A New Idea to Evade the Proton Decay in the Extension of the Zee Model to the SUSY GUT and Possible Forms of the Radiatively-Induced Neutrino Mass Matrix

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In order to evade the proton decay which appears when the Zee model is embedded into a SUSY GUT scenario with R-parity violation, a new idea based on a discrete symmetry Z_2 has recently been proposed. Under the symmetry Z_2, the quark and lepton mass matrices are tightly constrained. Possible forms of the radiatively-induced neutrino mass matrix are systematically investigated.

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I. INTRODUCTION

The origin of the neutrino mass generation is till mysterious problem in the unified understanding of the quarks and leptons. The Zee model [1] is one of promising models, because the model has only 3 free parameters and it can naturally lead to a large neutrino mixing [2], especially, to a bimaximal mixing [3]. However, the original Zee model is not on a framework of a grand unification theory (GUT)., and moreover, The most attractive idea [4] to embed the Zee model into GUT is to identify the Zee scalar h^+ as the slepton $\widetilde{e}_R$ in an R-parity violating supersymmetric (SUSY) model. However, usually, it is accepted that SUSY models with R-parity violation are incompatible with a GUT scenario, because the R-parity violating interactions induce the proton decay.

Recently, in order to suppress the proton decay due to the R-parity violating terms, a discrete Z_2 symmetry has been proposed [5]. As we review in Sec.II, the essential idea is that the R-parity violating interactions occur only when the field ψ_{10} of the third family is related under the Z_2 symmetry. Then, the quark and lepton mass matrices are tightly constrained. In the present paper, we will give a systematical study of the radiatively induced neutrino mass matrix forms under the Z_2 symmetry.

II. Z_2 SYMMETRY AND THE PROTON DECAY

We identify the Zee scalar h^+ as the slepton $\widetilde{e}_R^+$ which is a member of SU(5) 10-plet sfermions $\widetilde{\psi}_{10}$. Then, the Zee interactions correspond to the following R-parity violating interactions

$$\lambda_{ij}^k(\overline{\psi}_{\overline{5}}^c)_i^A(\psi_{\overline{5}})_j^B(\widetilde{\psi}_{10})_{kAB} \ , \tag{2.1}$$

where $\psi^c \equiv C\overline{\psi}^T$ and the indices $(i,j,\cdots)$, $(A,B,\cdots)$ and $(\alpha,\beta,\cdots)$ are family-, $SU(5)_{GUT}$- and $SU(3)_{color}$-indices, respectively. However, in GUT models, if the interactions (2.1) exist, the following R-parity violating interactions will also exist:

$$\lambda_{ij}^k(\overline{\psi}_{\overline{5}}^c)_i^A(\psi_{10})_{kAB}(\widetilde{\psi}_{\overline{5}})_j^B \ , \tag{2.2}$$

which contribute to the proton decay through the intermediate state $\widetilde{d}_R$.

In order to forbid the contribution of the interactions (2.2) to the proton decay, we assume that the R-parity violating interactions occur only when the field ψ_{10} of the third family is related, i.e., we assume the interactions

$$\lambda_{ij}^3(\overline{\psi}_{\overline{5}}^c)_i^A(\psi_{10})_{3AB}(\widetilde{\psi}_{\overline{5}})_j^B \ , \tag{2.3}$$

instead of the interaction (2.2). Then, the terms $\lambda_{ij}^3(\overline{d}_R)_i(\widetilde{d}_R^\dagger)_j(u_R^c)_3$ cannot contribute to the proton decay. In order to realize the constraints

$$\lambda_{12}^k = \lambda_{23}^k = \lambda_{31}^k = 0 \quad \text{for} \quad k=1,2 \ , \tag{2.4}$$

we introduce a discrete symmetry Z_2, which exactly holds at every energy scale, as follows:

$$\begin{array}{ll} (\psi_{\overline{5}})_i \to \eta_i(\psi_{\overline{5}})_i \ , & (\widetilde{\psi}_{\overline{5}})_i \to \eta_i(\widetilde{\psi}_{\overline{5}})_i \ , \\ (\psi_{10})_i \to \xi_i(\psi_{10})_i \ , & (\widetilde{\psi}_{10})_i \to \xi_i(\widetilde{\psi}_{10})_i \ , \end{array} \tag{2.5}$$

where η_i and ξ_i take

$$\eta = (+1,+1,+1) \ , \quad \xi = (-1,-1,+1) \ , \tag{2.6}$$

under the Z_2 symmetry. Then, the Z_2 invariance leads to the constraints (2.4).

However, if the renormalization group equation (RGE) effects cause a mixing between the first and third families, the interactions (2.3) can again contribute to the proton decay. If we assume that **5** and $\overline{\mathbf{5}}$ Higgs fields H_u and H_d transform as

$$H_u \to +H_u \ , \quad H_d \to +H_d \ , \tag{2.7}$$

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under the Z_2 symmetry, the up-quark mass matrix M_u is given by the form

$$M_u = \begin{pmatrix} c_u & d_u & 0 \\ d_u & b_u & 0 \\ 0 & 0 & a_u \end{pmatrix} \ . \qquad (2.8)$$

This guarantees that the top quark u_3 in the R-parity violating terms (2.3) does not mix with the other components (u_1 and u_2) even if we take the RGE effects into consideration, so that the interactions (2.3) cannot contribute to the proton decay at any energy scales.

On the other hand, the down-quark mass matrix M_d and the charged lepton mass matrix M_e, which are generated by the Higgs scalar H_d, have the form

$$M_d = M_e^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_1 & a_2 & a_3 \end{pmatrix} \ . \qquad (2.9)$$

The mass matrix form (2.9) cannot explain the observed masses and mixings. In order to give reasonable masses and mixings of the quarks and charged leptons, we must consider additional SU(5) 45-plet Higgs scalars, which do not contribute to the up-quark mass matrix because $\overline{\psi}_{10} M_u \psi_{10}^c$ belongs to $(\overline{10} \times \overline{10})_{symmetric}$. Then, we obtain the down-fermion mass matrices

$$M_d = \begin{pmatrix} c_1' & c_2' & c_3' \\ b_1' & b_2' & b_3' \\ a_1 + a_1' & a_2 + a_2' & a_3 + a_3' \end{pmatrix} \ , \qquad (2.10)$$

$$M_e^T = \begin{pmatrix} -3c_1' & -3c_2' & -3c_3' \\ -3b_1' & -3b_2' & -3b_3' \\ a_1 - 3a_1' & a_2 - 3a_2' & a_3 - 3a_3' \end{pmatrix} \ , \qquad (2.11)$$

where a_i' and (b_i', c_i') denote contributions from the 45-plet Higgs scalars $H_{45}^{(+)}$ and $H_{45}^{(-)}$ which transform $H_{45}^{(+)} \to +H_{45}^{(+)}$ and $H_{45}^{(-)} \to -H_{45}^{(-)}$ under the symmetry Z_2, respectively.

However, such additional Higgs scalars $H_{45}^{(\pm)}$ cause another problems. One is a problem of the flavor changing neutral currents (FCNC). This problem is a common subject to overcome not only in the present model but also in most GUT models. The conventional mass matrix models based on GUT scenario cannot give realistic mass matrices without assuming Higgs scalars more than two. For this problem, we optimistically consider that only one component of the linear combinations among those Higgs scalars survives at the low energy scale $\mu = \Lambda_L$ (Λ_L is the electroweak energy scale), while other components are decoupled at $\mu < \Lambda_X$ (Λ_X is a unification scale).

Another problem is that the **45** Higgs scalars can have vacuums expectation values (VEV) at the electroweak energy scale Λ_L, so that the Z_2 symmetry is broken at $\mu = \Lambda_L$. Therefore, the proton decay may occur through higher order Feynman diagrams. In the conventional GUT models, it is still a current topic whether the colored components of the SU(5) 5-plet Higgs scalar can become sufficiently heavy or not to suppress the proton decay. We again optimistically assume that the colored components of the 45-plet Higgs scalars are sufficiently heavy to suppress the proton decay, i.e., that such effects will be suppressed by a factor $(\Lambda_L/\Lambda_X)^2$.

III. RADIATIVELY INDUCED NEUTRINO MASSES

We define fields u_i, d_i and e_i as those corresponding to mass eigenstates, i.e.,

$$H_{mass} = \overline{u}_L U_L^{u\dagger} M_u U_R^u u_R + \overline{d}_L U_L^{d\dagger} M_d U_R^d d_R$$

$$+ \overline{e}_L U_L^{e\dagger} M_e U_R^e e_R + h.c. \ , \qquad (3.1)$$

and fields ν_{Li} as partners of the mass eigenstates e_{Li}, i.e., $\ell_{Li} = (\nu_{Li}, e_{Li})$. We define the neutrino mass matrix M_ν as

$$H_{\nu\ mass} = \overline{\nu}_L^c M_\nu \nu_L \ . \qquad (3.2)$$

Therefore, a unitary matrix U_L^ν which is defined by

$$U_L^{\nu T} M_\nu U_L^\nu = D_\nu \equiv \mathrm{diag}(m_1^\nu, m_2^\nu, m_3^\nu) \ , \qquad (3.3)$$

is identified as the Maki-Nakagawa-Sakata-Pontecorvo [6] neutrino mixing matrix $U_{MNSP} = U_L^\nu$.

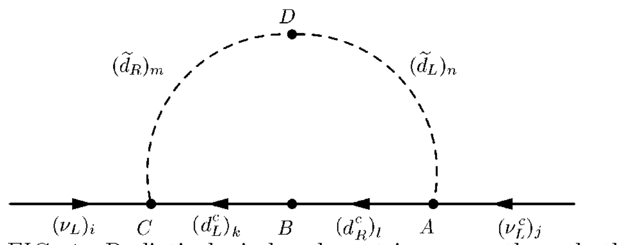

FIG. 1. Radiatively induced neutrino mass through the down-quark loop. The vertices A, B, C and D are given by $(U_R^{d\dagger} \lambda U_L^e)_{lj} (\widetilde{U}_L^d)_{3n}$, $(U_L^{d\dagger} M_d U_R^d)_{kl}$, $(\widetilde{U}_R^{d\dagger} \lambda U_L^e)_{mi} (U_L^d)_{3k}$, and $(\widetilde{U}_L^{d\dagger} \widetilde{m}_d^2 \widetilde{U}_R^d)_{kl}$, respectively.

In addition to the R-parity violating terms (2.1) and (2.2) [(2.3)], we assume SUSY breaking terms $\widetilde{\psi}_{\overline{5}} \widetilde{\psi}_{10} H_{\overline{5}}^d$ (and $\widetilde{\psi}_{\overline{5}} \widetilde{\psi}_{10} H_{\overline{45}}^d$). For the moment, we do not consider $\widetilde{e}_R^c$-H_d^+ mixing as in the original Zee model. Then, the neutrino masses are radiatively generated. In Fig. 1, we illustrate the Feynman diagram for the case with the down-quark loop. The amplitude is proportional to the coefficient

$$(U_R^{d\dagger} \lambda U_L^e)_{lj} (\widetilde{U}_L^d)_{3n} \cdot (U_L^{d\dagger} M_d U_R^d)_{kl}$$

$$\cdot (\widetilde{U}_R^{d\dagger} \lambda^T U_L^e)_{mi} (U_L^d)_{3k} \cdot (\widetilde{U}_L^{d\dagger} \widetilde{m}_d^2 \widetilde{U}_R^d)_{nm}$$

$$= (\widetilde{m}_d^2 \lambda U_L^e)_{3i} (M_d \lambda U_L^e)_{3j} \ , \tag{3.4}$$

where $(\widetilde{m}_d^2)_{ij}$ are coefficients of $(\widetilde{d}_L^\dagger)_i (\widetilde{d}_R)_j$, and $(\lambda)_{ij} = \lambda_{ij}^3$. Similarly, we obtain the contributions from the charged lepton loops. Therefore, the radiatively induced neutrino mass matrix M_ν is given by the following form

$$(M_\nu)_{ij} = (f_i^e g_j^e + f_j^e g_i^e) K_e + (f_i^d g_j^d + f_j^d g_i^d) K_d \ , \tag{3.5}$$

where K_f $(f = e, d)$ are common factors independently of the families, and

$$\begin{array}{ll} f_i^e = (M_e^T \lambda U_L^e)_{3i} \ , & g_i^e = (\widetilde{m}_e^{2T} \lambda U_L^e)_{3i} \ , \\ f_i^d = (M_d \lambda U_L^e)_{3i} \ , & g_i^d = (\widetilde{m}_d^2 \lambda U_L^e)_{3i} \ . \end{array} \tag{3.6}$$

On the other hand, the contributions due to the H_d^+-$\widetilde{e}^+$ mixing are as follows. There are no contributions from bilinear terms $H_u(5)[H_d(\overline{5}) + \sum_i \widetilde{\psi}(\overline{5})]$, because we can always eliminate the contributions from $H_u(5)\widetilde{\psi}(\overline{5})$ by re-definition of the scalar $H_d(\overline{5})$. Also, there are no contributions from $H_d(\overline{5}) H_d(\overline{5}) \widetilde{\psi}(10)$, because $\widetilde{\psi}(10)$ is anti-symmetric in SU(5) indices. However, the terms $H_d(\overline{5}) H_d(\overline{45}) \widetilde{\psi}(10)$ can cause the H_d^+ - $\widetilde{e}^+$ mixing. Therefore, the final result which includes the H_d^+ contributions is given by

$$M_{ij} = (f_i^e g_j^e + f_j^e g_i^e) K_e + (f_i^d g_j^d + f_j^d g_i^e) K_d + F_{ij}^e K_e^{'}, \tag{3.7}$$

where

$$F_{ij}^e = (U_L^{eT} \lambda M_e M_e^{'\dagger} U_L^e)_{ij} + (i \leftrightarrow j), \tag{3.8}$$

and $M_e^{'}$ denotes the contributions due to $\sum \overline{\nu}_L e_L H_d^+$ [H_d^+ denotes $(H_d(\overline{5}))^+$, $(H_d^{(+)}(\overline{45}))^+$ and $(H_d^{(-)}(\overline{45}))^+$]. We have already assumed that only one component of the linear combinations among those Higgs scalars survives at the low energy scale. Then, we can regard $M_e^{'}$ as $M_e^{'} = M_e$. Therefore, we can denote F_{ij}^e as

$$F_{ij}^e = (\lambda^{'} D_e^2)_{ij} + (i \leftrightarrow j) = \lambda_{ij}^{'} (m_j^{e2} - m_i^{e2}), \tag{3.9}$$

where $\lambda^{'} = U_L^{eT} \lambda U_L^e$ is antisymmetric tensor as well as λ , and $m_i^e = (m_e, m_\mu, m_\tau)$. Note that, in this case, the third term in (3.7) has the same mass matrix form as that in the original Zee model.

IV. PHENOMENOLOGY

In the SUSY GUT scenario, there are many origins of the neutrino mass generations. For example, the sneutrinos $\widetilde{\nu}_{iL}$ can have the VEV, and thereby, the neutrinos ν_{Li} acquire their masses (for example, see Ref. [7]). Although we cannot rule out a possibility that the observed neutrino masses can be understood from such compound origins, in the present paper, we do not take such the point of view, because the observed neutrino masses and mixings appear to be rather simple and characteristic. We simply assume that the radiative masses are only dominated even if there are other origins of the neutrino mass generations.

However, even if we neglect all the other contributions except for the radiatively induced neutrino masses, we still have too many parameters. In the present section, we investigate some special cases of the expression (3.7) by assuming simple relations among the parameters from the phenomenological point of view.

A. special case with $f_2 = f_3$ and $g_2 = g_3$

It is well known that the nearly bimaximal mixing is derived from the neutrino mass matrix with 2↔3 symmetry [8]. Therefore, first, we investigate a case with $f_2^f = f_3^f \gg f_1^f$ and $g_2^f = g_3^f \gg g_1^f$ $(f = e, d)$. For the third term of (3.7), for simplicity, we assume $\lambda'_{12} \simeq \lambda'_{13} \simeq \lambda'_{23}$, so that we neglect the term $F_{12}^e = \lambda'_{12}(m_\mu^2 - m_e^2)$ compared with $F_{23}^e = \lambda'_{23}(m_t^2 - m_\mu^2)$ and $F_{13}^e = \lambda'_{13}(m_\tau^2 - m_e^2)$. Then, the neutrino mass matrix (3.7) approximately becomes a simple form

$$M_\nu \simeq m_0 \begin{pmatrix} 0 & 0 & a \\ 0 & 1 & 1+a \\ a & 1+a & 1 \end{pmatrix} \ , \tag{4.1}$$

$$m_0 = 2(f_3^e g_3^e K_e + f_3^d g_3^d K_d) \ , \tag{4.2}$$

$$m_0 a = F_{13}^e K_e' \simeq F_{23}^e K_e' \ . \tag{4.3}$$

For a case $|a| \ll 1$, the mass eigenvalues are given by

$$m_1 = \left(\frac{\sqrt{3}-1}{2} a - \frac{3+\sqrt{3}}{24} a^2 + O(a^3) \right) m_0 \ ,$$

$$m_2 = - \left(\frac{\sqrt{3}+1}{2} a + \frac{3-\sqrt{3}}{24} a^2 + O(a^3) \right) m_0 \ , \tag{4.4}$$

$$m_3 = \left(2 + a + \frac{1}{4} a^2 + O(a^3) \right) m_0 \ ,$$

so that those give

$$\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = \sqrt{3} a^2 \left(1 + \frac{1}{6} a + O(a^2) \right) m_0^2 \ , \tag{4.5}$$

$$\Delta m_{32}^2 \equiv m_3^2 - m_2^2 = \left(4 + 4a + \frac{2-\sqrt{3}}{2} a^2 + O(a^3) \right) m_0^2 \ , \tag{4.6}$$

$$R \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{\sqrt{3}}{4}a^2\left(1-\frac{5}{6}a\right) + O(a^4) \ . \qquad (4.7)$$

The mixing matrix U is give by

$$U \simeq \left(\begin{array}{ccc} c & -s & \frac{1}{2\sqrt{2}}a \\ -\frac{1}{\sqrt{2}}s & -\frac{1}{\sqrt{2}}c & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & \frac{1}{\sqrt{2}} \end{array} \right) \ , \qquad (4.8)$$

where $c = [(\sqrt{3}+1)/2\sqrt{3}]^{\frac{1}{2}}$ and $s = [(\sqrt{3}-1)/2\sqrt{3}]^{\frac{1}{2}}$ and it gives

$$U^2_{13} = \frac{1}{8}a^2(1-a) + O(a^4) \ , \qquad (4.9)$$

$$\sin^2 2\theta_{solar} \equiv 4U^2_{11}U^2_{12} = \frac{2}{3}\left(1+\frac{1}{3}a\right) + O(a^2) \ , \quad (4.10)$$

$$\sin^2 2\theta_{atm} \equiv 4U^2_{23}U^2_{33} = 1 - \frac{1}{4}a^2 + O(a^2) \ . \qquad (4.11)$$

From the recent atmospheric and solar neutrino data [9,10]

$$R \simeq \frac{4.5\times 10^{-5}\mathrm{eV}^2}{2.5\times 10^{-3}\mathrm{eV}^2} = 1.8\times 10^{-2} \ , \qquad (4.12)$$

we estimate

$$a \simeq 0.23 \ , \qquad (4.13)$$

so that we obtain

$$U^2_{13} = 0.0050 \ , \qquad (4.14)$$

$$\sin^2 2\theta_{solar} = 0.72 \ , \qquad (4.15)$$

$$\sin^2 2\theta_{atm} = 0.99 \ . \qquad (4.16)$$

B. Model $M_{ij} = m_0(f_i g_j + f_j g_i)$ with inverse hierarchy

When the K'_e-terms are negligibly small and either the K_e- or K_d-terms are also negligible, the general form (3.7) becomes a simple form

$$(M_\nu)_{ij} = m_0(f_i g_i + f_j g_i) \ . \qquad (4.17)$$

Hereafter, for convenience, we will normalize f_i and g_i as

$$|f_1|^2+|f_2|^2+|f_3|^2 = 1, \quad |g_1|^2+|g_2|^2+|g_3|^2 = 1 \ . \quad (4.18)$$

In the most SUSY models, it is taken that the form of $\widetilde{m}^2_f$ $(f = e, d)$ is proportional to the fermion mass matrix M_f. Then, the coefficients g_i are proportional to f_i, so that the mass matrix (4.17) becomes $(M_\nu)_{ij} = 2m_0 f_i f_j$, which is a rank one matrix. Therefore, we rule out the case with $\widetilde{m}^2_f \propto M_f$.

For convenience, hereafter, we assume that f_i and g_i $(i = 1, 2, 3)$ are real. The mass eigenvalues and mixing matrix elements for the neutrino mass matrix (4.17) are given as follows [5]:

$$\begin{array}{l} m^\nu_1 = (1+\xi)m_0 \ , \\ m^\nu_2 = -(1-\xi)m_0 \ , \\ m^\nu_3 = 0 \ , \end{array} \qquad (4.19)$$

$$\begin{array}{l} U_{i1} = \dfrac{1}{\sqrt{2}}\dfrac{f_i+g_i}{\sqrt{1+\xi}} \ , \\ U_{i2} = \dfrac{1}{\sqrt{2}}\dfrac{f_i-g_i}{\sqrt{1-\xi}} \ , \\ U_{i3} = -\varepsilon_{ijk}\dfrac{f_j g_k}{\sqrt{1-\xi^2}} \ , \end{array} \qquad (4.20)$$

where

$$\xi = f_1 g_1 + f_2 g_2 + f_3 g_3 \ . \qquad (4.21)$$

As seen in (4.19), the mass level pattern of the present model shows the inverse hierarchy as well as that of the Zee model. From (4.19), we obtain

$$\begin{array}{l} \Delta m^2_{21} \equiv (m^\nu_2)^2 - (m^\nu_1)^2 = -4\xi m^2_0 \ , \\ \Delta m^2_{32} \equiv (m^\nu_3)^2 - (m^\nu_2)^2 = -(1-\xi)^2 m^2_0 \ , \end{array} \qquad (4.22)$$

$$R \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{4\xi}{(1-\xi)^2} \ . \qquad (4.23)$$

For a small R, i.e., $\xi \equiv \varepsilon \simeq 0$, the mixing parameters $\sin^2 2\theta_{solar}$, $\sin^2 2\theta_{atm}$ and U^2_{e3} are given by

$$\sin^2 2\theta_{solar} \equiv 4U^2_{11}U^2_{12} = \frac{1}{1-\varepsilon^2}(f^2_1 - g^2_1)^2 \ , \qquad (4.24)$$

$$\sin^2 2\theta_{atm} \equiv 4U^2_{23}U^2_{33}$$

$$= \frac{4}{(1-\varepsilon^2)^2}[f_2 f_3 + g_2 g_3 - \varepsilon(f_3 g_2 + f_2 g_3)]^2 \ , \qquad (4.25)$$

$$U^2_{13} = 1 - \frac{f^2_1 + g^2_1 - 2\varepsilon f_1 g_1}{1-\varepsilon^2} \ . \qquad (4.26)$$

In order to demonstrate that the mass matrix form (4.17) has reasonable parameter values for the observed data, we again assume the $2 \leftrightarrow 3$ symmetry:

$$\begin{array}{l} f_1 = s_\alpha, \ f_2 = f_3 = \frac{1}{\sqrt{2}}c_\alpha \ , \\ g_1 = c_\beta, \ g_2 = g_3 = -\frac{1}{\sqrt{2}}s_\beta \ , \end{array} \qquad (4.27)$$

where $c_\alpha = \cos\alpha$, $s_\alpha = \sin\alpha$ and so on. Then, the parameterization (4.27) gives

$$\xi \equiv \varepsilon = \sin(\alpha - \beta) \ , \qquad (4.28)$$

$$\sin^2 2\theta_{solar} = \cos^2(\alpha+\beta) \ , \tag{4.29}$$

$$\sin^2 2\theta_{atm} = 1 \ , \tag{4.30}$$

$$U_{13}^2 = 0 \ . \tag{4.31}$$

We assume that the values of α and β are highly close each other, i.e., $\sin(\alpha-\beta) \sim 10^{-2}$. The result (4.29) with $\alpha \simeq \beta$ means that we can fit the value of $\sin^2 2\theta_{solar}$ with the observed value [10] $\sin^2 2\theta_{solar} \sim 0.8$ from the solar neutrino data by adjusting the parameter α ($\simeq \beta$).

From the observed data (4.12), we estimate

$$\varepsilon = 4.5 \times 10^{-3} \ , \tag{4.32}$$

and

$$m_0 \simeq m_1^\nu \simeq |m_2^\nu| \simeq \sqrt{\Delta m^2_{atm}} = 0.050 \text{ eV} \ . \tag{4.33}$$

The effective neutrino mass $\langle m_\nu \rangle$ from the neutrinoless double beta decay experiment is given by

$$\langle m_\nu \rangle = (M_\nu)_{11} = 2 m_0 c_\alpha s_\beta$$

$$\simeq m_0 \sqrt{1-\sin^2 2\theta_{solar}} \simeq 2.2 \times 10^{-3} \text{ eV} \ , \tag{4.34}$$

where we have used the observed value [10] $\sin^2 2\theta_{solar} \simeq 0.8$.

C. Model $M_{ij} = m_0(f_i g_j + f_j g_i)$ with normal hierarchy

Note that, as seen in (4.23), the choice $1-\xi \simeq 0$ is also a possible solution which can explain the observed ratio (4.12). Instead of (4.19), we redefine

$$m_1^\nu = 0 \ ,$$

$$m_2^\nu = -(1-\xi) m_0 = -\varepsilon m_0 \ , \tag{4.35}$$

$$m_3^\nu = (1+\xi) m_0 = (2-\varepsilon) m_0 \ ,$$

where we have put $1-\xi = \varepsilon$. Since we may consider that M_f is approximately proportional to $\widetilde{m}_f^2$, it is likely that $f_1 g_1 + f_2 g_2 + f_3 g_3 \simeq 1$. Then, instead of (2.23)-(2.26), we obtain

$$R = \frac{\varepsilon^2}{4(1-\varepsilon)} \ , \tag{4.36}$$

$$\sin^2 2\theta_{solar} = \frac{(f_1-g_1)^2 (f_2 g_3 + f_3 g_2)^2}{\varepsilon(1-\frac{1}{2}\varepsilon)} \ , \tag{4.37}$$

$$\sin^2 2\theta_{atm} = \frac{(f_2+g_2)^2 (f_3+f_3)^2}{(2-\varepsilon)^2} \ , \tag{4.38}$$

$$U_{13}^2 = \frac{(f_1+g_1)^2}{4(1-\frac{1}{2}\varepsilon)} \ . \tag{4.39}$$

Again, we investigate a special case

$$f_1 = -g_1 = \sin\alpha \ ,$$

$$f_2 = -g_3 = \frac{1}{\sqrt{2}} \cos\alpha (\cos\beta - \sin\beta) \ , \tag{4.40}$$

$$f_3 = -g_2 = \frac{1}{\sqrt{2}} \cos\alpha (\cos\beta + \sin\beta) \ ,$$

which gives

$$U_{13}^2 = 0 \ , \quad \sin^2 2\theta_{atm} = 1 \ . \tag{4.41}$$

Then, (4.37) is given by

$$\sin^2 2\theta_{solar} = \frac{4\rho}{(1+\rho)^2} \ , \tag{4.42}$$

where

$$\rho = \sin^2\beta / \tan^2\alpha \ . \tag{4.43}$$

By the way, the parameterization (4.40) leads to the expression

$$M_\nu = 2 f_2 f_3 m_0 \begin{pmatrix} -b^2 & ab & -ab \\ ab & 1 & 1+a^2 \\ -ab & 1+a^2 & a \end{pmatrix} \ , \tag{4.44}$$

where

$$a = \frac{f_3 - f_2}{\sqrt{2 f_2 f_3}} = \frac{\sqrt{2}\sin\beta}{\sqrt{\cos 2\beta}} \ , \tag{4.45}$$

$$b = \frac{f_1}{\sqrt{2 f_2 f_3}} = \frac{\tan\alpha}{\sqrt{\cos 2\beta}} \ . \tag{4.46}$$

Although $f_2 \neq f_3$ in the present model, the mass matrix form (4.44) still keeps $2 \leftrightarrow 3$ symmetric (anti-symmetric). In the limit of $a = b = 0$ (i.e., $f_1^2 = (f_2-f_3)^2 = 0$), the mass matrix (4.44) becomes 2×2 democratic form. If we assume that the breaking parameters f_1^2 and $(f_2-f_3)^2$ of the democratic form satisfy the relation

$$f_1^2 = (f_3 - f_2)^2 \ , \tag{4.47}$$

which corresponds to

$$\tan^2\alpha = 2\sin^2\beta \ , \tag{4.48}$$

then, we obtain $\rho = 1/2$, so that we get

$$\varepsilon = 2\sin^2\alpha(1+\rho) = 3\sin^2\alpha \ , \tag{4.49}$$

$$\sin^2 2\theta_{solar} = 8/9 = 0.89 \ . \tag{4.50}$$

Note that the value of $\sin^2 2\theta_{solar}$ is independent of the value of α. The observed value of R, (4.12), is realized by taking $\varepsilon \simeq 1/4$, i.e., $\sin^2\alpha \simeq 1/12$.

V. CONCLUSION

In conclusion, we have proposed a neutrino mass matrix model based on a SUSY GUT model where only top quark takes R-parity violating interactions and the Z_2 symmetry plays an essential role, so that the proton decay due to the R-parity interactions can be evaded safely.

The general form of the radiatively induced neutrino mass matrix in the present model is given by the expression (3.7). Since the form (3.7) has too many parameters, we have investigated phenomenology for some special cases. A simple case (4.1) is very interesting, because the case leads to reasonable results $U_{13}^2 \simeq a^2/12$, $\sin^2 2\theta_{solar} \simeq 2/3$, and $\sin^2 2\theta_{atm} \simeq 1$ together with $R \simeq \sqrt{3}a^2/4$. The case (4.44) is also attractive, since it gives $U_{13}^2 = 0$, $\sin^2 2\theta_{solar} = 8/9$, and $\sin^2 2\theta_{atm} = 1$ when we assume a simple relation among the parameters. In order to know which model is reasonable, more careful study about SUSY breaking terms in the model is required together with phenomenological study of the quark and charged lepton mass matrices.

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