# Family Gauge Boson Model with Inverted Mass Hierarchy and Sumino's Cancellation Mechanism

**Yoshio Koide**

*Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*

*E-mail address: koide@kuno-g.phys.sci.osaka-u.ac.jp*

**Abstract**

An interesting family gauge boson (FGB) model has been proposed by Sumino. The model can give FGBs with a considerably low energy scale under a cancellation mechanism between radiative QED and FGB diagrams But his model is not anomaly free and causes effective interactions with $\Delta N_{\rm family} = 2$. In order to avoid those problems, a new FGB model with an inverted mass hierarchy has proposed, but it cannot satisfy the Sumino cancellation mechanism exactly. In this paper, we propose a revised FGB model, where the model still gives an inverted mass hierarchy, but it can exactly satisfy the Sumino mechanism. An effect of the revised model will be confirmed by $\mu^- N \to e^- N$ experiments.

PCAC numbers: 11.30.Hv, 12.60.-i,

## 1 Introduction:

The most challenging subjects in particle physics is to understand the origin of flavors. There is an attractive idea that the flavor physics is understood from the point of view of a family symmetry. Then, it is natural to consider that the symmetry is gauged, because if we consider a global symmetry, we will have unwelcome massless Nambu-Gloldstone scalars. So, we may expect existence of family gauge bosons (FGBs). However, in the conventional FGB models, it is considered that a scale of the family symmetry breaking is extremely high, because of constraints from the observed $P^0$-$\bar{P}^0$ mixings ($P = K, D, B, B_s$). Therefore, the observation of FGBs was not a realistic topic.

Against such a conventional view, in 2009, Sumino [2] gave a realistic role to FGBs. Thereby, Sumino has speculated that a scale of the FGBs is of an order of $10^3$ TeV.

Prior to a review of the Sumino FGB model, let us give a brief review of a charged lepton mass relation [3],

$$K \equiv \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\tau} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}. \tag{1.1}$$

The relation is satisfied by the pole masses [i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$], but not so well satisfied by the running masses [i.e. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$]. The formula (1.1) was first derived by the author in 1982 prior to observations of the precise tau lepton mass value in 1992 [4]. He has derived the formula (1.1) by assuming a U(3) family symmetry [3, 5]. Therefore, it is never likely that the formula (1.1) is satisfied by the pole masses, because the formula (1.1) should be satisfied only by running masses without QED correction. In other words, the relation (1.1) can never be satisfied by pole masses. This was an utter mystery.

The QED correction is given as [6]:

$$m_{e_i}(\mu) = m_{e_i}^{pole}\left[1 - \frac{\alpha_{em}(\mu)}{\pi}\left(1 + \frac{3}{4}\log\frac{\mu^2}{m_{e_i}^2(\mu)}\right)\right]. \tag{1.2}$$

However, note that ff the family-number dependent factor $\log m_{e_i}^2$ in Eq. (1.2) is absent, then the running masses $m_{e_i}(\mu)$ will also satisfy the formula (1.1). Noticing this fact, Sumino has proposed a U(3) family gauge model [2], where a factor $\log m_{e_i}^2$ in the QED correction for the charged lepton mass $m_{e_i}$ $(i = 1, 2, 3)$ is canceled by a factor $\log M_{ii}^2$ in a corresponding diagram due to the FGBs. Here, he has introduced a scalar $\Phi$ which is $(\mathbf{3}, \mathbf{3})$ of U(3)×O(3), and the vacuum expectation value (VEV) matrix is given by

$$\langle \Phi_i{}^\alpha \rangle = [\mathrm{diag}(v_{e1}, v_{e2}, v_{e3})]_i{}^\alpha, \tag{1.3}$$

where $i$ and $\alpha$ are indexes of U(3) and O(3), respectively. (Hereafter, since we consider $\Phi$ of $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)$'$ in an extended Sumino model later, we have denoted the components of $\Phi$ as $\Phi_i{}^\alpha$, not $\Phi_{i\alpha}$.) Then, the charged lepton mass matrix $M_e$ in his model is given by

$$(M_e)_i{}^j \propto \langle \Phi_i{}^\alpha \rangle \langle \bar{\Phi}_\alpha{}^j \rangle \propto \delta_i^j v_{ei}^2, \quad i.e. \ \ m_{ei} = k_e v_{ei}^2. \tag{1.4}$$

On the other hand, the scalar $\Phi$ also generates masses of FGBs $A_i{}^j$ as

$$M_{ij}^2 \propto \langle \Phi_i{}^\alpha \rangle \langle \bar{\Phi}_\alpha{}^i \rangle + \langle \Phi_j{}^\alpha \rangle \langle \bar{\Phi}_\alpha{}^j \rangle \propto v_i^2 + v_j^2 \propto m_{ei} + m_{ej}. \tag{1.5}$$

The basic idea by Sumino for the reason why Eq.(1.1) is well satisfied by "pole" masses is as follows: The factor $\log m_{ei}$ in (1.2) is canceled by a factor $\log M_{ii}$ in the radiative FGB diagram by taking a suitable relation between the electromagnetic gauge coupling constant $e$ and the family gauge coupling constant $g_F$ and by noticing $\log M_{ii}^2 = \log(k_e m_{ei}) = \log m_{ei} + \log k_e$. (Note that the family-independent term $\log k_e$ is not essential in the Sumino cancellation mechanism.) Note that this cancellation works only for the term $\log m_{ei}$ in Eq.(1.2), so that the running mass $m_{ei}(\mu)$ still has the energy scale dependence.

Note that not only the Sumino model gives a possible explanation for well-satisfied relation (1.1). but also the model gives a FGB model with a considerably low energy scale. (Sumino has speculated that the scale is of the order of $10^3$ TeV [2].) In the Sumino model, the FGB mass matrix is diagonal in the diagonal basis of the charged lepton mass matrix, so that family-number violation does not occur in the lepton sector. Family-number violation occurs only the quark mixing. In the limit of no quark mixing, the family-number violation is also forbidden.

Therefore, contribution of FGBs to the $P^0$-$\bar{P}^0$ mixing can be considerably reduced compared with the conventional FGB modes.

However, in the Sumino model (we call it Model A), in order to get a minus sign for the purpose of the cancellation, the charged leptons ($e_L$ and $e_R$) are assigned to **3** and $\mathbf{3}^*$ of U(3) family symmetry, respectively. Therefore, the Sumino model is not anomaly free. In order to avoid this anomaly problem, we may add some additional heavy leptons. As a result, the model becomes somewhat complicated. Besides, the model inevitably causes effective interaction with $\Delta N_F = 2$ ($N_F$ is a family number).

In order to evade this problem, Yamashita and the author [7] have proposed an extended model (we call it Model B) with anomaly free: $e_L$ and $e_R$ are assigned to **3** and **3** of U(3) family symmetry, respectively and, instead, they consider that the FGB masses have inverted masses

$$M_{ij}^2 = K_e (m_{ei}^{-1} + m_{ej}^{-1}), \tag{1.6}$$

in order to obtain a minus sign for the Sumino cancellation mechanism, i.e. $\log M_{ii}^2 = -\log m_{ei} + \log K_e$. Here, they have introduced additional scalar $\Psi$ which belongs to $(\mathbf{3}, \mathbf{3}^*)$ of U(3)$\times$U(3)$'$ and which has a VEV

$$\langle \Psi_i{}^\alpha \rangle = [\mathrm{diag}(v_{F1}, v_{F2}, v_{F3})]_i{}^\alpha. \tag{1.7}$$

They have proposed a superpotential which leads to a VEV relation

$$\langle \Psi_i{}^\alpha \rangle \langle \bar{\Psi}_\alpha{}^k \rangle \langle \Phi_k{}^\beta \rangle \langle \bar{\Phi}_\beta{}^j \rangle = k_\Psi k_\Phi \delta_i^k, \qquad \text{i.e.} \quad (v_{Fi})^2 = k_\Psi k_\Phi (v_{ei})^{-2}. \tag{1.8}$$

(For the explicit model for (1.8), see Ref.[7].) Since they assume that $\langle \Psi \rangle \gg \langle \Phi \rangle$, they can neglect contributions to FGB masses from $\langle \Phi \rangle$ and they obtain the inverse mass hierarchy of $M_{ij}$ (1.6). Thus, they have obtained the inverse hierarchy of FGB masses (1.6).

However, we have to note that only the loop diagram $e_i \to e_i + A_i{}^i \to e_i$ can contribute to $m_{ei}(\mu)$ in Model A, while, in Model B, another loop diagrams $e_i \to e_j + A_i{}^j \to e_i$ with $j \neq i$ too can contribute to $m_{ei}(\mu)$ in addition to $e_i \to e_i + A_i{}^i \to e_i$. This means that the original cancellation scenario $\delta_i \equiv \log m_{ei}^2 - \xi \log M_{ii}^2 = const$ is exchanged by

$$\delta_i \equiv \log m_{ei}^2 + \xi \sum_{j=1,2,3} \log M_{ij}^2 = \log m_{ei}^2 + \xi \log S_i, \tag{1.9}$$

where

$$S_i \equiv M_{i1}^2 M_{i2}^2 M_{i3}^2 = K_e^3 \left( \frac{1}{m_{ei}} + \frac{1}{m_{e1}} \right) \left( \frac{1}{m_{ei}} + \frac{1}{m_{e2}} \right) \left( \frac{1}{m_{ei}} + \frac{1}{m_{e3}} \right), \tag{1.10}$$

and $\xi$ is a family-number independent factor. From (1.10), the ratios $S_1/S_2$ and $S_2/S_3$ are given by $S_1/S_2 \simeq (m_\mu/m_e)^2$ and $S_2/S_3 \simeq m_\tau/m_\mu$, so that the Sumino's cancellation mechanism holds only approximately in Model B. Of course, the cancellation mechanism can be practically satisfied by adjusting a fine tuning parameter [7]. However, the cancellation mechanism is not so beautiful compared with the original one (Model A).

The purpose of the present paper is to propose a revised version of the FGB model with inverted hierarchical masses (Model B) and to discuss possible new effects of the family gauge bosons.

## 2 Basic idea:

In a new model (Model C), we change the relation (1.8) in Model B into

$$\langle \Psi_i^{\ \alpha} \rangle \langle \bar{\Psi}_\alpha^{\ k} \rangle \left( \langle \Phi_k^{\ \beta} \rangle \langle \bar{\Phi}_\beta^{\ j} \rangle + \langle (\Phi_0)_k^{\ \beta} \rangle \langle (\bar{\Phi}_0)_\beta^{\ j} \rangle \right) = k_\Psi k_\Phi \delta_i^k . \tag{2.1}$$

Here. we have introduced a new flavon $(\Phi_0)_i^{\ \alpha}$. We can replace $k_\Psi k_\Phi$ to $c_0 (v_{F0})^2 (v_0)^2$ without losing generality, where $(v_{F0})^2$ and $(v_{e0})^2$ are defined by

$$(v_{F0})^2 \equiv (v_{F1})^2 + (v_{F2})^2 + (v_{F3})^2, \qquad (v_{e0})^2 \equiv (v_{e1})^2 + (v_{e2})^2 + (v_{e3})^2, \tag{2.2}$$

and $c_0$ is a free parameter. Note that, in general, a relative ratio of the VEV term $\Phi_0 \bar{\Phi}_0$ to $\Phi \bar{\Phi}$ is free, so that we may, in general, denote $\Phi_0 \bar{\Phi}_0 + \Phi \bar{\Phi}$ as $v_{ei}^2 + c_0 (v_{e0})^2$. However, when we denote the relation (2.1) as

$$\frac{(v_{Fi})^2}{(v_{F0})^2} = \frac{c_0 v_{e0}^2}{(v_{ei})^2 + c_0 (v_{e0})^2}, \tag{2.3}$$

the parameter $c_0$ is not free any longer as we see Eq.(3.1) later. The VEV relation (2.3) is ad hoc assumption in this paper.

According to (1.3) and (2.2), we obtain $(v_{e0})^2 = [(v_{e1})^2 + (v_{e2})^2 + (v_{e3})^2]/k_e = (m_{e1} + m_{e2} + m_{e3})/k_e \equiv m_0/k_e$, so that the relation (2.3) can be expressed as

$$\frac{(v_{F0})^2}{(v_{Fi})^2} = \frac{m_{ei} + c_0 m_{e0}}{c_0 m_{e0}}. \tag{2.4}$$

Since we can express the right hand $(m_{ei} + c_0 m_{e0})/c_0 m_{e0}$ n Eq.(2.4) as $1 + m_{ei}/c_0 m_{e0}$ and the left hand $(v_{F0})^2/(v_{Fi})^2$ as

$$\frac{(v_{F0})^2}{(v_{Fi})^2} = \frac{(v_{Fi})^2 + (v_{Fj})^2 + (v_{Fk})^2}{(v_{Fi})^2} = 1 + \frac{(v_{Fj})^2 + (v_{Fk})^2}{(v_{Fi})^2}, \tag{2.5}$$

where $(i, j, k)$ denotes cyclic permutation of $(1, 2, 3)$, we obtain a relation

$$\frac{m_{ei}}{c_0 m_{e0}} = \frac{(v_{Fj})^2 + (v_{Fk})^2}{(v_{Fi})^2}. \tag{2.6}$$

On the other hand, the factor $S_i$ is given by

$$S_i = M_{i1}^2 M_{i2}^2 M_{i3}^2 = 2(v_{Fi})^2[(v_{Fi})^2 + (v_{Fj})^2][(v_{Fi})^2 + (v_{Fk})^2] = A\frac{(v_{Fi})^2}{(v_{Fj})^2 + (v_{Fk})^2}, \tag{2.7}$$

where $A \equiv 2[(v_{F1})^2 + (v_{F2})^2][(v_{F2})^2 + (v_{F3})^2][(v_3^F)^2 + (v_1^F)^2]$ is a family-number independent constant. Thus, from Eq(2.6), we can express $S_i$ as

$$S_i = A\, c_0 \frac{m_{e0}}{m_{ei}}. \tag{2.8}$$

Therefore, we get $\log S_i = -\log m_{ei} + \log(A\, c_0 m_{e0})$, and we can achieve a complete Sumino cancellation mechanism.

## 3 Possible effects in the new scenario

Note that the value $c_0$ in Eq.(2.4) is not free under the given values $(m_{e1}, m_{e2}, m_{e3})$. By using the relation (2.5), we obtain a constraint

$$1 = \frac{1}{(v_{F0})^2}[(v_{F1})^2 + (v_{F2})^2) + (v_{F3})^2] = \frac{c_0 m_{e0}}{m_{e1} + c_0 m_{e0}} + \frac{c_0 m_{e0}}{m_{e2} + c_0 m_{e0}} + \frac{c_0 m_{e0}}{m_{e3} + c_0 m_{e0}}. \tag{3.1}$$

Although Eq.(3.1) has, in general, three solutions, a positive solution is only one: $c_0 = 0.003774$. Since the solution gives $c_0 m_0 = 7.6219$ MeV, we can regard as $m_{e1} \ll c_0 m_{e0} \ll m_{e2} \ll m_{e3}$, so that we can approximate Eq.(3.1) as

$$\frac{1}{1 + (m_{e1}/c_0 m_{e0})} + \frac{c_0 m_{e0}}{m_{e2}} + \frac{c_0 m_{e0}}{m_{e3}} \simeq 1, \tag{3.2}$$

i.e.

$$\frac{c_0 m_{e0}}{m_{e2}} + \frac{c_0 m_{e0}}{m_{e3}} \simeq \frac{m_{e1}}{c_0 m_{e0}} \quad \Rightarrow \quad (c_0 m_{e0})^2 \simeq m_{e1} m_{e2}. \tag{3.3}$$

Therefore, from (2.4) and (3.3), we obtain approximate relations

$$(v_{F1})^2 \simeq (v_{F0})^2, \quad (v_{F2})^2 \simeq \left(\frac{\sqrt{m_{e1} m_{e2}}}{m_{e2}}\right)(v_{F0})^2, \quad (v_{F3})^2 \simeq \left(\frac{\sqrt{m_{e1} m_{e2}}}{m_{e3}}\right)^2 (v_{F0})^2. \tag{3.4}$$

Effect due to $c_0 \neq 0$ only appears in values of FGB masses $M_{ij}$: As well as in Model B, the relative mass ratios of the FGBs are given by

$$M_{33} : M_{32} : M_{22} : M_{31} : M_{21} : M_{11} = 1; \sqrt{\frac{a^2+1}{2}} : a : \sqrt{\frac{b^2+1}{2}} : \sqrt{\frac{b^2+a^2}{2}} : b, \tag{3.5}$$

where $a \equiv M_{22}/M_{33}$ and $b \equiv M_{11}/M_3$, but the parameters $a$ and $b$ in Model C are given by

$$a \equiv \frac{M_{22}}{M_{33.}} = \frac{v_{F2}}{v_{F3}} = \left(\frac{m_{e3}+c_0 m_{e0}}{m_{e2}+c_0 m_{e0}}\right)^{1/2}, \quad b \equiv \frac{M_{11}}{M_{33}} = \frac{v_{F1}}{v_{F3}} = \left(\frac{m_{e3}+c_0 m_{e0}}{m_{e1}+c_0 m_{e0}}\right)^{1/2}. \tag{3.6}$$

(The values in Model B are obtained by putting $c_0 = 0$ in Eq.(3.6). ) Let us show numerical values $a$ and $b$ in the present model (Model C) without approximation (3.3):

$$\begin{array}{l} a^C = 3.97347, \quad b^C = 15.0691, \\ (a^B = 4.10081, \quad b^B = 58.9674), \end{array} \tag{3.7}$$

Here, for the sake of comparison, we have also shown values $a$ and $b$ in Model B as $a^B$ and $b^B$, respectively. The value $a^C$ is almost same as the value $a^B$ while the value $b^C$ is considerably smaller than the value $b^B$, $b^C \sim \frac{1}{4} b^B$.

Lower bounds of $M_{ij}$ are constrained by the observed $P^0$-$\bar{P}^0$ mixing ($P = K, D, B, B_s$). The numerical results in Model B have been given in Ref.[1]. (However, take notice that Model B in the present paper corresponds to "Model A$_1$" in Ref.[1].) As seen in Eq.(3.5), the values $M_{33}$, $M_{32}$ and $M_{22}$ are independent on the parameter $b$, so that those values will almost be unchanged under the change of the value $c_0$. On the other hand, in a model with an inverse FGB masses, the constraint from the $P^0$-$\bar{P}^0$ mixing is almost determined by the value $M_{22}$ (and $M_{33}$ for $B^0_s$-$\bar{B}^0_s$ mixing) [1], so that the values of $M_{22}$, $M_{23}$ and $M_{33}$ are almost unchanged in Model C. (Exactly speaking, the constraint is fixed by a form of effective mass $\tilde{M}_{ij} \equiv M_{ij}/(g_F/\sqrt{2})$.) Therefore, we approximately put

$$\tilde{M}^C_{22} \equiv \frac{M^C_{22}}{g^C_F/\sqrt{2}} \simeq \tilde{M}^B_{22} \equiv \frac{M^B_{22}}{g^B_F/\sqrt{2}}. \tag{3.8}$$

In Model B [7], a cancellation condition $(g_F/\sqrt{2})^2 = (3/2)\zeta e^2$ has taken, where $\zeta$ is a fine tuning factor because the Sumino's cancellation is not complete, and the value $\zeta = 1.752$ was taken. In Model C, the Sumino's cancellation holds exactly, so that we have to put $\zeta = 1$. Therefore, the value $g^C_F/g^B_F$ is given by

$$\frac{g^C_F}{g^B_F} = \frac{1}{\sqrt{\zeta}} = \frac{1}{\sqrt{1.752}}. \tag{3.9}$$

Then. we can estimate values of $M^C_{ij}$ in Model C from the numerical results $M^C_{ij}$ in Model C (in Ref.[1]). The results are given Table 1.

For the sake of comparison, correspondingly to Ref.[1], we have added the case of $n = 2$ in addition to the case $n = 1$, where $n$ is defined by

$$M_{ij}^2 = K_e(m_{ei}^{-n} + m_{ej}^{-n}). \tag{3.10}$$

(The extension to $n = 2$ is also possible, although it needs somewhat complicated framework among flavons as stated in Ref. [1].) For Case $n = 2$, we obtain parameter values $a^C = 16.7763$ and $b^C = 241.447$, ($a^B = 16.8167$ and $b^B = 3477.15$), correspondingly to Eq.(3.7). Also, we use $(g_F^{n=1}/\sqrt{2})^B = 0.4339$ and $(g_F^{n=2}/\sqrt{2})^B = 0.3068$.

Table 1 Lower bound of $M_{ij}$ [TeV] from the observed $P^0$-$\bar{P}^0$ mixing

| Model | $n$ | $M_{11}$ | $M_{12}$ | $M_{13}$ | $M_{22}$ | $M_{23}$ | $M_{33}$ |
|---|---|---|---|---|---|---|---|
| Model C | $n = 1$ | $4.38\times10^2$ | $3.20\times10^2$ | $3.10\times10^2$ | 115 | 84.1 | 29.0 |
| Model B | $n = 1$ | $2.20\times10^3$ | $1.56\times10^3$ | $1.55\times10^3$ | 153 | 111 | 37.2 |
| Model C | $n = 2$ | $1.18\times10^3$ | $0.839\times10^3$ | $0.837\times10^3$ | 82.1 | 58.2 | 4.90 |
| Model B | $n = 2$ | $2.25\times10^4$ | $1.59\times10^4$ | $1.59\times10^4$ | 109 | 77.1 | 6.47 |

As seen in Table 1, we can obtain somewhat lower mass values compared with the previous values (in Model B) only for $M_{11}$, $M_{12}$ and $M_{13}$. However, the FGBs ${A_1}^1$, ${A_2}^1$ and ${A_3}^1$ have masses of $10^{2-3}$ TeV scale, it is usually not easy to observe the effects due to $c_0 \neq 0$. For example, let us see expect rare decays $K^+ \to \pi^+\mu^+e^-$ via ${A_1}^2$ and $B^+ \to \pi^+\tau^+e^-$ via ${A_1}^3$, which are proportional to $(g_F/M_{ij})^2$. From Table 1 and Eq.(3.9), we obtain

$$(M_{12}^B/M_{12}^B)^4 \simeq 1.84 \times 10^2, \quad (M_{13}^B/M_{13}^B)^4 \simeq 2.04 \times 10^2, \quad , \tag{3.11}$$

for $n = 1$. The values (3.11)is still insufficient to observe such rare decays. However, if we consider such observations for the case $n = 2$, we obtain

$$(M_{12}^B/M_{12}^B)^4 \simeq 4.26 \times 10^4, \quad (M_{13}^B/M_{13}^B)^4 \simeq 4.24 \times 10^4, \quad . \tag{3.12}$$

Since the FGB prediction in Model B was $Br(K^+ \to \pi^+e^-\mu^+) \simeq 2.3 \times 10^{-16}$ [7], we can predict

$$Br(K^+ \to \pi^+e^-\mu^+) \simeq 0.98 \times 10^{-11}. \tag{3.13}$$

Since the experiments have reported $Br(K^+ \to \pi^+e^-\mu^+) < 1.3 \times 10^{-11}$, the observation is promising in the near future. We also expect observation of $B^+ \to \pi^+\tau^+e^-$ in the near future. (Of course the values $M_{ij}$ in Table 1 are lower bound of FGB masses from the observation of $P^0$-$\bar{P}^0$ mixing, so that it is not likely that the actual masses are coincidentally the same as the lower bounds from the $P^0$-$\bar{P}^0$ mixing .) We hope further investigation of rare decays.

Besides, the case $n = 2$ predicts the lightest FGB mass as $M_{33} = 4.9$ TeV. This value is within reach of the 14 TeV LHC experiments, i.e. $p + p \to A_3{}^3 + X \to \tau^+\tau^- + X$.

Only the most certain visible effect will appear the so-called $\mu$-$e$ conversion experiments

$$R(N) \equiv \frac{\sigma(\mu^- N \to e^- N)}{\sigma(\mu \text{ capture})}. \tag{3.14}$$

For example, the COMET experiment [9] aims for a goal $R \sim 10^{-17}$. On the other hand, we roughly estimate $R(\text{Al})$ as

$$R(\text{Al}) \sim 10^{-16} \frac{1}{n^2} \left( \frac{10^3[\text{TeV}]}{\tilde{M}_{12}[\text{TeV}]} \right)^4 , \tag{3.15}$$

in Models B and C. (For $\mu$-$e$ conversion induced by exchange of the FGB $A_2{}^1$, for example, see Ref.[8].) Since the previous value of $M_{12}$ in the case $n = 1$ was $1.56 \times 10^3$ TeV, so that the value gives $R(\text{Al}) \sim 0.9 \times 10^{-16}$, the observation of $\mu$-$e$ conversion was critical in Model B. On the other hand, in Model C, the revised value (3.11) in the case $n = 1$ can give $R(\text{Al}) \sim 1.7 \times 10^{-14}$. Therefore, the observation of $\mu$-$e$ conversion due to FGB is promising in the experiments [9, 10, 11], even if we take into consideration that the mass values in Table 1 are nothing but lower bounds constrained from the observed $P^0$-$\bar{P}^0$ mixing.

## 4 Concluding remarks

In conclusion, we have proposed a FGB model with an inverted mass hierarchy. The model is anomaly free as well as Model B, while the Sumino cancellation mechanism exactly holds as well as the original Sumino model (Model A) differently from Model B. Therefore, we have obtained a FGB model, where all merits in the original Sumino model are retained, but all problems in the original model disappear.

The purpose of the present paper is not to discuss phenomenological results of the new FGB model, but to improve the formulation of the Models A and B. However, it is worth noticing that we can obtain considerably low mass scales of the FGBs $A_2{}^1$ and $A_3{}^1$ compared with the old model (Model B). If we suppose that the nature chooses not always a simple case, the case $n = 2$ is very interesting from the phenomenological point of view. If we suppose that the nature choose the simplest case $n = 1$, only visible effect which is distinguished from Model B is an observation in the $\mu^- N \to e^- N$ experiments. The observation s within our reach.

Although we have obtained interesting results in the revised model (Model C), a basic problem which should be solved still remain in the Sumino cancellation mechanism itself. In the Sumino cancellation mechanism, a specific relation between the family gauge coupling constant $g_F$ and QED coupling constant $e$ is required. Besides, in the present model (Model C), a specific value of $c_0$ defined by (3.1) is required. These problems are a future task to us. Those problems will be solved under a unified theory of the electroweak theory and family gauge theory as Sumino have conjectured in Ref.[2].

We hope that FGBs become more realistic and more familiar to us.

**Acknowledgments**

The author is supported by JSPS KAKENHI Grant number JP16K05325. He also indebted to M. Yamanaka for estimate of $Br((\mu^- N \to e^- N)$.

## References

[1] Y. Koide, Phys. Lett. B **736** (2014) 499.

[2] Y. Sumino, Phys. Lett. B **671** (2009) 477; JHEP **0905**(2009) 075 .

[3] Y. Koide, Lett. Nuovo Cim. **34** (1982) 201; Phys. Lett. B **120** (1983) 161; Phys. Rev. D **28** (1983) 252.

[4] H. Albrecht *et al.* ARGUS collab., Phys.=Lett. **B292**, (1992) 221; J. Z. Bai *el al.* BES collab., Phys. Rev. Lett. **69** (1982) 3021; M. Daoudi *et al.* CLEO collab., a talk given at the XXVI int. Conf. on High Energy Physics, Dallas, 1992.

[5] Y. Koide, Mod. Phys. Lett. A **28** (1990) 2319.

[6] H. Arason, *et al.*, Phys. Rev. **D 46** (1992) 3945.

[7] Y. Koide and T. Yamashita, Phys. Lett. B **711** (2012) 384.

[8] Y. Koide and M. Yamanaka, in preparation.

[9] Y. Kuno, Prog. Theor. Exp. Phys. 022C01 (2013); Y. G. Cui *et al.* [COMET Collaboration], KEK-2009-10.

[10] H. Natori [DeeMe Collaboration], Nucl. Phys. Proc. Suppl. **248-250** (2014) 52.

[11] L. Bartoszek *et al.* [Mu2e Collaboration], arXiv:1501.05241 [physics.ins-det].