu)$ and $\delta_3^d(\mu)$, but they are brought effectively by absorbing the unfactorizable phase parts in $Y_d(\mu)$. Thus, we can again use the texture (1.1) at an arbitrary energy scale μ from a practical point of view.

In the Yukawa coupling constants Y_e and Y_ν of the leptons, the RGE effects are not so large as in the quark sectors. In the charged lepton sector, since $m_\tau^2 \gg m_\mu^2 \gg m_e^2$, we can again demonstrate that the expression (1.1) is applicable at an arbitrary energy scale in a way similar to the quark sectors. For the neutrino Yukawa coupling constant $Y_\nu(\mu)$, the evolution equation is different from (A.1). We must use the RGE for the seesaw operator [26]. However, the calculation and result are essentially the same as those in $Y_u(\mu)$, $Y_d(\mu)$ and $Y_e(\mu)$, because $m_3^2 \gg m_2^2 > m_1^2$ in the present model.

Finally, we would like to add that these conclusions on the evolution of the mass matrices M_f $(f = u, d, e, \nu)$ are exactly confirmed by numerical study, without approximation.

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