Evolution of the Yukawa coupling constants and seesaw operators in the universal seesaw model

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The general features of the evolution of the Yukawa coupling constants and seesaw operators in the universal seesaw model with $\det M_F = 0$ are investigated. In the model, not only the magnitude of the Yukawa coupling constant $(Y_L^u)_{33}$ in the up-quark sector but also that of $(Y_L^d)_{33}$ in the down-quark sector is of the order of one, i.e., $(Y_L^u)_{33} \sim (Y_L^d)_{33} \sim 1$. The requirement that the model should be calculable perturbatively, i.e., $|Y_{ij}^f|^2/4\pi \leq 1$, puts some constraints on the values of the intermediate mass scales and $\tan\beta$ (in the SUSY model).

I. INTRODUCTION

Recently, there has been considerable interest in the evolution (energy-scale dependency) of the Yukawa coupling constants of quarks and leptons. If we intend to build a model which gives a unified description of quark and lepton mass matrices, we cannot avoid investigating evolutions of the Yukawa coupling constants. The recent study on the quark masses and mixings has been given, for example, in Ref. [1]. Especially, recently, the evolution of the neutrino seesaw mass matrix has been received considerable attention (for example, see Ref. [2]) in connection with the energy-scale dependence of the large mixing angle.

As one of such unified models, there is a non-standard model, the so-called "universal seesaw model" (USM) [3]. The model describes not only the neutrino mass matrix M_ν but also the quark mass matrices M_u and M_d and charged lepton mass matrix M_e by the seesaw-type matrices universally: The model has hypothetical fermions F_i ($F = U, D, N, E$; $i = 1, 2, 3$) in addition to the conventional quarks and leptons f_i ($f = u, d, \nu, e$; $i = 1, 2, 3$), and these fermions are assigned to $f_L = (2, 1)$, $f_R = (1, 2)$, $F_L = (1, 1)$ and $F_R = (1, 1)$ of $\mathrm{SU}(2)_L\times \mathrm{SU}(2)_R$. The 6×6 mass matrix which is sandwiched between the fields $(\overline{f}_L, \overline{F}_L)$ and (f_R, F_R) is given by

$$M^{6\times 6} = \left(\begin{array}{cc} 0 & m_L \\ m_R^\dagger & M_F \end{array} \right) , \tag{1.1}$$

where m_L and m_R are universal for all fermion sectors ($f = u, d, \nu, e$) and only M_F have structures dependent on the flavors F. For $\Lambda_L < \Lambda_R \ll \Lambda_S$, where $\Lambda_L = O(m_L)$, $\Lambda_R = O(m_R)$ and $\Lambda_S = O(M_F)$, the 3×3 mass matrices M_f for the fermions f are given by the well-known seesaw expression

$$M_f \simeq -m_L M_F^{-1} m_R^\dagger . \tag{1.2}$$

Thus, the model answers the question why the masses of quarks (except for top quark) and charged leptons are so small compared with the electroweak scale Λ_L ($\sim 10^2$ GeV).

Recently, in order to understand the observed fact $m_t \sim \Lambda_L$ (m_t is the top quark mass), the authors have proposed a universal seesaw mass matrix model with an ansatz [4–6] $\det M_F = 0$ for the up-quark sector ($F = U$). In the model, one of the fermion masses $m(U_i)$ is zero [say, $m(U_3) = 0$], so that the seesaw mechanism does not work for the third family, i.e., the fermions (u_{3L}, U_{3R}) and (U_{3L}, u_{3R}) acquire masses of $O(m_L)$ and $O(m_R)$, respectively. We identify (u_{3L}, U_{3_R}) as the top quark (t_L, t_R). Thus, we can understand the question why only the top quark has a mass of the order of Λ_L.

Our interest is as follows: In the conventional model, the Yukawa coupling constants y_f of the fermions f are given by $y_f = m_f/\langle\phi_L^0\rangle$. Only the Yukawa coupling constants y_t of the top quark t takes a large value $y_t = m_t/\langle\phi_L^0\rangle \sim 1$. The other Yukawa coupling constants y_f are sufficiently smaller than one. On the contrast to the conventional model, in this USM, the matrices $m_L^f = Y_L^f \langle\phi_L^0\rangle$ are universal for all fermion sectors $f = u, d, e, \nu$, i.e., $Y_L^u = Y_L^d = Y_L^e = Y_L^\nu$. Therefore, when $(Y_L^u)_{33}$ is of the order of one, the other $(Y_L^f)_{33}$ will also be of the order of one. We are afraid that in such a model the Yukawa coupling constants have Landau poles at energy scales lower than a unification energy scale $\mu = \Lambda_X$ (so that the model causes "burst" of Yukawa coupling constants before going to the unification energy scale). One of our interests is to investigate whether such a model can provide or not a set of reasonable parameter values under the conditions that the Yukawa coupling constants (and also the seesaw operators $m_L M_F^{-1} m_R^\dagger$) do not have the Landau poles below $\mu = \Lambda_X$.

We also take an interest in the "democratic" USM [4,5], which is an extended version of USM and has successfully given the quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) [7] matrix parameters in terms of the charged lepton masses. However, the study is only phenomenology at the energy scale $\mu = m_Z$ (m_Z is the neutral weak boson mass). Since the model is one of the promising models of the unified description of the quark and lepton mass matrices, it is important to investigate the evolutions of the mass matrices in the USM.

The democratic USM is as follows:
(i) The mass matrices m_L and m_R have the same structure except for their phase factors

$$m_R^f = \kappa m_L^f \equiv \kappa m_0 Z^f \ , \tag{1.3}$$

where κ is a constant and Z^f are given by

$$Z^f = \mathrm{diag}\left(z_1 \exp(i\delta_1^f), z_2 \exp(i\delta_2^f), z_3 \exp(i\delta_3^f)\right) \ , \tag{1.4}$$

with $z_1^2 + z_2^2 + z_3^2 = 1$. (The fermion masses m_i^f are independent of the parameters δ_i^f. Only the values of the CKM matrix parameters $|V_{ij}|$ depend on the parameters δ_i^f).
(ii) In the basis on which the matrices m_L^f and m_R^f are diagonal, the mass matrices M_F are given by the form

$$M_F = m_0 \lambda (\mathbf{1} + 3b_f X), \tag{1.5}$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \ . \tag{1.6}$$

(iii) The parameter b_f for the charged lepton sector is given by $b_e = 0$, so that in the limit of $\kappa/\lambda \ll 1$, the parameters z_i are given by

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}} \ . \tag{1.7}$$

Then, the up- and down-quark masses are successfully given by the choice of $b_u = -1/3$ and $b_d = -e^{i\beta_d}$ ($\beta_d = 18^\circ$), respectively. The CKM matrix is also successfully obtained by taking

$$\delta_1^u - \delta_1^d = \delta_2^u - \delta_2^d = 0 \ , \quad \delta_3^u - \delta_3^d \simeq \pi \ , \tag{1.8}$$

However, when we take the evolution of the Yukawa coupling constants into the consideration, we should consider that the assumptions (i) and (ii) are required not at the electroweak energy scale $\mu = \Lambda_L$, but at a unification energy scale $\mu = \Lambda_X$, i.e., the assumptions (i) and (ii) should be replaced with

$$Y_L^f(\Lambda_X) = Y_R^f(\Lambda_X) = \xi_{LR}^f Z^f \ , \tag{1.9}$$

$$\xi_{LR}^u = \xi_{LR}^d, \tag{1.10}$$

and

$$Y_S^f(\Lambda_X) = \xi_S^f (\mathbf{1} + 3b_f X) \ , \tag{1.11}$$

respectively, where mass matrices m_L, m_R and M_F are expressed by

$$m_L^f = Y_L^f \langle \phi_L^0 \rangle \ , \quad m_R^f = Y_R^f \langle \phi_R^0 \rangle \ , \quad M_F = Y_S^f \langle \Phi^0 \rangle \ , \tag{1.12}$$

respectively, and

$$\langle \phi_L^0 \rangle = \langle \phi_R^0 \rangle / \kappa = \langle \Phi^0 \rangle / \lambda \tag{1.13}$$

and ϕ_L, ϕ_R and Φ are Higgs scalars whose vacuum expectation values (VEV) break SU(2)$_L$, SU(2)$_R$, and an additional U(1) symmetry U(1)$_X$, respectively. (For simplicity, we have assumed that the values of $\langle \phi_L^0 \rangle$, $\langle \phi_R^0 \rangle$ and $\langle \Phi^0 \rangle$ are real.)

Another interest in the present paper is to check whether or not the phenomenological study in the previous paper [4] is still approximately valid under the evolution of the Yukawa coupling constants. For example, the model with $b_e = 0$ and $b_u = -1/3$ has led to the relation [8,4]

$$\frac{m_u}{m_c} \simeq \frac{3}{4}\frac{m_e}{m_\mu}, \tag{1.14}$$

almost independently of the value of the seesaw suppression factor κ/λ. One of the reasons to taking the value of b_f in the up-quark sector as $b_u = -1/3$ exists in the successful relation (1.14). Therefore, we have interest whether the relation (1.14) still holds even when we take the evolution into consideration.

Besides, even apart from such phenomenological interests, it is very important to investigate the general features of the evolution of the Yukawa coupling constants in the universal seesaw model with $\det M_F = 0$, because in the present model one of the fermions F_i does not decouple from the theory at $\mu < \Lambda_S$, so that the evolution shows peculiar behavior in contrast with the conventional seesaw model.

A similar study has been done in Ref. [9] by one of the authors (Y.K.). However, in Ref. [9], instead of the seesaw operators K^f which will be defined later in Eqs. (3.8) corresponding to $m_L M_F^{-1} m_R^\dagger$, the evolution of the seesaw forms of the Yukawa coupling constants $Y_L^f (Y_S^f)^{-1} (Y_R^f)^\dagger$ were investigated by calculating the Yukawa coupling constants Y_L^f, Y_R^f and Y_S^f individually under the assumption that the heavy particles with the masses of the order of $\Lambda_S \equiv \langle \Phi^0 \rangle$ do not contribute to the evolution of Y_A^f ($A = L, R, S$) below $\mu = \Lambda_S$. In the present paper, we will calculate the evolution of the Yukawa coupling constants Y_A^f above $\mu = \Lambda_S$ and that of the seesaw operators K^f below $\mu = \Lambda_S$, except for $(Y_L^f)_{i3}$ as discussed in Sec. III.

In Sec. II, we will discuss an additional symmetry which is introduced for the purpose of preventing that the fermions F acquire the masses M_F at the energy scale $\mu = \Lambda_S$. In Sec. III, we will give the general formulation of the evolution of the seesaw mass matrices with $\det M_U = 0$. In Sec. IV, we give the explicit coefficients of the renormalization group equations. In Sec. V, we discuss the evolution of an extended version of the USM, the "democratic seesaw model" [4,5]. The numerical results for a non-SUSY model and for a minimal SUSY model are given in Secs. VI and VII, respectively. It will be emphasized that the energy scale dependencies in the SUSY model are quite different from those in the non-SUSY model. The evolution of the neutrino mass matrix is given in Sec. VIII. It will be showed that, differently from the conventional seesaw model, the present neutrino mass matrix is form-invariant below $\mu = \Lambda_S$. Finally, Sec. IX will be devoted to the conclusions and remarks.

II. $U(1)_X$ SYMMETRY

In the present model, the gauge symmetries are broken as follows:

$$\begin{array}{c} SU(2)_L \times SU(2)_R \times U(1)_{LR} \times SU(3)_c \times U(1)_X \\ \downarrow \quad \mu = \Lambda_S \\ SU(2)_L \times SU(2)_R \times U(1)_{LR} \times SU(3)_c \\ \downarrow \quad \mu = \Lambda_R \\ SU(2)_L \times U(1)_Y \times SU(3)_c \\ \downarrow \quad \mu = \Lambda_L \\ U(1)_{em} \times SU(3)_c \ . \end{array} \tag{2.1}$$

Here, the symmetry $U(1)_X$, which is spontaneously broken at the energy scale $\mu = \Lambda_S$, has been introduced for the purpose of preventing that the fermions F acquire the masses M_F at $\mu > \Lambda_S$. Hereafter, we call the ranges $\Lambda_L < \mu \le \Lambda_R$, $\Lambda_R < \mu \le \Lambda_S$, and $\Lambda_S < \mu \le \Lambda_X$ as the ranges I, II, and III, respectively. In the present paper, the energy scale Λ_X does not always mean a gauge unification energy scale. We assume that at the energy scale Λ_X the mass matrices (Yukawa coupling constants) take simple forms discussed in the previous section.

The Yukawa coupling constants Y_L^f, Y_R^f and Y_S^f are defined as follows:

$$\begin{aligned} H_{int} = & Y^u_{Lij}\overline{q}_{Li}\widetilde{\phi}_L U_{Rj} + Y^d_{Lij}\overline{q}_{Li}\phi_L D_{Rj} + Y^\nu_{Lij}\overline{\ell}_{Li}\widetilde{\phi}_L N_{Rj} + Y^e_{Lij}\overline{\ell}_{Li}\phi_L E_{Rj} \\ & + Y^u_{Rij}\overline{q}_{Ri}\widetilde{\phi}_R U_{Lj} + Y^d_{Rij}\overline{q}_{Ri}\phi_R D_{Lj} + Y^\nu_{Rij}\overline{\ell}_{Ri}\widetilde{\phi}_R N_{Lj} + Y^e_{Rij}\overline{\ell}_{Ri}\phi_R E_{Lj} \\ & + Y^u_{Sij}\overline{U}_{Li}\Phi U_{Rj} + Y^d_{Sij}\overline{D}_{Li}\Phi^\dagger D_{Rj} + Y^\nu_{Sij}\overline{N}_{Li}\Phi N_{Rj} + Y^e_{Sij}\overline{E}_{Li}\Phi^\dagger E_{Rj} + h.c. \ , \end{aligned} \tag{2.2}$$

where

$$q_{L/R} = \begin{pmatrix} u \\ d \end{pmatrix}_{L/R} , \quad \ell_{L/R} = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_{L/R} ,$$
$$\phi_{L/R} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_{L/R} , \quad \widetilde{\phi}_{L/R} = \begin{pmatrix} \overline{\phi}^0 \\ -\phi^- \end{pmatrix}_{L/R} . \tag{2.3}$$

From Eq. (2.2), the $U(1)_X$ charge assignment should satisfy the following relations

$$\begin{aligned} X(U_R) &= X(U_L) - X(\Phi) \ , \\ X(D_R) &= X(D_L) + X(\Phi) \ , \end{aligned} \tag{2.4}$$

$$\begin{aligned} X(q_L) &= \frac{1}{2}\left[X(U_R) + X(D_R)\right] \ , \\ X(q_R) &= \frac{1}{2}\left[X(U_L) + X(D_L)\right] \ , \end{aligned} \tag{2.5}$$

$$\begin{aligned} X(\phi_L) &= \frac{1}{2}\left[X(U_R) - X(D_R)\right] \ , \\ X(\phi_R) &= \frac{1}{2}\left[X(U_L) - X(D_L)\right] \ , \end{aligned} \tag{2.6}$$

for quark sectors, and equations similar to Eqs. (2.4) - (2.6) for lepton sectors $f = \nu, e$. For simplicity, in the present paper, we choose

$$X(q_{L/R}) = X(\ell_{L/R}) = 0 \ , \quad X(\Phi) = +1 \ . \tag{2.7}$$

Then, the quantum numbers of the fermions f and F and Higgs scalars ϕ_L, ϕ_R and Φ for $SU(2)_L \times SU(2)_R \times U(1)_{LR} \times U(1)_X$ are given in Table I.

Note that the quantum number of the fermion N_L is identical with that of the fermion N_R^c [$\equiv (N_R)^c \equiv C\overline{N}_R^T$]. Therefore, the neutral fermions N_L and N_R can acquire the following Majorana mass terms at $\mu = \Lambda_S$:

$$H_{Majorana} = \left(Y^L_{Sij}\overline{N}_{Li}N^c_{Lj} + Y^R_{Sij}\overline{N}^c_{Ri}N_{Rj}\right)\Phi + h.c. \ . \tag{2.8}$$

Then, the neutrino mass matrix is given as follows

$$\left(\overline{\nu}_L \ \overline{\nu}^c_R \ \overline{N}_L \ \overline{N}^c_R\right)\begin{pmatrix} 0 & 0 & 0 & m_L \\ 0 & 0 & m_R^{\dagger T} & 0 \\ 0 & m_R^\dagger & M_L & M_D \\ m_L^T & 0 & M_D^T & M_R \end{pmatrix}\begin{pmatrix} \nu^c_L \\ \nu_R \\ N^c_L \\ N_R \end{pmatrix} , \tag{2.9}$$

where $M_D = Y_S^\nu\langle\Phi\rangle$, $M_L = Y_S^L\langle\Phi\rangle$ and $M_R = Y_S^R\langle\Phi\rangle$. Since $O(M_D) \sim O(M_L) \sim O(M_R) \gg O(m_R) \gg O(m_L)$, we obtain the mass matrix M_ν for the active neutrinos ν_L

$$M_\nu \simeq -m_L M_R^{-1} m_L^T \ . \tag{2.10}$$

III. GENERAL FEATURES OF THE EVOLUTIONS

In the present section, we give a general formulation of the evolution of the seesaw matrix with $\det M_F = 0$. The evolution of the neutrino seesaw mass matrix is well known. However, in such a model with $\det M_F = 0$ as the present model (the democratic seesaw model), a careful treatment is required.

Without losing the generality, we can express the Yukawa coupling constants Y_L^f and Y_R^f ($f = u, d, \nu, e$) as

$$Y_L^f(\mu) = \xi_L^f(\mu) Z_L^f(\mu) \ , \quad Y_R^f(\mu) = \xi_R^f(\mu) Z_R^f(\mu) \ , \tag{3.1}$$

where $Z_A^f(\mu)$ ($A = L, R$) are defined by

$$Z_A^f(\mu) = \mathrm{diag}(z_{A1}^f(\mu), z_{A2}^f(\mu), z_{A3}^f(\mu)) \ , \tag{3.2}$$

$$|z_{A1}^f(\mu)|^2 + |z_{A2}^f(\mu)|^2 + |z_{A3}^f(\mu)|^2 = 1 \ , \tag{3.3}$$

on the basis on which $Y_A^f(\mu)$ are diagonal. In the present model, the word "universal" means the following initial conditions

$$\xi_L^f(\Lambda_X) = \xi_R^f(\Lambda_X) \equiv \xi_{LR} \ , \tag{3.4}$$

$$|z_{Li}^f(\Lambda_X)| = |z_{Ri}^f(\Lambda_X)| \equiv z_i \ , \tag{3.5}$$

for all fermion sectors $f = u, d, \nu, e$ universally.

In the range III ($\Lambda_S < \mu \leq \Lambda_X$), the evolutions of the Yukawa coupling constants Y_L^f, Y_R^f and Y_S^f are given by the one loop renomalization group equations (RGE) as follows:

$$16\pi^2 \frac{dY_A^f}{dt} = \left(T_A^f - G_A^f + H_A^f\right) Y_A^f \ , \quad (A = L, R, S) \ , \tag{3.6}$$

where $t = \log \mu$, and T_A^f, G_A^f and H_A^f ($A = L, R, S$) denote contributions from fermion loop corrections, vertex corrections due to the gauge bosons, and vertex corrections due to the Higgs boson, respectively. Note that the matrices T_A^f and G_A^f are proportional to the unit matrix. As stated in the next section, the coefficients H_A^f ($A = L, R, S$) take diagonal forms on the basis on which Y_A^f are diagonal. Therefore, if we take a basis on which Y_L^f (and Y_R^f) or Y_S^f are diagonal at $\mu = \Lambda_X$, then the Yukawa coupling constants Y_L^f (and Y_R^f) or Y_S^f can keep the forms diagonal in the range III. Sometimes, the basis on which Y_L^f (and Y_R^f) are diagonal is useful, but sometimes, another basis on which Y_S^f are diagonal is useful, as we discuss later.

In the present model, it is assumed that we can choose a flavor basis on which $Y_S^f(\Lambda_X)$ are simultaneously diagonal for all $f = u, d, \nu, e$. Then, on this basis, since the Yukawa coupling constants $Y_S^f(\mu)$ can keep the forms diagonal in the range III, we can find that all Y_S^f are diagonal at $\mu = \Lambda_S$. We can denote those as

$$Y_S^f(\Lambda_S) = \mathrm{diag}(y_{1S}^f, y_{2S}^f, y_{3S}^f) \ . \tag{3.7}$$

At the energy scale $\mu = \Lambda_S$, the fermions F_i (except for U_3) acquire the heavy masses $(M_F)_{ii} = y_{iS}^f \langle \Phi^0 \rangle$. In the conventional seesaw model with $\det M_F \neq 0$, the energy scale behaviors of the fermion masses in $\mu < \Lambda_S$ are described by evolutions of the following operators

$$(K^f)_{ij} = \left[Y_L^f (Y_S^f)^{-1} (Y_R^f)^\dagger\right]_{ij} = \sum_{k=1}^{3} \frac{1}{y_{kS}^f} (Y_L^f)_{ik} (Y_R^f)_{jk}^* \ , \tag{3.8}$$

and

$$(K^\nu)_{ij} = \left[Y_L^\nu (Y_S^\nu)^{-1} (Y_L^\nu)^T\right]_{ij} = \sum_{k=1}^{3} \frac{1}{y_{kS}^\nu} (Y_L^\nu)_{ik} (Y_L^\nu)_{jk} \ . \tag{3.9}$$

(Hereafter, for convenience, we will denote the Yukawa coupling constants Y_S^R in the Majorana mass matrix $M_R = Y_S^R \langle \Phi^0 \rangle$ as Y_S^ν.) The quark and lepton mass matrices M_f are given by

$$M_f = K^f \langle \phi_L^0 \rangle \langle \phi_R^0 \rangle / \langle \Phi^0 \rangle \ , \quad (f = u, d, e) \ , \tag{3.10}$$

$$M_\nu = K^\nu \langle \phi_L^0 \rangle^2 / \langle \Phi^0 \rangle \ . \tag{3.11}$$

As explicitly shown in Sec. IV, the evolutions of the operators K^f are described by the one-loop RGE's with the following forms

$$16\pi^2 \frac{dK^f}{dt} = \left(T_K^f - G_K^f\right) K^f + H_{KL}^f K^f + K^f H_{KR}^{f\dagger} \ , \quad (f = u, d, e), \tag{3.12}$$

$$16\pi^2 \frac{dK^\nu}{dt} = \left(T_K^\nu - G_K^\nu\right) K^\nu + H_{KL}^\nu K^\nu + K^\nu H_{KL}^{\nu T} \ , \tag{3.13}$$

where T_K^f, G_K^f and (H_{KL}^f, H_{KR}^f) denote contributions from fermion loop corrections, vertex corrections due to the gauge bosons, and vertex corrections due to the Higgs bosons ϕ_L and ϕ_R, respectively.

However, in the seesaw mass matrix with $\det M_F = 0$, since one of the eigenvalues of Y_S^f ($f = u$) is zero (say, $y_{3S}^u = 0$), we must calculate the following operator

$$(K^u)_{ij} = \left[Y_L^u (Y_S^u)^{-1} Y_R^{u\dagger}\right]_{ij} = \sum_{k=1}^{2} \frac{1}{y_{kS}^u} (Y_L^u)_{ik} (Y_R^u)_{jk}^* \ , \tag{3.14}$$

where $Y_S^u = \mathrm{diag}(y_{1S}^u, y_{2S}^u)$. Note that the matrices Y_U and Y_L^u (Y_R^u) in Eq. (3.14) are 2×2 and 3×2 matrices, respectively.

Note that in (3.14) we have taken the sum over $k = 1$ and 2 only. In the range II, the evolutions of the Yukawa coupling constants Y_{Li3}^u and Y_{Ri3}^u ($i = 1, 2, 3$) are still described by the equation (3.6). At the energy scale $\mu = \Lambda_R$, we obtain a new mass term

$$H_{mass} = \sum_i (Y_R^u)_{i3}^* \overline{U}_{L3} u_{Ri} \langle \phi_R^0 \rangle \ . \tag{3.15}$$

By defining a mixing state

$$u'_{R3} = \frac{(Y_R^u)_{13}^* u_{R1} + (Y_R^u)_{23}^* u_{R2} + (Y_R^u)_{33}^* u_{R3}}{\sqrt{|(Y_R^u)_{31}|^2 + |(Y_R^u)_{32}|^2 + |(Y_R^u)_{33}|^2}} \ , \tag{3.16}$$

we obtain a mass $m_{t'}$ of the fourth up-quark $t' = (t'_L, t'_R) = (U_{L3}, u'_{R3})$,

$$m_{t'} = \langle \phi_R^0 \rangle \sqrt{|(Y_R^u)_{13}|^2 + |(Y_R^u)_{23}|^2 + |(Y_R^u)_{33}|^2} \ . \tag{3.17}$$

Similarly, in the approximation in which the terms suppressed by y_{1S}^u and y_{2S}^u are neglected, the mass m_t of the third up-quark (i.e., top quark) $t = (t_L, t_R) = (u'_{L3}, U_{R3})$ is given by

$$m_t \simeq \langle \phi_L^0 \rangle \sqrt{|(Y_L^u)_{13}|^2 + |(Y_L^u)_{23}|^2 + |(Y_L^u)_{33}|^2} \ , \tag{3.18}$$

where

$$u'_{L3} \simeq \frac{(Y_L^u)_{13}^* u_{L1} + (Y_L^u)_{23}^* u_{L2} + (Y_L^u)_{33}^* u_{L3}}{\sqrt{|(Y_L^u)_{13}|^2 + |(Y_L^u)_{23}|^2 + |(Y_L^u)_{33}|^2}} \ . \tag{3.19}$$

More precisely speaking, the masses $(m_u, m_c, m_t, m_{t'})$ are obtained by diagonalizing the following mass matrix

$$M^u = \langle \phi_L^0 \rangle \begin{pmatrix} -(\kappa/\lambda)K_{11}^u & -(\kappa/\lambda)K_{12}^u & -(\kappa/\lambda)K_{13}^u & Y_{L13}^u \\ -(\kappa/\lambda)K_{21}^u & -(\kappa/\lambda)K_{22}^u & -(\kappa/\lambda)K_{23}^u & Y_{L23}^u \\ -(\kappa/\lambda)K_{31}^u & -(\kappa/\lambda)K_{32}^u & -(\kappa/\lambda)K_{33}^u & Y_{L33}^u \\ \kappa Y_{R13}^{u*} & \kappa Y_{R23}^{u*} & \kappa Y_{R33}^{u*} & 0 \end{pmatrix} \ , \tag{3.20}$$

which is sandwiched by the fields $(\overline{u}_{L1}, \overline{u}_{L2}, \overline{u}_{L3}, \overline{U}_{L3})$ and $(u_{R1}, u_{R2}, u_{R3}, U_{R3})$, where $\kappa = \langle \phi_R^0 \rangle / \langle \phi_L^0 \rangle$ and $\lambda = \langle \Phi^0 \rangle / \langle \phi_L^0 \rangle$ as defined in Eq. (1.13). Of the Yukawa coupling constants $(Y_L^u)_{ij}$ and $(Y_R^u)_{ij}$, the twelve components $(Y_L^u)_{ik}$ and $(Y_R^u)_{ik}$ ($i = 1, 2, 3; j = 1, 2$) are absorbed into the operator K^u defined by (3.14), while the rest $(Y_L^u)_{i3}$ and $(Y_R^u)_{i3}$ are still described by the equation (3.6).

Finally, we denote the effective Hamiltonian in each range: The effective Hamiltonian H_{int}^{III} in the range III ($\Lambda_X \geq \mu > \Lambda_S$) is still given by the form (2.2), and H_{int}^{II} in the range II ($\Lambda_S \geq \mu > \Lambda_R$) and H_{int}^{I} in the range I ($\Lambda_R \geq \mu > \Lambda_L$) are given by

$$\begin{aligned} H_{int}^{II} = & \sum_{i=1}^{3} Y_{Li3}^u (\overline{q}_{Li} \widetilde{\phi}_L U_{R3}) \\ & + \sum_{i=3}^{3} Y_{Ri3}^u (\overline{q}_{Ri} \widetilde{\phi}_R U_{L3}) \\ & + \sum_{i,j \neq 3} \frac{1}{\langle \Phi^0 \rangle} K_{ij}^u (\overline{q}_{Li} \widetilde{\phi}_L)(\widetilde{\phi}_R^\dagger q_{Rj}) \\ & + \sum_{i,j} \frac{1}{\langle \Phi^0 \rangle} K_{ij}^d (\overline{q}_{Li} \phi_L)(\phi_R^\dagger q_{Rj}) \\ & + \sum_{i,j} \frac{1}{\langle \Phi^0 \rangle} K_{ij}^e (\overline{\ell}_{Li} \phi_L)(\phi_R^\dagger \ell_{Rj}) + h.c. \\ & + \sum_{i,j} \frac{1}{\langle \Phi^0 \rangle} K_{Lij}^\nu (\overline{\ell}_{Li} \widetilde{\phi}_L)(\widetilde{\phi}_L^T \ell_{Lj}^c) \\ & + \sum_{i,j} \frac{1}{\langle \Phi^0 \rangle} K_{Rij}^\nu (\overline{\ell}_{Ri} \widetilde{\phi}_R)(\widetilde{\phi}_R^T \ell_{Rj}^c) \ , \end{aligned} \tag{3.21}$$

and

$$\begin{aligned} H_{int}^{I} = & \sum_{i=1}^{3} Y_{Li3}^u (\overline{q}_{Li} \widetilde{\phi}_L U_{R3}) \\ & + \sum_{i,j \neq 3} \frac{\langle \phi_R^0 \rangle}{\langle \Phi^0 \rangle} K_{ij}^u (\overline{q}_{Li} \widetilde{\phi}_L u_{Rj}) \\ & + \sum_{i,j} \frac{\langle \phi_R^0 \rangle}{\langle \Phi^0 \rangle} K_{ij}^d (\overline{q}_{Li} \phi_L d_{Rj}) \\ & + \sum_{i,j} \frac{\langle \phi_R^0 \rangle}{\langle \Phi^0 \rangle} K_{ij}^e (\overline{\ell}_{Li} \phi_L e_{Rj}) + h.c. \\ & + \sum_{i,j} \frac{1}{\langle \Phi^0 \rangle} K_{Lij}^\nu (\overline{\ell}_{Li} \widetilde{\phi}_L)(\widetilde{\phi}_L^T \ell_{Lj}^c) \ , \end{aligned} \tag{3.22}$$

respectively.

IV. COEFFICIENTS OF THE RGE

In the present section, we give the coefficients of the renomalization group equations (RGE) (3.6), (3.12) and (3.13).

A. Evolution in the range III

In the non-SUSY model, the terms T^f_A, G^f_A and H^f_A ($A = L, R, S$) are given as follows:

$$\begin{aligned} T^u_A &= T^d_A = T^\nu_A = T^e_A \\ &= 3\mathrm{Tr}\left(Y^u_A Y^{u\dagger}_A + Y^d_A Y^{d\dagger}_A\right) + \mathrm{Tr}\left(Y^\nu_A Y^{\nu\dagger}_A + Y^e_A Y^{e\dagger}_A\right) \ , \end{aligned} \tag{4.1}$$

$$\begin{aligned} G^u_A &= \frac{17}{8} g_1^2 + \frac{9}{4} g_{2A}^2 + 8 g_3^2 + \frac{3}{4} g_X^2 \ , \\ G^d_A &= \frac{5}{8} g_1^2 + \frac{9}{4} g_{2A}^2 + 8 g_3^2 + \frac{3}{4} g_X^2 \ , \\ G^\nu_A &= \frac{9}{8} g_1^2 + \frac{9}{4} g_{2A}^2 + \frac{3}{4} g_X^2 \ , \\ G^e_A &= \frac{45}{8} g_1^2 + \frac{9}{4} g_{2A}^2 + \frac{3}{4} g_X^2 \ , \end{aligned} \tag{4.2}$$

$$\begin{aligned} H^u_A &= -H^d_A = \frac{3}{2}\left(Y^u_A Y^{u\dagger}_A - Y^d_A Y^{d\dagger}_A\right) \ , \\ H^\nu_A &= -H^e_A = \frac{3}{2}\left(Y^\nu_A Y^{\nu\dagger}_A - Y^e_A Y^{e\dagger}_A\right) \ , \end{aligned} \tag{4.3}$$

where $A = L, R$, and

$$\begin{aligned} T^u_S &= T^d_S = T^\nu_S = T^e_S \\ &= 3\mathrm{Tr}\left(Y^u_S Y^{u\dagger}_S + Y^d_S Y^{d\dagger}_S\right) + \mathrm{Tr}\left(Y^\nu_S Y^{\nu\dagger}_S + Y^e_S Y^{e\dagger}_S\right) \ , \end{aligned} \tag{4.4}$$

$$\begin{aligned} G^u_S &= 4 g_1^2 + 8 g_3^2 + \frac{3}{2} g_X^2 \ , \\ G^d_S &= g_1^2 + 8 g_3^2 + \frac{3}{2} g_X^2 \ , \\ G^\nu_S &= + \frac{3}{2} g_X^2 \ , \\ G^e_S &= 9 g_1^2 + \frac{3}{2} g_X^2 \ , \end{aligned} \tag{4.5}$$

$$H^f_S = Y^f_S Y^{f\dagger}_S \ , \qquad (f = u, d, \nu, e) \ . \tag{4.6}$$

The coefficients T^f_A, G^f_A and H^f_A in the minimal SUSY model are given in the Appendix.

As seen from Eq. (4.6), since the matrix H^f_A is diagonal on the diagonal basis of $M_F(\Lambda_X)$, the Yukawa coupling constants $Y^f_S(\mu)$ can keep the forms diagonal. Similarly, when we choose the diagonal basis of $M_L(\Lambda_X)$ [$M_R(\Lambda_X)$], the matrices $Y^f_L(\mu)$ [$Y^f_R(\mu)$] keep their forms diagonal.

For a model with $g_{2L}(\mu) = g_{2R}(\mu)$ and $Y^f_L(\mu) = Y^f_L(\mu)$ at $\mu = \Lambda_X$, we can assert that

$$Y^f_L(\mu) = Y^f_R(\mu) \ , \tag{4.7}$$

in the range III ($\Lambda_S < \mu \leq \Lambda_X$), because on the diagonal basis of Y_L we obtain

$$16\pi^2 \frac{d}{dt} \ln \frac{(Y^f_L)_{ii}}{(Y^f_R)_{ii}} = (T^f_L - G^f_L + H^f_L)_{ii} - (T^f_R - G^f_R + H^f_R)_{ii} \tag{4.8}$$

The case $g_{2L} = g_{2R}$ is likely in the L-R symmetric model. For convenience, in the numerical evaluation in the present paper, we will take $g_{2L}(\mu) = g_{2R}(\mu)$ in the range III ($\Lambda_S < \mu \leq \Lambda_X$).

B. Evolution in the ranges I and II

In the ranges I and II, all the fermions F_i except for U_3 are decoupled from the equation (3.6). In the present section, we will take the diagonal basis of M_F. Therefore, it is convenient that we define a spurion

$$S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ . \tag{4.9}$$

Then, the surviving Yukawa coupling constants $(Y^u_A)_{i3}$ are expressed as $(Y^u_A S)_{ij} = (Y^u_A)_{i3}\delta_{3j}$. The evolution of $Y^u_A S$ is still described by the RGE (3.6) by substituting $Y^u_A S$ for Y^f_A. Here, the terms T^u_A, G^u_A and H^u_A ($A = L, R$) are expressed as follows [Y^f_A ($f = d, e, \nu$) are already absorbed into the operators K^f]:

$$T^u_A = 3\mathrm{Tr}\left(Y^u_A S Y^{u\dagger}_A\right) \ , \tag{4.10}$$

$$G^u_A = \frac{17}{8} g_1^2 + \frac{9}{4} g_{2A}^2 + 8 g_3^2 \ , \tag{4.11}$$

$$H^u_A = \frac{3}{2} Y^u_A S Y^{u\dagger}_A \ , \tag{4.12}$$

($A = L, R$) in the range II, and

$$T^u_L = 3\mathrm{Tr}\left(Y^u_L S Y^{u\dagger}_L\right) \ , \tag{4.13}$$

$$G^u_L = \frac{17}{20} g_1^2 + \frac{9}{4} g_{2L}^2 + 8 g_3^2 \ , \tag{4.14}$$

$$H^u_L = \frac{3}{2} Y^u_L S Y^{u\dagger}_L \ , \tag{4.15}$$

in the range I. Here, the coupling constant $g_1 \equiv g_{1LR}$ in the range II is that for the U(1) operator $(1/2)Y_{LR}$ which is defined by the relation

$$Q = I^L_3 + I^R_3 + \frac{1}{2} Y_{LR} \ , \tag{4.16}$$

for the symmetry $SU(2)_L \times SU(2)_R \times U(1)_{LR}$, while the coupling constant $g_1 \equiv g_{1Y}$ in the range I is that for the U(1) operator $(1/2)Y$ which is defined by the relation

$$Q = I_3^L + \frac{1}{2}Y \ , \tag{4.17}$$

for the symmetry $SU(2)_L \times U(1)_Y$, and they are connected by

$$\alpha_{em}^{-1}(\Lambda_L) = \alpha_{2L}^{-1}(\Lambda_L) + \frac{5}{3}\alpha_{1LR}^{-1}(\Lambda_L) \ , \tag{4.18}$$

$$\frac{5}{3}\alpha_{1Y}^{-1}(\Lambda_R) = \alpha_{2R}^{-1}(\Lambda_R) + \frac{2}{3}\alpha_{1LR}^{-1}(\Lambda_R) \ , \tag{4.19}$$

where $\alpha_i = g_i^2/4\pi$.

Similarly, the terms T_K^f, G_K^f H_{KL}^f and H_{KR}^f ($f = u, d, e$) are given by

$$T_K^u = T_K^d = T_K^e = 3\mathrm{Tr}\left(Y_L^u S Y_L^{u\dagger} + Y_R^u S Y_R^{u\dagger}\right) \ , \tag{4.20}$$

$$\begin{aligned} G_K^u = G_K^d &= \frac{5}{2}g_1^2 + \frac{9}{4}g_{2L}^2 + \frac{9}{4}g_{2R}^2 + 8g_3^2 \ , \\ G_K^e &= \frac{9}{2}g_1^2 + \frac{9}{4}g_{2L}^2 + \frac{9}{4}g_{2R}^2 \ , \end{aligned} \tag{4.21}$$

$$H_{KA}^u = H_{KA}^d = \frac{3}{2}Y_A^u S Y_A^{u\dagger} \ , \quad H_{KA}^e = 0 \ , \quad (A = L, R) \ , \tag{4.22}$$

in the range II, and

$$T_K^u = T_K^d = T_K^e = 3\mathrm{Tr}\left(Y_L^u S Y_L^{u\dagger}\right) \ , \tag{4.23}$$

$$\begin{aligned} G_K^u &= \frac{17}{20}g_1^2 + \frac{9}{4}g_{2L}^2 + 8g_3^2 \ , \\ G_K^d &= \frac{5}{20}g_1^2 + \frac{9}{4}g_{2L}^2 + 8g_3^2 \ , \\ G_K^e &= \frac{45}{20}g_1^2 + \frac{9}{4}g_{2L}^2 \ , \end{aligned} \tag{4.24}$$

$$H_{KL}^u = H_{KL}^d = \frac{3}{2}Y_L^u S Y_L^{u\dagger} \ , \quad H_{KR}^f = 0 \ , \quad f = u, d \ , \tag{4.25}$$

$$H_{KL}^e = H_{KR}^e = 0 \ , \tag{4.26}$$

in the range I.

The terms T_K^ν, G_K^ν and H_{KL}^ν have rather simple forms in contrast with those in the conventional neutrino seesaw model, because the partners of the fermions f_L which couple to the Higgs scalar ϕ_L are not f_R, but F_R which are already decoupled at $\mu = \Lambda_R$:

$$T_K^\nu = 6\mathrm{Tr}\left(Y_L^u S Y_L^{u\dagger}\right) \ , \tag{4.27}$$

$$G_K^\nu = 3g_{2L}^2 \ , \tag{4.28}$$

$$H_{KL}^\nu = \lambda_{HL} \ , \tag{4.29}$$

in the ranges I and II, where λ_{HL} is the coupling constant of the Higgs scalar ϕ_L defined by

$$H_\phi = \frac{1}{2}\lambda_{HL}(\phi_L^\dagger \phi_L)^2 \ , \tag{4.30}$$

and the mass of the physical Higgs scalar H_L^0 is given by

$$m_{HL}^2 = 2\lambda_{HL}\langle\phi_L^0\rangle^2 \ . \tag{4.31}$$

The similar coefficients in the minimal SUSY model are given in the Appendix.

V. CASE OF THE DEMOCRATIC SEESAW MODEL

In the democratic seesaw model, on the diagonal basis of $Y_L^f(\Lambda_X)$ and $Y_R^f(\Lambda_X)$, the Yukawa coupling constants of heavy fermions $Y_S^f(\Lambda_X)$ are given by the democratic form (1.11). Since on this basis the Yukawa coupling constants Y_S^f keep the forms democratic:

$$Y_S^f(\mu) = \xi_S^f(\mu)\left(\mathbf{1} + 3b_f(\mu)X\right) \ , \tag{5.1}$$

we will call this basis the "democratic basis of M_F" hereafter. On the other hand, if we take a basis on which Y_S^f are diagonal, i.e., the matrix forms are given by

$$\widetilde{Y}_S^f(\mu) = \xi_S^f(\mu)\left(\mathbf{1} + 3b_f(\mu)\widetilde{X}\right) \ , \tag{5.2}$$

$$\widetilde{X} = AXA^T = \mathrm{diag}(0, 0, 1) \ , \tag{5.3}$$

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \ . \tag{5.4}$$

Especially, on this basis, the Yukawa coupling constants $(\widetilde{Y}_S^e)_{ii}$ and $(\widetilde{Y}_S^u)_{ii}$ of the fermions E_i and U_i satisfy the relations

$$[\widetilde{Y}_S^e(\mu)]_{11} = [\widetilde{Y}_S^e(\mu)]_{22} = [\widetilde{Y}_S^e(\mu)]_{33} = \xi_S^e(\mu) \ , \tag{5.5}$$

$$[\widetilde{Y}_S^u(\mu)]_{11} = [\widetilde{Y}_S^u(\mu)]_{22} = \xi_S^u(\mu) \ , \quad [\widetilde{Y}_S^u(\mu)]_{33} = 0 \ , \tag{5.6}$$

in the range III ($\Lambda_S < \mu \leq \Lambda_X$), i.e.,

$$b_e(\mu) = 0 \ , \quad b_u(\mu) = -1/3 \ . \tag{5.7}$$

On the other hand, on this basis, the Yukawa coupling constants $\widetilde{Y}_L^f(\mu)$ and $\widetilde{Y}_R^f(\mu)$ are not diagonal. However, we can easily obtain their diagonal forms by $A^T\widetilde{Y}_L^f(\mu)A$ and $A^T\widetilde{Y}_R^f(\mu)A$.

At the energy scale $\mu = \Lambda_S$, the fermions F_i (except for U_3) acquire the heavy masses $(M_F)_{ii}$. Therefore, for $\mu < \Lambda_S$, the operators K^f are given as follows:

$$(K^u)_{ij} = \left[\widetilde{Y}_L^u(\widetilde{Y}_S^u)^{-1}\widetilde{Y}_R^{u\dagger}\right]_{ij} = \frac{1}{\xi_S^u(\Lambda_S)}\sum_{k=1,2}(\widetilde{Y}_L^u)_{ik}(\widetilde{Y}_R^u)^*_{jk} \ , \tag{5.8}$$

$$(K^d)_{ij} = \left[\widetilde{Y}_L^d(\widetilde{Y}_S^d)^{-1}\widetilde{Y}_R^{d\dagger}\right]_{ij} = \frac{1}{\xi_S^d(\Lambda_S)}\left(\sum_{k=1,2}(\widetilde{Y}_L^d)_{ik}(\widetilde{Y}_R^d)^*_{jk} + \frac{1}{1+3b_d(\Lambda_S)}(\widetilde{Y}_L^d)_{i3}(\widetilde{Y}_R^d)^*_{j3}\right) \ , \tag{5.9}$$

$$(K^e)_{ij} = \left[\widetilde{Y}_L^e(\widetilde{Y}_S^e)^{-1}\widetilde{Y}_R^{e\dagger}\right]_{ij} = \frac{1}{\xi_S^e(\Lambda_S)}\sum_{k=1,2,3}(\widetilde{Y}_L^e)_{ik}(\widetilde{Y}_R^e)^*_{jk} \ , \tag{5.10}$$

$$(K^\nu)_{ij} = \left(\widetilde{Y}_L^\nu(\widetilde{Y}_S^\nu)^{-1}\widetilde{Y}_L^{\nu T}\right)_{ij} = \frac{1}{\xi_S^\nu(\Lambda_S)}\left(\sum_{k=1,2}(\widetilde{Y}_L^\nu)_{ik}(\widetilde{Y}_L^\nu)_{jk} + \frac{1}{1+3b_\nu(\Lambda_S)}(\widetilde{Y}_L^\nu)_{i3}(\widetilde{Y}_L^\nu)_{j3}\right) \ . \tag{5.11}$$

In Eq. (5.11), we have assumed that the structure of the Majorana mass term $M_R(\Lambda_S) = Y_S^R(\Lambda_S)\langle\Phi^0\rangle$ for the neutral fermions N_R has a structure similar to the Dirac mass matrices M_F which is given by Eq. (5.1).

Since the Yukawa coupling constants $Y_A^f(\mu)$ $(A = L, R)$ in the range III keep their forms diagonal on the democratic basis of M_F, it is convenient to express $Y_A^f(\mu)$ as follows,

$$Y_A^f(\mu) = \xi_A^f(\mu) Z_A^f(\mu) \ , \tag{5.12}$$

where the diagonal matrix $Z_A^f(\mu)$ is given by Eq. (3.2). Then, the matrix $\widetilde{Y}_A^f$ on the diagonal basis of M_F is given by

$$\widetilde{Y}_A^f(\mu) = \xi_A^f(\mu)\widetilde{Z}^f(\mu) \ , \tag{5.13}$$

where

$$\widetilde{Z}^f = AZ^fA^T = \frac{1}{6}\begin{pmatrix} 3(z_2+z_1) & -\sqrt{3}(z_2-z_1) & -\sqrt{6}(z_2-z_1) \\ -\sqrt{3}(z_2-z_1) & 4z_3+z_2+z_1 & -\sqrt{2}(2z_3-z_2-z_1) \\ -\sqrt{6}(z_2-z_1) & -\sqrt{2}(2z_3-z_2-z_1) & 2(z_3+z_2+z_1) \end{pmatrix} \ , \tag{5.14}$$

(we have dropped the indices A and f, and for simplicity, we have taken $\delta_i^f = 0$). Although the Yukawa coupling constants $\widetilde{Y}_L^u$ and $\widetilde{Y}_R^u$ in the range II and $\widetilde{Y}_L^f$ in the range I have the physical meaning only for the one column matrix components $(\widetilde{Y}_A^f)_{i3}$ $(i = 1, 2, 3)$, we still use the expressions (5.12) and (5.13), because the matrix $K^e(\mu)$ $(\Lambda_L < \mu \le \Lambda_S)$ which is proportional to $Y_L^e(\mu)Y_R^{e\dagger}(\mu)$ is still diagonal on the democratic basis of M_F as discussed in Sec. IV, so that we regard that $Y_A^u(\mu)$ is also "diagonal". Then, the top quark mass $m_t(\mu)$ is approximately expressed as

$$m_t(\mu) \simeq \langle\phi_L^0\rangle\sqrt{\sum_i |(\widetilde{Y}_L^u(\mu))_{i3}|^2} = \langle\phi_L^0\rangle\xi_L^u(\mu)\sqrt{\frac{1}{3}\sum_i |z_{Li}^u|^2} = \frac{1}{\sqrt{3}}\xi_L^u(\mu)\langle\phi_L^0\rangle \ . \tag{5.15}$$

The expression (5.15) is valid in the whole ranges $\Lambda_L < \mu \le \Lambda_X$.

Since

$$\sum_{i=1}^{3}\sum_{k=1}^{2}\left(\widetilde{Z}_{ik}\right)^2 = \frac{2}{3}(z_1^2+z_2^2+z_3^2) = \frac{2}{3} \ , \tag{5.16}$$

we obtain

$$m_c(\mu) + m_u(\mu) \simeq \frac{2}{3}\frac{\xi_L^u(\mu)\xi_R^u(\mu)}{\xi_S^u(\mu)}\frac{\langle\phi_L^0\rangle\langle\phi_R^0\rangle}{\langle\Phi^0\rangle} \ , \tag{5.17}$$

from Eq. (5.8). Note that the expression (5.17) is valid only in the range III. In the ranges I and II, the ratio $\xi_L^u\xi_R^u/\xi_S^u$ behaves as a operator $K^u(\mu)$ which obeys Eq. (3.12). From Eq. (5.17), the ratio m_c/m_t is given by

$$\frac{m_c(\mu)}{m_t(\mu)} \simeq \frac{2}{\sqrt{3}} \frac{\xi_R^u(\mu)}{\xi_S^u(\mu)} \frac{\langle \phi_R^0 \rangle}{\langle \Phi^0 \rangle} \ . \tag{5.18}$$

Since $H_{KL}^e = H_{KR}^e = 0$ in the ranges I and II, the form of $K^e(\mu)$ is invariant in the ranges, i.e.,

$$Z_L^e(\Lambda_L) Z_R^{e\dagger}(\Lambda_L) = Z_L^e(\Lambda_S) Z_R^{e\dagger}(\Lambda_S) \ , \tag{5.19}$$

especially, since

$$Z_L^f(\mu) = Z_R^f(\mu) \equiv Z^f(\mu) \ , \tag{5.20}$$

for a model with $g_{2R}(\Lambda_R) = g_{2L}(\Lambda_R)$, we obtain

$$Z^e(\Lambda_L) = Z^e(\Lambda_S) \ . \tag{5.21}$$

Therefore, in preliminary evaluations prior to fixing the final values of the parameters, we will sometimes use the values of $z_i(m_Z)$ which are obtained from the observed charge lepton masses $m_i^e(m_Z)$ by using Eq. (1.7) instead of the values of $z_i(\Lambda_X)$ which are defined in Eq. (1.9) as the initial condition at $\mu = \Lambda_X$.

VI. NUMERICAL RESULTS IN THE NON-SUSY MODEL

We define

$$\Lambda_L = \langle \phi_L^0 \rangle \ , \quad \Lambda_R = \langle \phi_R^0 \rangle \ , \quad \Lambda_S = \langle \Phi^0 \rangle \ . \tag{6.1}$$

However, for convenience, in the numerical evaluations, instead of physical quantities at $\mu = \Lambda_L$, we will use those at $\mu = m_Z$ (m_Z is the neutral weak boson mass).

First, in order to overlook the behavior of the Yukawa coupling constant $Y_L^f(\mu)$, we illustrate the behavior of $\xi_L^u(\mu)$ in the non-SUSY model in Fig. 1. Here, we have used the approximate relation (5.15) and the input values $m_t(m_Z) = 181$ GeV and $\langle \phi_L^0 \rangle = 174$ GeV:

$$\xi_L^u(m_Z) = \sqrt{3} \frac{m_t(m_Z)}{\langle \phi_L^0 \rangle} = 1.80. \tag{6.2}$$

In other words, the behavior of $\xi_L^u(\mu)$ corresponds to that of $m_t(\mu)$ because of $\xi_L^u(\mu) = (m_t(\mu)/m_t(m_Z))\xi_L^u(m_Z)$. In the ranges I and II, since the terms T_L^u and H_L^u are expressed only in terms of $\widetilde{Y}_L^u S \widetilde{Y}_L^{u\dagger}$, the evolution of the factor $|\xi_L^u(\mu)|^2 = 3\mathrm{Tr}[\widetilde{Y}_L^u S \widetilde{Y}_L^{u\dagger}]$ is described by the equation

$$16\pi^2 \frac{d}{dt} |\xi_L^u|^2 = 2 \left[\left(\frac{1}{3} |\xi_L^u|^2 - G_L^u \right) |\xi_L^u|^2 + \frac{1}{2} |\xi_L^u|^4 \right] \ . \tag{6.3}$$

However, in the range III, the terms T_L^u and H_L^u contain other factors $Y_L^f Y_L^{f\dagger}$ in addition to $Y_L^u Y_L^{u\dagger}$, so that the evolution of ξ_L^u cannot be expressed so simply such as (6.3). For the evaluation of ξ_L^u in the range III, we have tentatively substituted the values $z_i(m_Z)$ given by (1.7) for the initial values $z_i(\Lambda_X)$. For simplicity, as we discussed in (4.7), we have taken as $g_{2L}(\Lambda_R) = g_{2R}(\Lambda_R)$. In Fig. 1, the ratio Λ_S/Λ_R has been taken as $\Lambda_S/\Lambda_R = 107$, which has determined from the fitting of the observed ratio m_t/m_c as we discuss later. The behavior of $\xi_L^u(\mu)$ is insensitive to the ratio Λ_S/Λ_R. As seen in Fig. 1, in a case with a lower Λ_S ($\Lambda_S < 10^5$ GeV), $\xi_L^u(\mu)$ has the Landau pole below $\mu = \Lambda_X$, so that the case is ruled out. On the other hand, a case with a higher Λ_S ($\Lambda_S > 10^{19}$ GeV) causes $\alpha_1(\mu) \to \infty$ at $\mu \to \Lambda_S$, so that the case is also ruled out.

Taking account of the behavior of $\xi_L^u(\mu)$ shown in Fig. 1, as a trial, we take

$$\Lambda_X = 2 \times 10^{16} \text{ GeV} \ , \tag{6.4}$$

which is known as the unification energy scale in the minimal SUSY model. (However, in the present paper, we do not consider the gauge unification.) As a value of Λ_S, we tentatively take

$$\Lambda_S = 3 \times 10^{13} \text{ GeV} \ , \tag{6.5}$$

which leads to the mass-squared difference $\Delta m_{32}^2 \equiv m_{\nu_3}^2 - m_{\nu_2}^2 \sim (10^{-3} - 10^{-2})\mathrm{eV}^2$ as we demonstrate later. For the values (6.4) and (6.5), we obtain $\xi_L^u(\Lambda_X) = 1.2$.

Next, we determine the values of $\xi_S^u(\Lambda)$ and Λ_S/Λ_R. Since we have already obtained the value $\xi_L^u(\Lambda_X) = 1.2$, it seems that we can fix the value of $\xi_S^u(\Lambda)\Lambda_S/\Lambda_R$ from the observed value of $m_t(m_Z)/m_c(m_Z)$ because of the relation (5.18). However, the value of $\xi_S^u(\Lambda_X)$ [also $\xi_S^f(\Lambda_X)$] is sensitive to the value of $\xi_S^u(\Lambda_S)$ [$\xi_S^f(\Lambda_S)$] [in other words, a small deviation of $\xi_S^f(\Lambda_S)$ causes a large deviation of $\xi_S^f(\Lambda_X)$]. Therefore, we cannot fix the values $\xi_S^u(\Lambda_X)$ unless we put a tentative model for ξ_{LR}^f and ξ_S^f. The basic assumption in the universal seesaw model is to consider that the mass matrices m_L and m_R in Eq. (1.1) are "universal" (common) for all fermion sectors (quarks and leptons). Therefore, we put the following initial condition

$$\xi_{LR}^u(\Lambda_X) = \xi_{LR}^d(\Lambda_X) = \xi_{LR}^e(\Lambda_X) = \xi_{LR}^\nu(\Lambda_X) \equiv \xi_{LR}(\Lambda_X) \ . \tag{6.6}$$

Then, a model with $\xi_S^u(\Lambda_X) = \xi_S^d(\Lambda_X) = \xi_S^e(\Lambda_X)$ is obviously ruled out because we cannot give the observed values of quark and charged lepton masses simultaneously. We must consider

$$\xi_S^u(\Lambda_X) = \xi_S^d(\Lambda_X) \equiv \xi_S^q(\Lambda_X) \neq \xi_S^e(\Lambda_X) \ . \tag{6.7}$$

We tentatively put $\xi_S^e(\Lambda_X) = \xi_{LR}(\Lambda_X)$. The numerical results are as follows:

$$\xi_{LR}(\Lambda_X) = \xi_S^e(\Lambda_X) = 1.20 \ , \quad \xi_S^q(\Lambda_X) = 0.80 \ , \tag{6.8}$$

$$\Lambda_S/\Lambda_R = 107 \ , \tag{6.9}$$

$$z_1 = 0.01617 \ , \quad z_2 = 0.2349 \ , \quad z_3 = 0.9719 \ . \qquad (6.10)$$

In the quark and charged lepton mass expressions (3.19) the factors ξ_S^e and ξ_S^q appear only in terms of the combinations $\xi_S^q \Lambda_S$ and $\xi_S^e \Lambda_S$, respectively, so that the absolute values of ξ_S^e and ξ_S^q depend on the choice of the input value of Λ_S. Only the ratio ξ_S^e/ξ_S^q is substantial for the fitting of the quark and charged lepton mass. (However, as we state in the Sec. VIII, the neutrino mass difference between $m_{\nu 3}$ and $m_{\nu 2}$ rapidly varies in the range III. Therefore, in the neutrino mass matrix, the choice of the input value Λ_S is important.) We can obtain

$$\xi_S^e(\Lambda_X)/\xi_S^q(\Lambda_X) \simeq 1.5 \ , \qquad (6.11)$$

for any initial values of $\xi_S^e(\Lambda_X)$ with $O(1)$. The values (6.10) are nearly in agreement with the values $z_1 = 0.01622$, $z_2 = 0.2357$, and $z_3 = 0.9717$ which are obtained from Eq. (1.7) at $\mu = m_Z$. We can see that the effect of the evolution is not so large for Z^e.

The value of the parameter $b_d(\Lambda_X)$ is determined from the fitting of the observed down-quark mass ratios m_d/m_s and m_s/m_b and the CKM matrix parameter $|V_{us}(m_Z)| = 0.22$. In Fig. 2, we illustrate the mass ratios $m_d(\mu)/m_s(\mu)$ and $m_s(\mu)/m_b(\mu)$ and the CKM parameter $|V_{us}(\mu)|$ at $\mu = m_Z$ versus the parameters b_d and β_d, where we have re-defined the complex parameter b_d by $b_d e^{i\beta_d}$ with two real parameters. For convenience, in Fig. 2, the quantities are expressed in the unit of the corresponding observed values at $\mu = m_Z$ (for example, in Fig. 2, the curve m_d/m_s denotes $[m_d(\mu)/m_s(\mu)]_{\mu=m_Z}/[m_d/m_s]_{observed}$). We obtain

$$b_d(\Lambda_X) = -1.20 \ , \quad \beta_d(\Lambda_X) = 19.2^\circ \ , \qquad (6.12)$$

which give the following predictions at $\mu = m_Z$:

$$\begin{aligned} m_u(m_Z) &= 2.60 \times 10^{-3} \text{ GeV} \ , \\ m_c(m_Z) &= 6.92 \times 10^{-1} \text{ GeV} \ , \\ m_t(m_Z) &= 182 \text{ GeV} \ , \\ m_d(m_Z) &= 4.38 \times 10^{-3} \text{ GeV} \ , \\ m_s(m_Z) &= 9.84 \times 10^{-2} \text{ GeV} \ , \\ m_b(m_Z) &= 3.02 \text{ GeV} \ , \\ m_e(m_Z) &= 4.90 \times 10^{-4} \text{ GeV} \ , \\ m_\mu(m_Z) &= 1.03 \times 10^{-1} \text{ GeV} \ , \\ m_\tau(m_Z) &= 1.76 \text{ GeV} \ . \end{aligned} \qquad (6.13)$$

The experimental values corresponding to the results (6.13) are as follows [10]:

$$\begin{aligned} m_u(m_Z) &= (2.33^{+0.42}_{-0.45}) \times 10^{-3} \text{ GeV} \ , \\ m_c(m_Z) &= (6.85^{+0.56}_{-0.61}) \times 10^{-1} \text{ GeV} \ , \\ m_t(m_Z) &= (181 \pm 13) \text{ GeV} \ , \\ m_d(m_Z) &= (4.69^{+0.60}_{-0.66}) \times 10^{-3} \text{ GeV} \ , \\ m_s(m_Z) &= (0.934^{+0.118}_{-0.130}) \times 10^{-1} \text{ GeV} \ , \\ m_b(m_Z) &= (3.00 \pm 0.11) \text{ GeV} \ , \\ m_e(m_Z) &= (4.8684727 \pm 0.00000014) \times 10^{-4} \text{ GeV} \ , \\ m_\mu(m_Z) &= (1.0275138 \pm 0.0000033) \times 10^{-1} \text{ GeV} \ , \\ m_\tau(m_Z) &= (1.7467 \pm 0.0003) \text{ GeV} \ . \end{aligned} \qquad (6.14)$$

The results (6.13) is in agreement with the observed values (6.14) within the experimental errors.

The predicted values of $|V_{ij}|$ depends on the phase parameters δ_i^f given by Eq. (1.4). Only when we take those as (1.8) (at $\mu = \Lambda_X$), we can obtain reasonable values of $|V_{ij}|$. For example, for $\delta_3^u - \delta_3^d = \pi$, we obtain the predictions at $\mu = m_Z$

$$\begin{aligned} &|V_{us}| = 0.220 \ , \quad |V_{cb}| = 0.0668 \ , \\ &|V_{ub}/V_{cb}| = 0.0558 \ , \\ &|V_{td}| = 0.0177 \ , \\ &J = 3.25 \times 10^{-5} \ . \end{aligned} \qquad (6.15)$$

The observed values [11] are

$$\begin{aligned} |V_{us}| &= 0.2196 \pm 0.0023 \ , \\ |V_{cb}| &= 0.0402 \pm 0.0019 \ , \\ |V_{ub}/V_{cb}| &= 0.090 \pm 0.025 \ . \end{aligned} \qquad (6.16)$$

Although the results (6.15) are roughly consistent with experiments, the value $|V_{cb}| = 0.066$ is somewhat large compared with the observed value $|V_{cb}| = 0.040$. This discrepancy can be adjusted by considering a small deviation from π of the relative phase $\delta_3^u - \delta_3^d$ as demonstrated in Ref. [5].

Related to the phenomenological requirement (1.8), it is interesting to consider that Y_L^u which is the coefficient of the Higgs scalar $\widetilde{\phi}_L$ is related to Y_L^d which is the coefficient of the scalar ϕ_L as

$$Y_L^u(\Lambda_X) = [Y_L^d(\Lambda_X)]^\dagger \ . \qquad (6.17)$$

Then, the relations (1.8) mean that $(Y_L^f)_{11}$ and $(Y_L^f)_{22}$ are real, while $(Y_L^f)_{33}$ is almost pure imaginary. We take

$$(Z^u)^\dagger = Z^d = \mathrm{diag}(z_1, z_2, z_3 e^{i\delta_3}) \ . \qquad (6.18)$$

The parameter δ_3 $(= \delta_3^d = -\delta_3^u)$ does not affect the masses, but only the CKM mixings. It is interesting to consider that the parameter $\delta_3(\Lambda_X)$ takes its value such as the CKM mixings become minimum, i.e., such as the value $\sum_{i\neq j}|V_{ij}(\Lambda_X)|^2$ takes the minimum. This requirement gives the initial value $\delta_3(\Lambda_3) = 84^\circ$ (see Fig. 3). Then, we obtain the predictions of $|V_{ij}|$ at $\mu = m_Z$

$$\begin{aligned} &|V_{us}| = 0.220 \ , \quad |V_{cb}| = 0.0418 \ , \\ &|V_{ub}/V_{cb}| = 0.0726 \ , \\ &|V_{td}| = 0.0109 \ , \\ &J = 2.38 \times 10^{-5} \ . \end{aligned} \qquad (6.19)$$

which is in excellent agreement with the experimental values (6.16). In Fig. 4, we illustrate the predicted values $|V_{ij}(m_Z)|$ versus $\delta_3(\Lambda_X)$. As seen in Fig. 4, the value of $\delta_3(\Lambda_X)$ at which $\sum_{i\neq j}|V_{ij}(\Lambda_X)|^2$ takes the minimum also gives the minimum of the CKM mixings at $\mu = m_Z$.

VII. NUMERICAL RESULTS IN THE SUSY MODEL

The behavior of $\xi_L^u(\mu)$ in the SUSY model is somewhat different from that in the non-SUSY model. Since in the SUSY model, the top quark mass $m_t(\mu)$ is given by

$$m_t(\mu) = \frac{1}{\sqrt{3}} \xi_L^u(\mu) \frac{v_L}{\sqrt{2}} \sin\beta \ , \tag{7.1}$$

where $v_L/\sqrt{2} = 174$ GeV and $\tan\beta \equiv \tan\beta_L = v_L^u/v_L^d$, the initial value of $\xi_L^u(m_Z)$ in the SUSY model corresponds to

$$[\xi_L^u(m_Z)]_{SUSY} = [\xi_L^u(m_Z)]_{nonSUSY} \frac{1}{\sin\beta} \ . \tag{7.2}$$

However, this does not mean $[\xi_L^u(\Lambda_X)]_{SUSY} = [\xi_L^u(\Lambda_X)]_{nonSUSY}/\sin\beta$, because the behavior of $[\xi_L^u(\mu)]_{SUSY}$ is considerably different from that of $[\xi_L^u(\mu)]_{nonSUSY}$. In Fig. 5, we illustrate the behavior of $\xi_L^u(\mu)$ in the SUSY model for the case of $\tan\beta = 3.5$. If we take $\tan\beta < 2.5$, the initial value of $\xi_L^u(m_Z)$ becomes $\xi_L^u(m_Z, \tan\beta > 2.5) > \xi_L^u(m_Z, \tan\beta = 2.5)$ from Eq. (7.2), so that the curve of $\xi_L^u(\mu)$ will be illustrated in the upper side of the curve given in Fig. 5. Therefore, for a case with a small value of $\tan\beta$, the Landau pole of $\xi_L^u(\mu)$ appears at a relatively lower energy scale. We consider that the model should be calculable perturbatively, so that a case with such a large value of ξ_L^u should be ruled out. As seen in Fig. 5, since the model gives, in general, $\xi_L^u(\Lambda_X) > \xi_L^u(\mu)$ $(m_Z < \mu < \Lambda_X)$, the value $\xi_L^u(\Lambda_X)$ should, at least, be $[\xi_L^u(\Lambda_X)]^2/4\pi < 1$, i.e., $\xi_L^u(\Lambda_X) < \sqrt{4\pi} = 3.54$. However, when we take contributions from the higher order corrections into consideration, even the value $\xi_L^u(\Lambda_X) = 3.0$ is still dangerous. Therefore, we put the constraint $\xi_L^u(\Lambda_X) = 2.0$ for the results of the present one loop calculation. In Fig. 6, we illustrate the predicted value of $m_t(m_Z)$ for the initial values $\xi_L^u(\Lambda_X) = \sqrt{4\pi} = 3.54$ and $\xi_L^u(\Lambda_X) \lesssim 2.0$, where we have used the input values

$$\Lambda_X = 2 \times 10^{16} \text{ GeV} \ , \quad \Lambda_S = 6 \times 10^{13} \text{ GeV} \ . \tag{7.3}$$

The value of Λ_S has been chosen as the neutrino mass-squared difference Δm_{32}^2 is of the order of $(10^{-3} - 10^{-2})$ eV^2.

From Fig. 6, we conclude that the value of $\tan\beta$ must be

$$\tan\beta \gtrsim 3 \ . \tag{7.4}$$

Prior to the numerical investigation of the evolutions in the SUSY model, in order to see the difference between the parameter structures in the non-SUSY and SUSY models, let us give a rough sketch for the parameters in the case of the SUSY model by neglecting the evolution effects. The quark mass matrices M_u and M_d are given by

$$(M_u)_{ij} = \sum_{k=1}^{2} \left(\widetilde{Z}\right)_{ik} \left(\widetilde{Z}\right)_{jk} (O^u)_{kk} \frac{\xi_L^u \xi_R^u}{\xi_S^u} \frac{\Lambda_L \Lambda_R}{\Lambda_S} \sin\beta \ , \tag{7.5}$$

$$(M_d)_{ij} = \sum_{k=1}^{3} \left(\widetilde{Z}\right)_{ik} \left(\widetilde{Z}\right)_{jk} (O^d)_{kk} \frac{\xi_L^d \xi_R^d}{\xi_S^d} \frac{\Lambda_L \Lambda_R}{\Lambda_S} \cos\beta \ , \tag{7.6}$$

where $O^u = \mathrm{diag}(1,1)$, $O^d = \mathrm{diag}(1,1,1/(1+3b_d))$, and $\widetilde{Z}$ is given by Eq. (5.14). Here, for simplicity, we have assumed $\beta_L = \beta_R = \beta_S \equiv \beta$. For $\tan\beta > 3$, the factors $\sin\beta$ and $\cos\beta$ are approximated as $\sin\beta \simeq 1$ and $\cos\beta \simeq 1/\tan\beta$, respectively. Obviously, the model with $\xi_S^u = \xi_S^d$ in addition to the constraint

$$\xi_{LR}^u(\Lambda_X) = \xi_{LR}^d(\Lambda_X) \equiv \xi_{LR}^q(\Lambda_X) \ , \tag{7.7}$$

is ruled out, because we cannot fit the up- and down-quark masses simultaneously due to the existence of the factor $\cos\beta$. Therefore, we must consider a model with $\xi_S^u \neq \xi_S^d$ differently from the constraint (6.6) in the non-SUSY model. If we consider

$$\xi_S^u \simeq \xi_S^q \sin\beta \ , \quad \xi_S^d \simeq \xi_S^q \cos\beta \ , \tag{7.8}$$

then the model becomes similar to the case of the non-SUSY model, because

$$\frac{\xi_L^u \xi_R^u}{\xi_S^u} \frac{\Lambda_L \Lambda_R}{\Lambda_S} \sin\beta \simeq \frac{\xi_L^d \xi_R^d}{\xi_S^d} \frac{\Lambda_L \Lambda_R}{\Lambda_S} \cos\beta \ , \tag{7.9}$$

and we will obtain reasonable fittings for the quark masses and CKM matrix parameters as well as in the non-SUSY model. Note that for a large value of $\tan\beta$, the value of $K^d \equiv \xi_L^d \xi_R^d/\xi_S^d$ becomes large because $K^d \simeq K^u \tan\beta$ from the relation (7.9), so that we cannot evaluate the RGE (3.12) perturbatively. We must take the value of $\tan\beta$ near to the lower bound given by Eq. (7.4).

When we take the evolution effects into consideration, the situation is further complicated. The evolutions of $\xi_L^f(\mu)$, $\xi_R^f(\mu)$ and $\xi_S^f(\mu)$ in the SUSY model are quite different from those in the non-SUSY model. We illustrate the behaviors of $m_i^f(\mu)/m_i^f(\Lambda_X)$ which correspond to the behaviors of $[\xi_L^f(\mu)\xi_R^f(\mu)/\xi_S^f(\mu)]/[\xi_L^f(\Lambda_X)\xi_R^f(\Lambda_X)/\xi_S^f(\Lambda_X)]$ in the non-SUSY model and those in the SUSY model in Figs. 7 and 8, respectively. In Fig. 8, we see that the values $m_u(\mu)$ and $m_c(\mu)$ cause rapid changes in the range III. In the non-SUSY model, the charged lepton mass ratios are almost invariant, i.e., $m_e(\mu)/m_\mu(\mu) \simeq$ constant and $m_\mu(\mu)/m_\tau(\mu) \simeq$ constant, while, in the SUSY model, the mass ratio $m_\mu(\mu)/m_\tau(\mu)$ shows a considerable change (although $m_e(\mu) \simeq m_\mu(\mu)$ still holds).

The situation is critical for the input values. If we adhere to the input value $m_t(m_Z) = 181$ GeV, then it is hard to obtain reasonable values of the other quark mass values m_c, m_u, m_b, m_s and m_d for any parameter values of Λ_S/Λ_R and b_d. However, if we take a slightly lower value of $m_t(m_Z)$, for example, $m_t(m_Z)=168$ GeV [cf. $[m_t(m_Z)]_{observed}=181\pm 13$ GeV], we can find the following parameter values

$$\tan\beta = 3.5\ , \quad \Lambda_S/\Lambda_R = 38\ , \tag{7.10}$$

$$z_1 = 0.01449\ , \quad z_2 = 0.2117\ , \quad z_3 = 0.9772\ , \tag{7.11}$$

$$\xi^u_{LR}(\Lambda_X) = \xi^d_{LR}(\Lambda_X) \equiv \xi^q_{LR}(\Lambda_X) = 1.3\ , \quad \xi^e_{LR}(\Lambda_X) = 1.0\ , \tag{7.12}$$

$$\xi^u_S(\Lambda_X) = 1.7\ , \quad \xi^d_S(\Lambda_X) = 0.50\ , \quad \xi^e_S(\Lambda_X) = 1.0\ , \tag{7.13}$$

$$b_d = -1.2\ , \quad \beta_d = 19.4^\circ\ , \tag{7.14}$$

which leads to the following quark and charged lepton masses and CKM matrix parameters:

$$\begin{aligned}
m_u(m_Z) &= 2.47\times 10^{-3}\ \mathrm{GeV}\ ,\\
m_c(m_Z) &= 6.46\times 10^{-1}\ \mathrm{GeV}\ ,\\
m_t(m_Z) &= 167\ \mathrm{GeV}\ ,\\
m_d(m_Z) &= 4.49\times 10^{-3}\ \mathrm{GeV}\ ,\\
m_s(m_Z) &= 1.00\times 10^{-1}\ \mathrm{GeV}\ ,\\
m_b(m_Z) &= 2.83\ \mathrm{GeV}\ ,\\
m_e(m_Z) &= 4.87\times 10^{-4}\ \mathrm{GeV}\ ,\\
m_\mu(m_Z) &= 1.03\times 10^{-1}\ \mathrm{GeV}\ ,\\
m_\tau(m_Z) &= 1.75\ \mathrm{GeV}\ ,
\end{aligned} \tag{7.15}$$

$$\begin{aligned}
&|V_{us}| = 0.220\ , \quad |V_{cb}| = 0.0665\ ,\\
&|V_{ub}/V_{cb}| = 0.0603\ ,\\
&|V_{td}| = 0.0179\ ,\\
&J = 3.38\times 10^{-5}\ .
\end{aligned} \tag{7.16}$$

The values $|V_{ij}|^2$ are again desirably adjustable by the phase parameter δ_3 defined by (6.18).

VIII. EVOLUTION OF THE NEUTRINO MASS MATRIX

The evolution of the neutrino mass matrix $M_\nu = K^\nu\langle\phi^0_L\rangle^2/\langle\Phi^0\rangle$ is described by the RGE (3.13). Since the coefficient H^ν_{KL} in the ranges I and II is given by $H^\nu_{KL}=\lambda_{HL}$, (4.28), for the non-SUSY model, and by $H^\nu_{KL}=0$, (A.15) and (A.24), for the SUSY model, the form of the matrix K^ν at $\mu=\Lambda_L$ does not vary from that at $\mu=\Lambda_S$, so that the mass ratios and mixing matrix U_ν are also invariant. Since the coefficients H^e_{KL} and H^e_{KR} in the charged lepton sector are given by $H^e_{KL}=H^e_{KR}=0$ in the ranges I and II for the non-SUSY and SUSY models, the form of the charged lepton mass matrix M_e is also invariant below $\mu=\Lambda_S$. Therefore, the Maki-Nakagawa-Sakata (MNS) [12] matrix $U=U^\dagger_{eL}U_\nu$ is invariant in the ranges I and II. Note that in the conventional model, the neutrino seesaw mass matrix can vary the from. The neutrino mass matrix in the present model can vary the form only in the range III ($\Lambda_S<\mu\le\Lambda_X$). The reason is that in the conventional model the scalar ϕ^+_L couples to $\overline{\nu}_L e_R$, while that in the present model couples to $\overline{\nu}_L E_R$, so that the contribution of ϕ_L to H^ν_{KL} in the latter case is decoupled below $\mu=\Lambda_S$.

For the numerical study, the case with $b_\nu=-1/2$ is most interesting, because the inverse matrix of $Y^\nu_S(\Lambda_X)=\xi^\nu_S(\Lambda_X)[\mathbf{1}+3b_\nu(\Lambda_X)X]$ with $b_\nu(\Lambda_X)=-1/2$ has the form

$$[Y^\nu_S(\Lambda_X)]^{-1} = -\frac{1}{\xi^\nu_S(\Lambda_X)}\begin{pmatrix} 0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{pmatrix}\ , \tag{8.1}$$

so that

$$Y^\nu_L(Y^\nu_S)^{-1}(Y^\nu_L)^T = -\frac{\xi^\nu_L(\Lambda_X)]^2}{\xi^\nu_S(\Lambda_X)}\begin{pmatrix} 0 & z_1z_2 & z_1z_3\\ z_1z_2 & 0 & z_2z_3\\ z_1z_3 & z_2z_3 & 0\end{pmatrix}\ . \tag{8.2}$$

The form (8.2) is well known as the Zee-type [13] mass matrix, which can lead to a large mixing [14].

The mass eigenvalues $m_{\nu i}$ and mixing matrix U at $\mu=\Lambda_X$ are given by [15]

$$\begin{aligned}
m_{\nu 1} &\simeq -2z_1^2 m^\nu_0\ ,\\
m_{\nu 2} &\simeq -\left[z_2z_3 - \left(1-\frac{z_3}{2z_2}\right)z_1^2\right]m^\nu_0\ ,\\
m_{\nu 3} &\simeq \left[z_2z_3 + \left(1+\frac{z_3}{2z_2}\right)z_1^2\right]m^\nu_0\ ,
\end{aligned} \tag{8.3}$$

$$m^\nu_0 = \frac{(\xi^\nu_L)^2}{\xi^\nu_S}\frac{\Lambda_L^2}{\Lambda_S}\ , \tag{8.4}$$

$$U = \begin{pmatrix} 1 & \frac{1}{\sqrt{2}}\frac{z_1}{z_2}(1-z_2) & \frac{1}{\sqrt{2}}\frac{z_1}{z_2}(1+z_2)\\ -\frac{z_1}{z_2} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ -z_1 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}\ . \tag{8.5}$$

The model with $b_d=-1/2$ gives highly degenerate mass-squared levels $m^2_{\nu 2}\simeq m^2_{\nu 3}$ and a large mixing between ν_μ and ν_τ at $\mu=\Lambda_X$. Therefore, the model has a possibility that it can give a reasonable explanation for the atmospheric neutrino data [16].

In Figs. 9 and 10, we illustrate the behaviors of the mass-squared differences $\Delta m^2_{ij}=m^2_i-m^2_j$ in the non-SUSY and SUSY models, respectively. As seen in Figs. 9

and 10, the mass-squared difference Δm^2_{32} rapidly increase according as the energy scale decreases in the range III. The numerical results are given in Table II. We can see that the neutrino mass ratios are invariant in the ranges I and II.

As we stated already, the values (z_1, z_2, z_3) (therefore, the mass ratios m_e/m_μ and m_μ/m_τ) are almost invariant in the range III, while the ratio $\Delta m^2_{32}/\Delta m^2_{21}$ is rapidly vary in the range III. Although the relations (8.3) give $\Delta m^2_{32} \simeq 4z_1^2 z_2 z_3 (m_0^\nu)^2$ and $\Delta m^2_{21} \simeq (z_2 z_3^2)^2 (m_0^\nu)^2$, the rapid decrease in the ratio $\Delta m^2_{32}/\Delta m^2_{21}$ does not mean the rapid decrease in the ratio $z_1^2/z_2 z_3$. The rapid decrease comes from the slight deviation of the parameter $b_\nu(\mu)$ from the value $b_\nu(\Lambda_X) = -1/2$. The value of $b_f(\mu)$ is not invariant in the range III, although the form of $Y_S^f(\mu)$, "the unit matrix plus a democratic matrix", is invariant. When we denote

$$b_\nu(\mu) = -\frac{1}{2}\left(1 + \varepsilon_\nu(\mu)\right) , \tag{8.6}$$

the expression (8.1) is replaced with

$$[Y_S^\nu(\mu)]^{-1} \simeq -\frac{1}{\xi_S^\nu(\mu)} \begin{pmatrix} -2\varepsilon_\nu & 1 & 1 \\ 1 & -2\varepsilon_\nu & 1 \\ 1 & 1 & -2\varepsilon_\nu \end{pmatrix} , \tag{8.7}$$

so that

$$Y_L^\nu (Y_S^\nu)^{-1} (Y_L^\nu)^T \simeq -\frac{[\xi_L^\nu(\mu)]^2}{\xi_S^\nu(\mu)} \begin{pmatrix} -2\varepsilon_\nu z_1^2 & z_1 z_2 & z_1 z_3 \\ z_1 z_2 & -2\varepsilon_\nu z_2^2 & z_2 z_3 \\ z_1 z_3 & z_2 z_3 & -2\varepsilon_\nu z_3^2 \end{pmatrix} . \tag{8.8}$$

Therefore, the mass eigenvalues in the range III are given by

$$\begin{aligned} m_{\nu 1} &\simeq -2(1+3\varepsilon_\nu) z_1^2 m_0^\nu , \\ m_{\nu 2} &\simeq -\left[z_2 z_3 - \left(1 - \frac{z_3}{2z_2}\right) z_1^2 + \varepsilon_\nu \right] m_0^\nu , \\ m_{\nu 3} &\simeq \left[z_2 z_3 + \left(1 + \frac{z_3}{2z_2}\right) z_1^2 - \varepsilon_\nu \right] m_0^\nu , \end{aligned} \tag{8.9}$$

instead of (8.3), and the mass squared differences Δm^2_{21} and Δm^2_{32} are given by

$$\begin{aligned} \Delta m^2_{21} &\simeq (z_2 z_3)^2 (m_0^\nu)^2 , \\ \Delta m^2_{32} &\simeq 4 z_2 z_3 (z_1^2 - \varepsilon_\nu)(m_0^\nu)^2 . \end{aligned} \tag{8.10}$$

Note that the approximate expression (8.19) tell us that $\Delta m^2_{32}(\mu)$ takes a zero between $\mu = \Lambda_X$ and $\mu = \Lambda_S$ because $\varepsilon_\nu(\Lambda_X) = 0 < z_1^2(\Lambda_X) \simeq z_1^2(\Lambda_S) < \varepsilon_\nu(\Lambda_S)$, e.g., $\varepsilon_\nu(\Lambda_S) = 7.3 \times 10^{-2}$ and $z_1^2(\Lambda_X) \simeq z_1^2(\Lambda_S) \simeq 2.6 \times 10^{-4}$ for the non-SUSY and $\varepsilon_\nu(\Lambda_S) = 1.1 \times 10^{-2}$, $z_1^2(\Lambda_X) \simeq 2.1 \times 10^{-4}$ and $z_1^2(\Lambda_S) \simeq 2.6 \times 10^{-4}$ for the SUSY model. In fact, we can see this at a point which is very close to $\mu = \Lambda_X$ in Figs. 9 and 10. Thus, the value of $\Delta m^2_{32}(\mu)$ is highly sensitive to the value of $\varepsilon_\nu(\mu)$, although $\Delta m^2_{21}(\mu)$ is not so.

In general, since the mixing angle θ_{23} is given by

$$\sin 2\theta_{23} \simeq \frac{2(M_\nu)_{23}}{m_{\nu 3} - m_{\nu 2}} , \tag{8.11}$$

the mixing angle θ_{23} in the conventional democratic type neutrino mass matrix model is sensitive to the energy scale [17], because $\Delta m^2_{32}(\mu)$ has a large energy scale dependency. In contrast to the conventional model, the mixing angle θ_{23} in the present model does not so drastically vary. The reason is as follows: the neutrino mass matrix M_ν in the present "democratic" seesaw model is not democratic, i.e., the form of M_ν is given by Eq. (8.2). In fact, the present model gives not $m_{\nu 2} \simeq m_{\nu 3}$, but $m_{\nu 2} \simeq -m_{\nu 3}$, so that the evolution effect on U_{23} is not so sensitive as seen in Eq. (8.11).

As seen in Table II, the model can fit the value Δm^2_{32} to the atmospheric neutrino data [16] $(\Delta m^2_{32})_{observ} = 3.2 \times 10^{-3}$ eV^2 by adjusting the value of Λ_S, but it cannot give any explanation of the solar neutrino data [18], because of $\Delta m^2_{21} \gg \Delta m^2_{32} \equiv \Delta m^2_{atm}$. We must introduce a further mechanism for the explanation of the solar neutrino data, for example, as discussed in Ref. [19]. However, since the purpose of the present model is not to propose a plausible neutrino mass matrix model in the framework of the USM, but to see the characteristic features of the neutrino mass matrix evolution in contrast to the conventional seesaw model. Therefore, we do not touch the numerical fitting furthermore.

IX. CONCLUSIONS

In conclusion, we have investigated the evolutions of the quark and lepton mass matrices M_f $(f = u, d, \nu, e)$ in the universal seesaw model with $\det M_F = 0$ in the up-quark sector $F = U$.

The assumptions which have made in the present paper are classified into the following three categories:
(A) Basic assumptions in the universal seesaw model with $\det M_U = 0$;
(B) Basic assumptions in the democratic seesaw model [4,5] (we have taken the model as a more concrete one of the universal seesaw model with $\det M_U = 0$ in order to give an explicit evaluation of the universal seesaw model);
(C) Tentative assumptions for convenience of the numerical evaluation.

The assumptions in the category (A) are as follows:
(A1) $\mathrm{SU(3)}_c \times \mathrm{SU(2)}_L \times \mathrm{SU(2)}_R \times \mathrm{U(1)}_Y \times \mathrm{U(1)}_X$ gauge symmetries with the symmetry breaking pattern (2.1);
(A2) Hypothetical heavy fermions $F = (U, D, N, E)$ which belong to $(1,1)$ of $\mathrm{SU(2)}_L \times \mathrm{SU(2)}_R$ and acquire masses of the order of Λ_S at the energy scale $\mu = \Lambda_S$ except for U_{3L} and U_{3L}.

In the present model, therefore, the quark and charged lepton mass matrices M_f $(f = u, d, e)$ and neutrino mass matrix M_ν are given by

$$M_f = Y_L^f (Y_S^f)^{-1} (Y_R^f)^\dagger (\Lambda_L \Lambda_R / \Lambda_S) \ , \tag{9.1}$$

$$M_\nu = Y_L^\nu (Y_S^\nu)^{-1} (Y_L^\nu)^T (\Lambda_L^2 / \Lambda_S) \ , \tag{9.2}$$

except for the top quark, where $\Lambda_L = \langle \phi_L^0 \rangle$, $\Lambda_R = \langle \phi_R^0 \rangle$ and $\Lambda_S = \langle \Phi^0 \rangle$. The evolutions below $\mu = \Lambda_S$ are described by the RGE (3.12) and (3.13) for the seesaw operators. On the other hand, the top quark mass $m_t(\mu)$ given by the expression (3.18) is still described by RGE (3.6) for the Yukawa coupling constants below $\mu = \Lambda_S$. Although the heavy fermions F do not contribute to the evolutions below $\mu = \Lambda_S$, the third family "would-be" heavy up-quark U_3 can contribute to the RGE even below $\mu = \Lambda_S$. However, as far as the H_{KL}^f ($F = \nu, e$) and H_{KR}^e terms in the lepton sectors are concerned, the would-be heavy quark U_3 cannot contribute to those, so that the forms of the mass matrices $M_\nu(\mu)$ and $M_e(\mu)$ are invariant below $\mu = \Lambda_S$.

The assumptions in the category (B) are as follows:
(B1) At a unification scale $\mu = \Lambda_X$, the Yukawa coupling constants Y_L^f and Y_R^f have the same form, i.e., $Y_L^f(\Lambda_X) = Y_R^f(\Lambda_X) \equiv Y_{LR}^f(\Lambda_X)$.
(B2) At $\mu = \Lambda_X$, the heavy fermion mass matrices M_F (therefore, the Yukawa coupling constants Y_S^f) [and also the Majorana masses M_L (M_R) of the neutral fermions N_L (N_R)] take a simple diagonal form (5.1), the form " the unit matrix pulse a democratic matrix", on the basis on which the Yukawa coupling constants $Y_{LR}^f(\Lambda_X)$ are diagonal. Then, the form (5.1) is invariant under the evolution in the range III.
(B3) The values of the parameter b_f in the matrix Y_S^f given by Eq. (5.1) are given by $b_e = 0$, $b_\nu = -1/2$ and $b_u = -1/3$ at $\mu = \Lambda_X$. (The value b_d is kept as a free parameter in order to fit the up- and down-quark masses and CKM matrix parameters reasonably.)

In this model, the top quark mass $m_t(\mu)$ is given by (5.15). The behavior of $m_t(\mu)$, i.e., $\xi_L^u(\mu)$, is given in Figs. 1 and 5 for the non-SUSY and SUSY models, respectively. We can obtain the constraint on the values of the intermediate energy scales Λ_R and Λ_S by considering that the model should be calculable perturbatively. In the non-SUSY model, since $\Lambda_S/\Lambda_R \sim 10^2$ from the ratio m_t/m_c, we find the constraint

$$10^{10} \text{ GeV} < \Lambda_S < 10^{19} \text{ GeV} \ , \tag{9.3}$$

for $\Lambda_X \sim 10^{16}$ GeV. In the SUSY model, the results highly depend on the input parameter $\tan\beta$. From the numerical study, we obtain the constraints

$$3 < \tan\beta < 4 \ , \tag{9.4}$$

$$10^{10} \text{ GeV} < \Lambda_S < 10^{19} \text{ GeV} \ , \tag{9.5}$$

for $\Lambda_X \sim 10^{16}$ GeV. (The above numerical results are slightly dependent on the assumptions stated below, but the dependence is not so large.)

The assumptions in the category (C) are as follows:
(C1) For convenience of the numerical evaluation, $g_{2L}(\mu) = g_{2R}(\mu)$ has been assumed. Then, we can assert $Y_L^f(\mu) = Y_R^f(\mu)$ in the range III ($\Lambda_S < \mu \leq \Lambda_X$) as we have shown in Eq. (4.8).
(C2) For evaluation of the non-SUSY model, the initial condition

$$\xi_{LR}^u(\Lambda_X) = \xi_{LR}^d(\Lambda_X) = \xi_{LR}^e(\Lambda_X) = \xi_{LR}^\nu(\Lambda_X) \tag{9.6}$$

has been assumed together with the initial condition (3.5), i.e.,

$$Z_{L/R}^u = Z_{L/R}^d = Z_{L/R}^e = Z_{L/R}^\nu \ . \tag{9.7}$$

However, since there is no solution of the parameter values for the SUSY model under such a constraint (9.6), the constraint corresponding to (9.6) in the SUSY model has been loosened as

$$\xi_{LR}^u(\Lambda_X) = \xi_{LR}^d(\Lambda_X) \neq \xi_{LR}^e(\Lambda_X) = \xi_{LR}^\nu(\Lambda_X) \ , \tag{9.8}$$

although the initial condition (9.7) has still been required in the SUSY model.
(C3) For the non-SUSY model, we have assumed $\xi_S^u(\Lambda_X) = \xi_S^d(\Lambda_X) \neq \xi_S^e(\Lambda_X) = \xi_S^\nu(\Lambda_X)$, but, for the SUSY model, we have assumed that each value of $\xi_S^f(\Lambda)$ may be different among them, because the previous condition is too strong for the SUSY model and the up$\leftrightarrow$down symmetry is already broken due to the factor $\tan\beta \neq 1$ in the SUSY model.

In the conventional model for quark and charged lepton masses (i.e., not seesaw model), the following approximate relations are satisfied in the non-SUSY and SUSY models:

$$\frac{(m_u/m_c)_L}{(m_u/m_c)_X} \simeq \frac{(m_d/m_s)_L}{(m_d/m_s)_X} \simeq \frac{(m_e/m_\mu)_L}{(m_e/m_\mu)_X} \simeq \frac{(m_\mu/m_\tau)_L}{(m_\mu/m_\tau)_X} \simeq 1 \tag{9.9}$$

$$\frac{(m_u/m_t)_L}{(m_u/m_t)_X} \simeq \frac{(m_c/m_t)_L}{(m_c/m_t)_X} \simeq 1 + \varepsilon_u \ , \tag{9.10}$$

$$\frac{(m_d/m_b)_L}{(m_d/m_b)_X} \simeq \frac{(m_s/m_b)_L}{(m_s/m_b)_X} \simeq 1 + \varepsilon_d \ , \tag{9.11}$$

$$\frac{|V_{cb}(\Lambda_L)|}{|V_{cb}(\Lambda_X)|} \simeq \frac{|V_{ub}(\Lambda_L)|}{|V_{ub}(\Lambda_X)|} \simeq \frac{|V_{td}(\Lambda_L)|}{|V_{td}(\Lambda_X)|} \simeq 1 + \varepsilon_d \ , \tag{9.12}$$

where $(m_u/m_c)_L$ denotes $m_u(\Lambda_L)/m_c(\Lambda_L)$, and so on. The relations (9.9)-(9.12) are due to that the Yukawa coupling constant y_t of the top quark in the conventional model is very large compared with the other Yukawa coupling constants. In the present model, as seen in Figs. 7 and 8, the relations (9.9)-(9.12) are also satisfied in the range I ($\Lambda_L < \mu \leq \Lambda_R$) (so that we read the relations (9.9)-(9.12) as $X \to R$). The values of ε_u and ε_d are approximately given by $\varepsilon_u \sim \varepsilon_d$ for the non-SUSY model, and by $\varepsilon_u \simeq -3\varepsilon_d$ for the SUSY model. In the range II ($\Lambda_R < \mu \leq \Lambda_S$), the relations (9.9)-(9.12) are slightly broken. In the SUSY model, the values show not $\varepsilon_u \simeq -3\varepsilon_d$, but $\varepsilon_u \sim \varepsilon_d$ in the range II. However, in the model with $\Lambda_L/\Lambda_R \gg 1$, which is required in order to make the neutrino masses tiny, the evolution effects in the range II are not so large, so that we can regard that the relations (9.9)-(9.12) are still satisfied in the range $\Lambda_L < \mu \leq \Lambda_S$, i.e.,

$$D_u(\Lambda_L) \simeq \frac{m_t(\Lambda_L)}{m_t(\Lambda_S)}(1+\varepsilon_u)(\mathbf{1} - \varepsilon_u S) D_u(\Lambda_S) \ , \quad (9.13)$$

$$D_d(\Lambda_L) \simeq \frac{m_b(\Lambda_L)}{m_b(\Lambda_S)}(1+\varepsilon_d)(\mathbf{1} - \varepsilon_d S) D_d(\Lambda_S) \ , \quad (9.14)$$

$$V(\Lambda_L) \simeq (\mathbf{1} + \varepsilon_V S) V(\Lambda_S)(\mathbf{1} + \varepsilon_V S) - 2\varepsilon_V V_{td} S \ , \quad (9.15)$$

where $D_u = \mathrm{diag}(m_u, m_c, m_t)$, $D_d = \mathrm{diag}(m_d, m_s, m_b)$, and S is defined by Eq. (4.9). In the present model, the value of ε_V is not always given by $\varepsilon_V \simeq \varepsilon_d$ because of the presence of the range II.

Also in the ranges I and II, differently from the conventional seesaw model (for example, see Ref. [2]), the neutrino mass ratios and mixing angles are not affected by the evolution effects:

$$\frac{m_{\nu i}(\Lambda_L)/m_{\nu j}(\Lambda_L)}{m_{\nu i}(\Lambda_S)/m_{\nu j}(\Lambda_S)} \simeq 1 \ , \quad (9.16)$$

$$\frac{V_{ij}(\Lambda_L)}{V_{ij}(\Lambda_S)} \simeq 1 \ . \quad (9.17)$$

Note that the relation (9.16) does not mean $\Delta m^2_{ij}(\Lambda_L)/\Delta m^2_{ij}(\Lambda_S) \simeq 1$. However, the ratio $\Delta m^2_{21}/\Delta m^2_{32}$ is again invariant in the ranges I and II.

In the range III ($\Lambda_S < \mu \leq \Lambda_X$), the relations (9.9)-(9.12) [(9.13)-(9.15)] and (9.16)-(9.17) are not satisfied at all. For example, the behavior of $\Delta m^2_{32}(\mu)$ is highly sensitive to the value $\varepsilon_\nu(\mu)$ and is given by Eq. (8.10). In other words, the differences of the numerical behaviors of the quark masses, CKM matrix parameters and neutrino mass squared differences from those in the conventional model are substantially formed in the range III.

Note that the mass ratios m_e/m_μ and m_u/m_c are almost constant (although the ratio m_u/m_c is slightly changed in the SUSY model), so that the phenomenologically well-satisfied relation (1.14) still holds under the evolutions.

For the neutrino mass matrix M_ν, we have investigate the model with $b_\nu(\Lambda_X) = -1/2$, which leads to a large mixing $\sin^2\theta_{23} \simeq 1$. Although the mass-squared difference $\Delta m^2_{32}(\mu)$ is highly sensitive to the energy scale μ in the range III ($\Lambda_S < \mu \leq \Lambda_X$), the mixing angle θ_{23} is not sensitive to the energy scale. In contrast to the conventional seesaw neutrino mass matrix, note that the present neutrino mass matrix M_ν is form-invariant below $\mu = \Lambda_S$, so that the neutrino mass ratios and mixings are invariant below $\mu = \Lambda_S$.

In the present paper, we have assumed $\mathrm{SU(3)}_c \times \mathrm{SU(2)}_L \times \mathrm{SU(2)}_R \times \mathrm{U(1)}_{LR} \times \mathrm{U(1)}_X$ symmetries above $\mu = \Lambda_S$. As seen in Figs. 1 and 5, in general, the rapid increasing of the Yukawa coupling constant $Y_L^u(\mu)$ causes above $\mu = \Lambda_S$, although we have been able to find a set of the reasonable parameter values without having the Landau pole below $\mu = \Lambda_X$. The rapid increasing is mainly due to the rapid increasing of the gauge coupling constant g_1 above $\mu = \Lambda_S$. If we want to build a unification model with a unified gauge symmetry G, we may consider that the U(1) symmetry is embedded into the unified symmetry G. (For example, see an $\mathrm{SO(10)}_L \times \mathrm{SO(10)}_R$ model [20], where $\mathrm{SO(10)}_L \times \mathrm{SO(10)}_R$ is broken into $[\mathrm{SU(2)} \times \mathrm{SU(2)}' \times \mathrm{SU(4)}]_L \times [\mathrm{SU(2)} \times \mathrm{SU(2)}' \times \mathrm{SU(4)}]_R$.) Then, the gauge structure above $\mu = \Lambda_S$ is different from the present model, so that the evolutions will be also different from the present results. (Of course, the evolutions below $\mu = \Lambda_S$ are still the same as those in the present paper.) It is likely that the gauge structure above $\mu = \Lambda_S$ is different from the present model. Our next task is to investigate what gauge structure above $\mu = \Lambda_S$ is promising for a unified description of the quark and lepton masses and mixings.

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Appendix

In Secs. 4 and 5, the coefficients of RGE (3.6), (3.12) and (3.13) have been given only for the case of non-SUSY scenario with one SU(2)-doublet Higgs scalar. In the present Appendix, we give the coefficients of RGE in the minimal SUSY scenario.

[Range III]

$$\begin{aligned} T_A^u &= T_A^\nu = 3\mathrm{Tr}(Y_A^u Y_A^{u\dagger}) + \mathrm{Tr}(Y_A^\nu Y_A^{\nu\dagger}), \\ T_A^d &= T_A^e = 3\mathrm{Tr}(Y_A^d Y_A^{d\dagger}) + \mathrm{Tr}(Y_A^e Y_A^{e\dagger}), \end{aligned} \quad (A.1)$$

$$\begin{aligned} G_A^u &= \frac{13}{6}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2 + g_X^2, \\ G_A^d &= \frac{7}{6}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2 + g_X^2, \\ G_A^\nu &= \frac{9}{6}g_1^2 + 3g_2^2 + g_X^2, \end{aligned} \quad (A.2)$$

$$G_A^e = \frac{27}{6} g_1^2 + 3g_2^2 + g_X^2,$$

$$\begin{aligned} H_A^u &= 3Y_A^u Y_A^{u\dagger} + Y_A^d Y_A^{d\dagger}, \\ H_A^d &= 3Y_A^d Y_A^{d\dagger} + Y_A^u Y_A^{u\dagger}, \\ H_A^\nu &= 3Y_A^\nu Y_A^{\nu\dagger} + Y_A^e Y_A^{e\dagger}, \\ H_A^e &= 3Y_A^e Y_A^{e\dagger} + Y_A^\nu Y_A^{\nu\dagger}, \end{aligned} \tag{A.3}$$

$$\begin{aligned} T_S^u = T_S^\nu &= 3\mathrm{Tr}(Y_S^u Y_S^{u\dagger}) + \mathrm{Tr}(Y_S^\nu Y_S^{\nu\dagger}), \\ T_S^d = T_S^e &= 3\mathrm{Tr}(Y_S^d Y_S^{d\dagger}) + \mathrm{Tr}(Y_S^e Y_S^{e\dagger}), \end{aligned} \tag{A.4}$$

$$\begin{aligned} G_S^u &= \frac{8}{3} g_1^2 + \frac{16}{3} g_3^2 + 3g_X^2, \\ G_S^d &= \frac{2}{3} g_1^2 + \frac{16}{3} g_3^2 + 3_X^2, \\ G_S^\nu &= 3g_X^2, \\ G_S^e &= \frac{18}{3} g_1^2 + 3g_X^2, \end{aligned} \tag{A.5}$$

$$H_S^f = 2Y_S^f Y_S^{f\dagger}, \tag{A.6}$$

where $A = L, R$ and $f = u, d, \nu, e$.

[Range II]

$$T_A^u = 3\mathrm{Tr}(Y_A^u S Y_A^{u\dagger}), \tag{A.7}$$

$$G_A^u = \frac{13}{6} g_1^2 + 3g_{2A}^2 + \frac{16}{3} g_3^2, \tag{A.8}$$

$$H_A^u = 3Y_A^u S Y_A^{u\dagger}, \tag{A.9}$$

$$\begin{aligned} T_K^u &= 3\mathrm{Tr}(Y_L^u S Y_L^{u\dagger} + Y_R^u S Y_R^{u\dagger}), \\ T_K^d &= T_K^e = 0, \end{aligned} \tag{A.10}$$

$$G_K^u = G_K^d = G_K^e = \frac{9}{2} g_1^2 + \frac{9}{2}(g_{2L}^2 + g_{2R}^2), \tag{A.11}$$

$$\begin{aligned} H_{KA}^u &= H_{KA}^d = \frac{2}{3} Y_A S Y_A^{u\dagger}, \\ H_{KA}^e &= 0, \end{aligned} \tag{A.12}$$

$$T_K^\nu = 6\mathrm{Tr}(Y_L^u S Y_L^{u\dagger}), \tag{A.13}$$

$$G_K^\nu = \frac{9}{2} g_1^2 + 9g_{2L}^2, \tag{A.14}$$

$$H_{KL}^\nu = 0, \tag{A.15}$$

where $A = L, R$.

[Range I]

$$T_L^u = 3\mathrm{Tr}(Y_L^u S Y_L^{u\dagger}), \tag{A.16}$$

$$G_L^u = \frac{13}{15} g_1^2 + 3g_{2A}^2 + \frac{16}{3} g_3^2, \tag{A.17}$$

$$H_L^u = 3Y_u S Y_u^\dagger, \tag{A.18}$$

$$\begin{aligned} T_K^u &= 3\mathrm{Tr}(Y_L^u S Y_L^{u\dagger}), \\ T_K^d &= T_K^e = 0, \end{aligned} \tag{A.19}$$

$$G_K^u = G_K^d = G_K^e = \frac{9}{10} g_1^2 + \frac{9}{2} g_{2L}^2, \tag{A.20}$$

$$\begin{aligned} H_{KL}^u &= H_{KL}^d = \frac{2}{3} Y_L^u S Y_L^{u\dagger}, \\ H_{KR}^u &= H_{KR}^d = 0, \\ H_{KL}^e &= H_{KR}^e = 0, \end{aligned} \tag{A.21}$$

$$T_K^\nu = 6\mathrm{Tr}(Y_L^u S Y_L^{u\dagger}), \tag{A.22}$$

$$G_K^\nu = \frac{9}{10} g_1^2 + 9g_{2L}^2, \tag{A.23}$$

$$H_{KL}^\nu = 0. \tag{A.24}$$

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[1] S. R. Juárex W., S.F. Herrera H., P. Kielanowski and G. Mora H., talk presented at the *IX Mexican School on Particles and Fields*, August 9-19, 2000, Pue., Mexico. hep-ph/0009148 (2000); C. R. Das and M. K. Parida, NEHU/PHYS-MP-03/2000, hep-ph/0010004 (2000).

[2] P. H. Chankowski and Z. Pluciennik, Phys. Lett. **B316**, 312 (1993); K. S. Babu, C. N. Leung and J. Pantaleone, Phys. Lett. **B319**, 191 (1993). For a very recent work, for example, see, Amol S. Dighe and A. S. Joshipura, CERN-TH/2000-301, hep-ph/0010079 (2000).

[3] Z. G. Berezhiani, Phys. Lett. **129B**, 99 (1983); Phys. Lett. **150B**, 177 (1985); D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **58**,1600 (1987); A. Davidson and K. C. Wali, Phys. Rev. Lett. **59**, 393 (1987); S. Rajpoot, Mod. Phys. Lett. **A2**, 307 (1987); Phys. Lett. **191B**, 122 (1987); Phys. Rev. **D36**, 1479 (1987); K. B. Babu and R. N. Mohapatra, Phys. Rev. Lett. **62**, 1079 (1989); Phys. Rev. **D41**, 1286 (1990); S. Ranfone, Phys. Rev. **D42**, 3819 (1990); A. Davidson, S. Ranfone and K. C. Wali, Phys. Rev. **D41**, 208 (1990); I. Sogami and T. Shinohara, Prog. Theor. Phys. **66**, 1031 (1991); Phys. Rev. **D47**, 2905 (1993); Z. G. Berezhiani and R. Rattazzi, Phys. Lett. **B279**, 124 (1992); P. Cho, Phys. Rev. **D48**, 5331 (1994); A. Davidson, L. Michel, M. L, Sage and K. C. Wali, Phys. Rev. **D49**, 1378 (1994); W. A. Ponce, A. Zepeda and R. G. Lozano, Phys. Rev. **D49**, 4954 (1994).

[4] Y. Koide and H. Fusaoka, Z. Phys. **C71**, 459 (1996); Prog. Theor. Phys. **97**, 459 (1997).

[5] Y. Koide and H. Fusaoka, Prog. Theor. Phys. **97**, 459 (1997).

[6] T. Morozumi, T. Satou, M. N. Rebelo and M. Tanimoto, Phys. Lett. **B410**, 233 (1997).

[7] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1996); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[8] Y. Koide, Mod. Phys. Lett. **22**, 2071 (1993).

[9] Y. Koide, Phys. Rev. **D58**, 053008 (1998).

[10] H. Fusaoka and Y. Koide, Phys. Rev. **D57**, 3986 (1998).

[11] Particle data group, Eur. Phys. Jour. **C15**, 1 (2000).

[12] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[13] A. Zee, Phys. Lett. **93B**, 389 (1980).

[14] For example, see, A. Yu. Smirnov and M. Tanimoto, Phys. Rev. **D55**, 1665 (1997); C. Jarlskog, M. Matsuda, S. Skadhauge, M. Tanimoto, Phys. Lett. **B449**, 240 (1999).

[15] Y. Koide, Mod. Phys. Lett. **11**, 2849 (1996); Phys. Rev. **D57**, 5836 (1998).

[16] Y. Fukuda *et al.*, Phys. Lett. **B335**, 237 (1994); Super-Kamiokande collaboration, Y. Fukuda, *et. al.*, Phys. Rev. Lett. **81**, 1562 (1998); H. Sobel, Talk presented at *Neutrino 2000*, Sudbury, Canada, June 2000 (http://nu2000.sno.laurentian.ca/).

[17] N. Haba, Y. Matsui, N. Okamura and T. Suzuki, Phys. Lett. **B489**, 184 (2000).

[18] Y. Suzuki, Talk presented at *Neutrino 2000*, Sudbury, Canada, June 2000 (http://nu2000.sno.laurentian.ca/). Also see, M. Gonzalez-Garcia, Talk presented at *Neutrino 2000*, Sudbury, Canada, June 2000 (http://nu2000.sno.laurentian.ca/).

[19] Y. Koide and H. Fusaoka, Phys. Rev. **D59**, 053004 (1999); Y. Koide and A. Ghosal, Phys. Lett. **B488**, 344 (2000).

[20] Y. Koide, Phys. Rev. **D61**, 035008 (2000).

TABLE I. Quantum numbers of the fermions f and F and Higgs scalars ϕ_L, ϕ_R and Φ for $SU(2)_L \times SU(2)_R \times U(1)_{LR} \times U(1)_X$.

	I_3^L	I_3^R	$\frac{1}{2}Y_{LR}$	X		I_3^L	I_3^R	$\frac{1}{2}Y_{LR}$	X
u_L	$+\frac{1}{2}$	0	$+\frac{1}{6}$	0	u_R	0	$+\frac{1}{2}$	$+\frac{1}{6}$	0
d_L	$-\frac{1}{2}$	0	$+\frac{1}{6}$	0	d_R	0	$-\frac{1}{2}$	$+\frac{1}{6}$	0
ν_L	$+\frac{1}{2}$	0	$-\frac{1}{2}$	0	ν_R	0	$+\frac{1}{2}$	$-\frac{1}{2}$	0
e_L	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	e_R	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
U_L	0	0	$+\frac{2}{3}$	$+\frac{1}{2}$	U_R	0	0	$+\frac{2}{3}$	$-\frac{1}{2}$
D_L	0	0	$-\frac{1}{3}$	$-\frac{1}{2}$	D_R	0	0	$-\frac{1}{3}$	$+\frac{1}{2}$
N_L	0	0	0	$+\frac{1}{2}$	N_R	0	0	0	$-\frac{1}{2}$
E_L	0	0	-1	$-\frac{1}{2}$	E_R	0	0	-1	$+\frac{1}{2}$
ϕ_L^+	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$-\frac{1}{2}$	ϕ_R^+	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$
ϕ_L^0	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$-\frac{1}{2}$	ϕ_R^0	0	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$
Φ^0	0	0	0	$+1$					

TABLE II. The squared mass difference $\Delta m_{ij}^2 = m_{\nu i}^2 - m_{\nu j}^2$. The values of the input parameters are the same as in Figs. 9 and 10. The absolute values of $|\Delta m_{ij}^2|$ should not be taken rigidly, because we can adjust those by the value of Λ_S.

	non-SUSY model			SUSY model		
	at $\mu = \Lambda_L$	at $\mu = \Lambda_S$	at $\mu = \Lambda_X$	at $\mu = \Lambda_L$	at $\mu = \Lambda_S$	at $\mu = \Lambda_X$
$\lvert\Delta m_{32}^2\rvert$ [eV^2]	2.39×10^{-3}	9.32×10^{-3}	3.49×10^{-4}	2.72×10^{-3}	2.51×10^{-3}	4.08×10^{-4}
$\lvert\Delta m_{21}^2\rvert$ [eV^2]	1.83×10^{-2}	7.15×10^{-1}	7.67×10^{-2}	1.35×10^{-2}	1.25×10^{-2}	1.01×10^{-2}
$\lvert\Delta m_{32}^2/\Delta m_{21}^2\rvert$	1.30×10^{-1}	1.30×10^{-1}	4.56×10^{-3}	2.02×10^{-1}	2.02×10^{-1}	4.04×10^{-2}
$\lvert V_{23}\rvert^2$	0.485	0.485	0.500	0.478	0.478	0.500
$\lvert V_{12}\rvert^2$	0.00484	0.00484	0.00471	0.00492	0.00492	0.00466

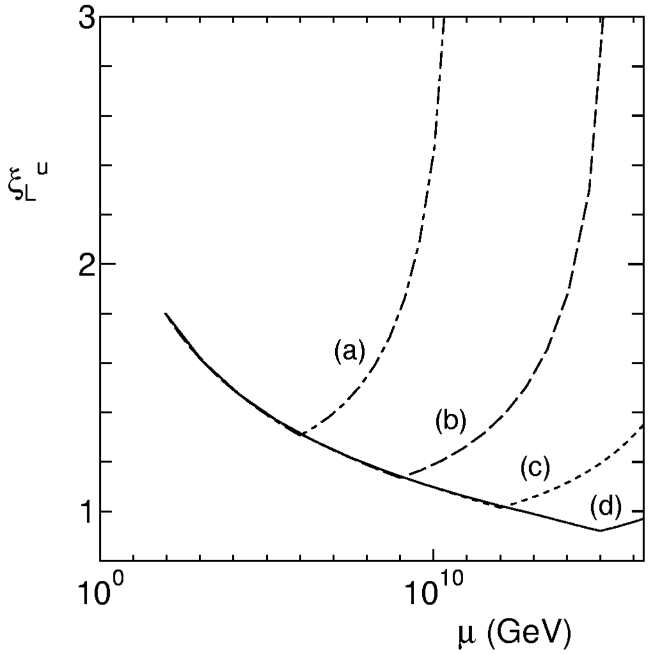

FIG. 1. Behaviors of $\xi^u(\mu)$ in a non-SUSY model for the cases (a) $\Lambda_S = 10^6$ GeV, (b) $\Lambda_S = 10^9$ GeV, (c) $\Lambda_S = 10^{12}$ GeV, and (d) $\Lambda_S = 10^{15}$ GeV. The input values are $m_t(m_Z) = 181$ GeV and $\Lambda_S/\Lambda_R = 107$.

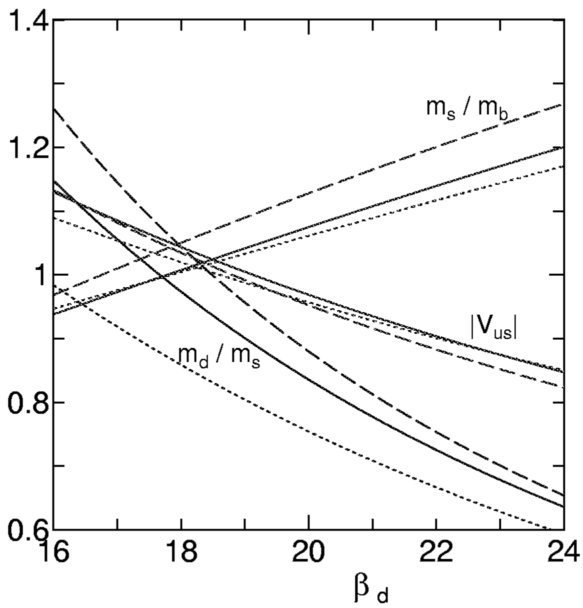

FIG. 2. Predictions of m_d/m_s, m_s/m_b and $|V_{us}|$, and their dependency on the parameters b_d and β_d. Here, the mass ratios are denoted in the unit of the corresponding observed values which are quoted from Ref. [10]. The dashed, solid and dotted lines denote $b_d = -1.1$, -1.2 and -1.3, respectively.

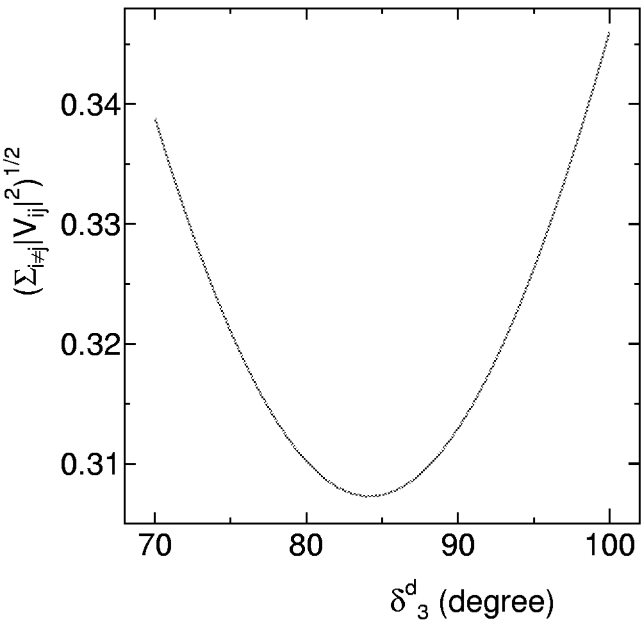

FIG. 3. $\sum_{i\neq j} |V_{ij}(\Lambda_X)|^2$ versus $\delta_3^d(\Lambda_X)$.

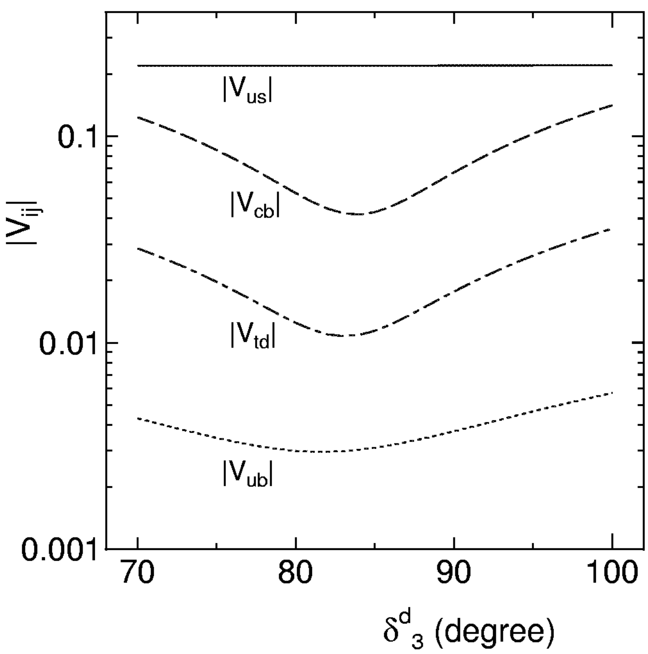

FIG. 4. Predicted values of the CKM matrix parameters $|V_{ij}(m_Z)|$ versus the parameter $\delta_3^d(\Lambda_X)$. Other input values of the parameters are $\Lambda_S = 3 \times 10^{13}$ GeV, $\Lambda_S/\Lambda_R = 107$, $b_d = -1.2$ and $\beta_d = 19.2°$.

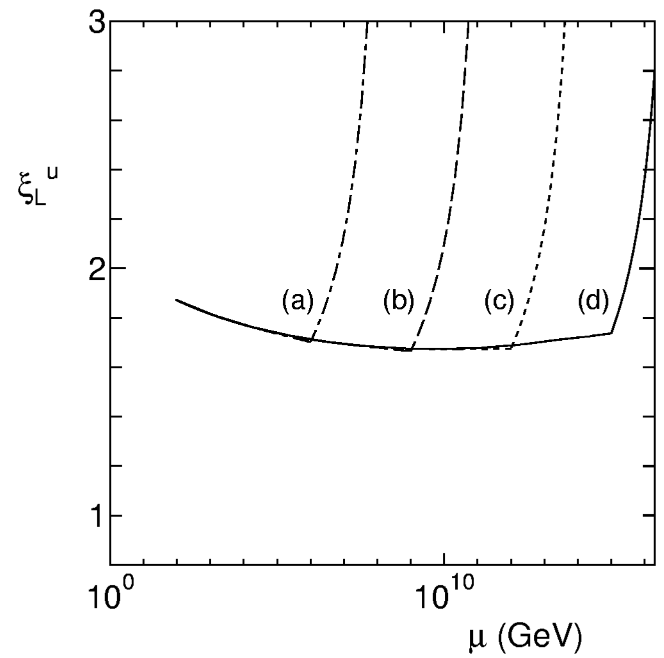

FIG. 5. Behaviors of $\xi^u(\mu)$ in a SUSY model for the cases (a) $\Lambda_S = 10^6$ GeV, (b) $\Lambda_S = 10^9$ GeV, (c) $\Lambda_S = 10^{12}$ GeV, and (d) $\Lambda_S = 10^{15}$ GeV. The input values are $m_t(m_Z) = 181$ GeV, $\Lambda_S/\Lambda_R = 38$ and $\tan\beta = 3.5$.

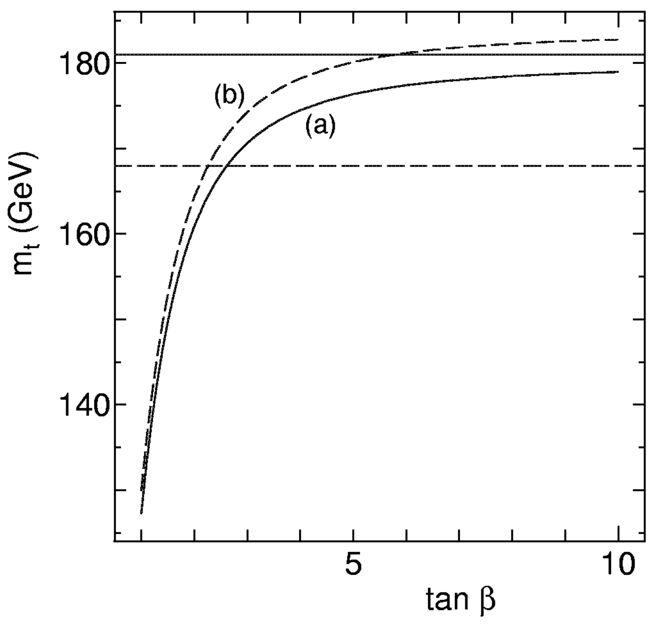

FIG. 6. The top-quark mass $m_t(m_Z)$ versus $\tan\beta$ in a SUSY model. The solid and broken lines denote the cases with the initial conditions (a) $\xi_L^u(\Lambda_X) = 2.0$ and (b) $\xi_L^u(\Lambda_X) = \sqrt{4\pi} = 3.54$, respectively. The other input values are $\Lambda_S = 6\times 10^{13}$ GeV and $\Lambda_S/\Lambda_R = 38$. The horizontal solid and broken lines denote the center and lower values of the observed top quark mass at $\mu = m_Z$, respectively.

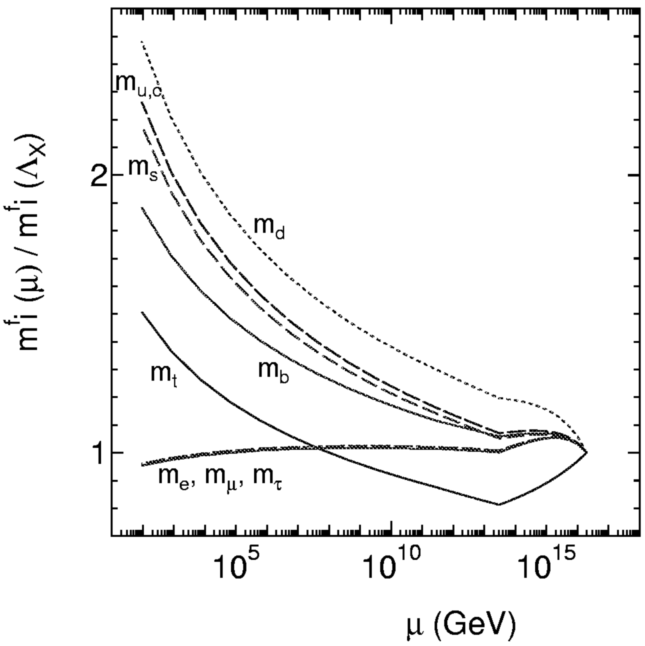

FIG. 7. Behaviors of $m_i^f(\mu)/m_i^f(\Lambda_X)$ ($f = u, d, \nu, e$; $i = 1, 2, 3$) in the non-SUSY model. The dotted, broken and solid lines denote the first, second and third fermion masses, respectively. The input parameter values are $\Lambda_S = 3\times 10^{13}$ GeV, $\Lambda_S/\Lambda_R = 107$ and $b_d(\Lambda_X) = -1.2e^{i19.2°}$.

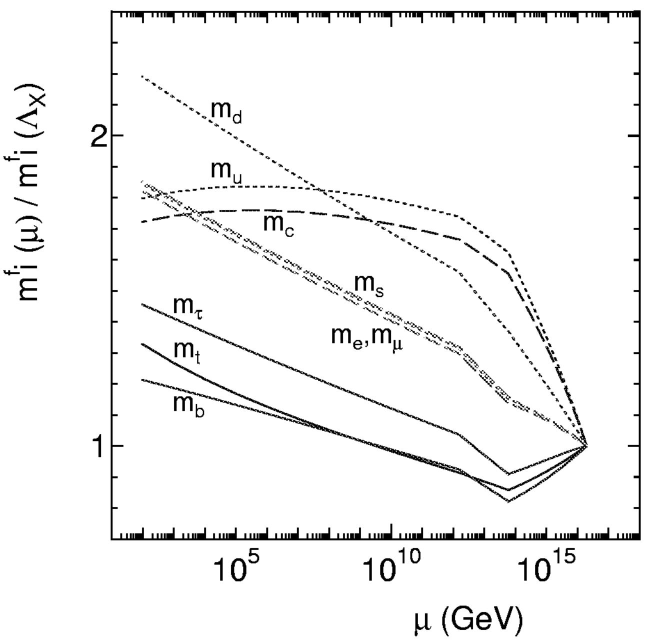

FIG. 8. Behaviors of $m_i^f(\mu)/m_i^f(\Lambda_X)$ ($f = u, d, \nu, e$; $i = 1, 2, 3$) in the SUSY model. The dotted, broken and solid lines denote the first, second and third fermion masses, respectively. The input parameter values are $\Lambda_S = 6\times 10^{13}$ GeV, $\Lambda_S/\Lambda_R = 38$, $\tan\beta = 3.5$ and $b_d(\Lambda_X) = -1.2e^{i19.4°}$.

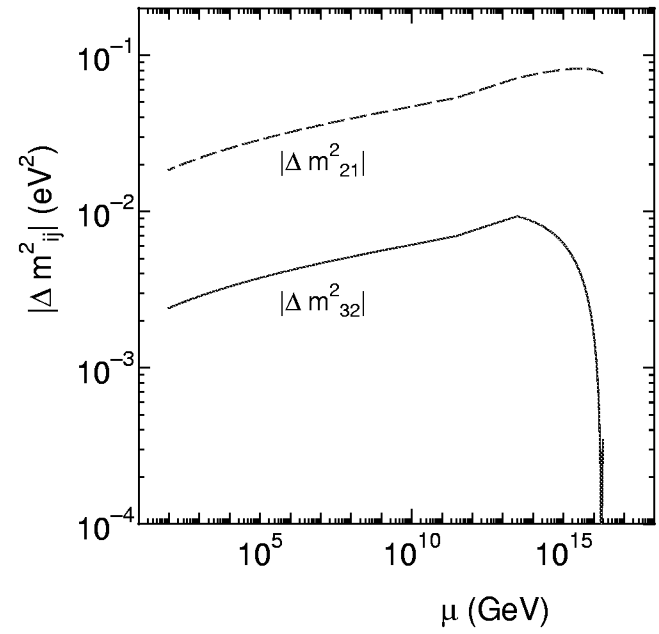

FIG. 9. Behavior of $|\Delta m^2_{ij}(\mu)|$ in the non-SUSY model. The input parameter values are the same as in Fig. 7 with $\xi^\nu_A = \xi^e_A$ $(A = L, R, S)$.

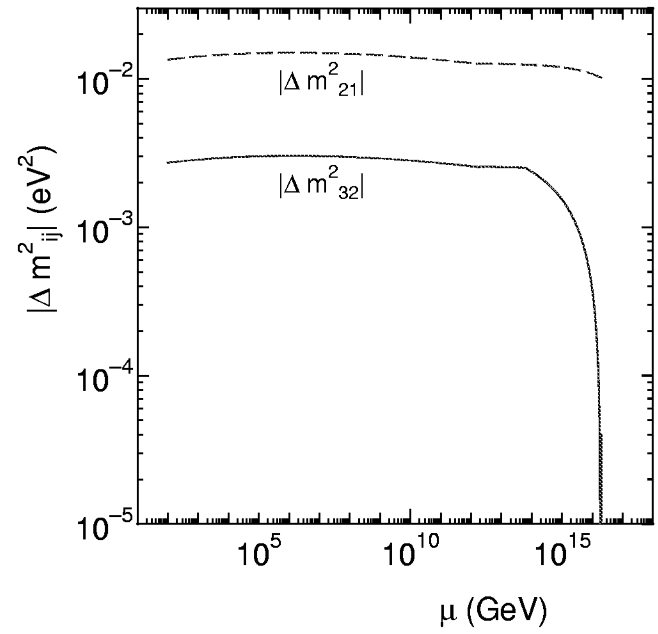

FIG. 10. Behavior of $|\Delta m^2_{ij}(\mu)|$ in the SUSY model. The input parameter values are the same as in Fig. 8 with $\xi^\nu_A = \xi^e_A$ $(A = L, R, S)$.