Quark and Lepton Mass Matrices
Described by Charged Lepton Masses

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Abstract

Recently, we proposed a unified mass matrix model for quarks and leptons, in which, mass ratios and mixings of the quarks and neutrinos are described using only the observed charged lepton mass values as family-number-dependent parameters and only six family-number-independent free parameters. In spite of quite few parameters, the model gives remarkable agreement with observed data (i.e. CKM mixing, PMNS mixing and mass ratios). Taking this phenomenological success seriously, we give a formulation of the so-called Yukawaon model in details from a theoretical aspect, especially for the construction of superpotentials and R charge assignments of fields. The model is considerably modified from the previous one, while the phenomenological success is kept unchanged.

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1 Introduction

It is a big concern in the flavor physics to investigate the origin of the observed hierarchical structures of masses and mixings of quarks and leptons. Recently, a unified mass matrix model for quarks and leptons was proposed [1]: In the model, mass ratios and mixings of the quarks and neutrinos are described using only the observed charged lepton mass values as "family-number-dependent" parameters and only six "family-number-independent" free parameters. In spite of quite few parameters, the model gives remarkable coincidence with observed all data, i.e. Cabibbo-Kobayashi-Maskawa (CKM) mixing [2] in quark sector and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [3] mixing in lepton sector, and quark and lepton mass ratios. Besides, the model gives very interesting predictions for leptonic CP violation parameter $\delta^{\ell}_{CP} \simeq -70^\circ \simeq -\delta^{q}_{CP}$ and effective Majorana neutrino mass $\langle m \rangle \simeq 21$ meV.

The previous paper has focused on only phenomenological aspect of the model and shown the phenomenological success which should be taken seriously. However, discussions on the theoretical aspect of the model was somewhat not sufficient. So, in this paper, we shall give a formulation of a new Yukawaon model from the theoretical aspect.

In the so-called Yukawaon model [4], we regard Yukawa coupling constants $(Y_f)_{ij}$ ($f = u, d, \nu, e$) as effective quantities $(Y_f)^{eff})_{ij}$ which are given by vacuum expectation values (VEVs) of scalares Y_f as $(Y_f^{eff})_{ij} = y_f \langle (Y_f) \rangle_{ij}/\Lambda$. The model is a sort of flavon model [5]. The model is based on family symmetries U(3)×U(3)′. The U(3)×U(3)′ symmetries are broken at $\mu = \Lambda$ and $\mu = \Lambda'$. (We assume $\Lambda \ll \Lambda'$.) The symmetry U(3)′ is broken into S_3 at an energy scale Λ', so that vacuum expectation values (VEVs) of flavons $(S_f)_\alpha^{\ \beta}$ (f is sector names $f = u, d, \nu, e$) take a form "unit matrix plus democratic matrix". (Here, the flavons S_f play an essential role as we discuss in Eq.(2.3).) Here and hereafter, indices i, j and α, β are those in U(3) and U(3)′, respectively.

The VEV relations in the model are expected to be derived from superpotentials which are invariant under the U(3)×U(3)′ and constructed by using suitable R charge assignments. When once we give superpotential form with U(3)×U(3)′ and R charge conservation, we can uniquely obtain our desirable VEV relation as seen in Sec.3. The R charge assignment is crucial for the phenomenological success. However, the explicit forms of superpotentials and R charge assignments were not presented in the previous paper [1]. The purpose of the present paper is to give explicit superpotential forms and R charge assignments in details. We will seek for more natural R charge assignment. The new R charges assignment is given in Table 1. As a result, we will change the formulation given in the previous paper, too. Especially, a flavon P with VEV of phase matrix type $\langle P \rangle = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ plays an essential role in the Yukawaon model and the phases (ϕ_1, ϕ_2, ϕ_3) are described in term of charged lepton mass values [6]. In the the previous model, we did not discuss the explicit mechanism only by citing Ref. [6], while the mechanism given in Ref. [6] cannot be straightforwardly applied to the model [1] because the R charge assignment in the model [1] is different from that in the model [6]. The new R charge assignment in the Table is different from [6] and [1], so that we are let to a new relation among the phase parameters and the observed charged lepton masses. As a result, we obtain different parameter values $(\phi_1, \phi_2, \phi_3) \equiv (\tilde{\phi}_1 + \phi_0, \tilde{\phi}_2 + \phi_0, \phi_0)$, while, as far as the values $(\tilde{\phi}_1, \tilde{\phi}_2)$ are concerned, we can obtain the same values as those in the previous paper [1]. (However, this does not mean that present model is identical with the previous one [1]. Note that, in the CKM fitting, only the values $(\tilde{\phi}_1, \tilde{\phi}_2)$ are observable and ϕ_0 is not observable, while the parameter ϕ_0 is observable in the U(3) family model.) The details are given in Sec.4.

The present paper is arranged as follows: In Sec.2, the basic postulation in the Yukawaon model [4] and the VEV relations in the previous paper [1] are reviewed without showing the explicit superpotentials. In Sec.3, we will discuss superpotentials which give special VEV forms, which play an essential role in the phenomenological investigation. In Sec.4, we will discuss the relation of phase parameters defined by Eq.(2.14) to the charged lepton masses. In Sec.5, we give a possible R charge assignment in this model. Although the formulation of the model in the present paper is changed from that in the previous model [1], the resulting VEV relations are the same as in the previous paper [1]. Therefore, we give only a brief review of the numerical results of the model in Sec.6. Finally, Sec.7 is devoted to summary and concluding remarks.

2 Basic assumptions in the Yukawaon model and its VEV relations

We investigate flavor physics from the point of view of family symmetry. It is unnatural that the Yukawa coupling constants Y_f explicitly break the family symmetry. Therefore, in order that the Hamiltonian is invariant under the symmetry, we must consider that Y_f are effective coupling constants Y_f^{eff} which are given by vacuum expectation values (VEVs) of scalars ("Yukawaons" [4]) Y_f with 3×3 components for each sector f:

$$(Y_f^{eff})_i^{\ j} = \frac{y_f}{\Lambda} \langle Y_f \rangle_i^{\ j} \qquad (f = u, d, \nu, e), \tag{2.1}$$

where Λ is an energy scale of the effective theory. All the flavons in the Yukawaon model are expressed by 3×3 components. Would-be Yukawa interactions are given by

$$H_Y = \frac{y_\nu}{\Lambda} (\bar{\ell}_L)^i (\hat{Y}_\nu)_i^{\ j} (\nu_R)_j H_u + \frac{y_e}{\Lambda} (\bar{\ell}_L)^i (\hat{Y}_e)_i^{\ j} (e_R)_j H_d + y_R (\bar{\nu}_R)^i (Y_R)_{ij} (\nu_R^c)^j$$

$$+ \frac{y_u}{\Lambda} (\bar{q}_L)^i (\hat{Y}_u)_i^{\ j} (u_R)_j H_u + \frac{y_d}{\Lambda} (\bar{q}_L)^i (\hat{Y}_d)_i^{\ j} (d_R)_j H_d, \tag{2.2}$$

where we have assumed a U(3) family symmetry, and $\ell_L = (\nu_L, e_L)$ and $q_L = (u_L, d_L)$ are $\mathrm{SU}(2)_L$ doublets. H_u and H_d are two Higgs doublets. Those Yukawaons $\hat{Y}_f$ are distinguished from each other by R charges. Hereafter, for convenience, we use notations $\hat{A}$, A, and $\bar{A}$ for fields with $\mathbf{8+1}$, $\mathbf{6}$, and $\mathbf{6}^*$ of U(3), respectively.

In the present model, we have flavons $(\hat{Y}_f)_i^{\ j}$, $(\Phi_{0f})_i^{\ \alpha}$, $(P_f)_i^\alpha$, $(\hat{S}_f)_\alpha^{\ \beta}$, and $(\Theta_f)_i^{\ \alpha}$. We assume that VEV matrices of those flavons take diagonal forms (and democratic form for flavons with $(\mathbf{1}, \mathbf{8+1})$ of U(3)$\times$U(3)$'$) at our basic flavor basis as we discuss later.

In addition to those flavons, we consider other flavons $(\Phi_0)_{ij}$, $(\bar{Y}_R)^{ij}$, $(\Theta_R)_{ij}$, E_{ij}, $\hat{E}_i^{\ j}$, and $\hat{\Theta}_\phi$. The transformation properties and the R charges of those flavons are listed in Table 1. A role of each flavon will be discussed step by step below.

Let us list VEV relations which are our goal: The following (i)-(v) are used in the previous model [1], while (vi) is revised from the previous paper. We will derive these relations from superpotentials presented in this paper.

(i) The VEV of the Yukawaons $\langle \hat{Y}_f \rangle$ are given by the following relations:

$$\langle \hat{Y}_f \rangle_i^{\ j} = k_f \ \langle \Phi_{0f} \rangle_i^{\ \alpha} \langle (S_f)^{-1} \rangle_\alpha^{\ \beta} \langle \bar{\Phi}_{0f}^T \rangle_\beta^{\ j} \qquad (f = u, d, \nu, e). \tag{2.3}$$

The factor S_f^{-1} in Eq.(2.3) comes from a seesaw-like scenario: Let us assume new heavy fermions F_α and consider the following 6×6 mass matrix model:

$$(\bar{f}_L^i \ \ \bar{F}_L^\alpha) \begin{pmatrix} (\hat{Y}_f)_i^{\ j} & (\Phi_{0f})_i^{\ \beta} \\ (\bar{\Phi}_{0f}^T)_\alpha^{\ j} & -(S_f)_\alpha^{\ \beta} \end{pmatrix} \begin{pmatrix} f_{Rj} \\ F_{R\beta} \end{pmatrix}. \tag{2.4}$$

Here $f_{L(R)}$ and $F_{L(R)}$ are, respectively, left (right) handed light and heavy fermions fields. Exactly speaking, we have to read $\bar{f}_L$ in Eq.(2.4) as $\bar{f}_L H_{u/d}/\Lambda$. However, for convenience, we

Table 1: Transformation properties and R charges of flavons in the present model. (Quarks and leptons and Higgs scalars are omitted in this table, because these quantum numbers are obvious.) Except for $\bar{Y}_R$ and Θ_R, we always consider a flavon $\bar{A}$, for a flavon A.

flavon	$\hat{Y}_e$	$\hat{Y}_\nu$	$\hat{Y}_d$	$\hat{Y}_u$	Φ_{0e}	$\Phi_{0\nu}$	Φ_{0d}	Φ_{0u}	P_e	P_ν	P_d	P_u
U(3)	$\mathbf{8}+\mathbf{1}$	$\mathbf{8}+\mathbf{1}$	$\mathbf{8}+\mathbf{1}$	$\mathbf{8}+\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
U(3)′	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
R charge	1	1	$\frac{1}{2}$	$\frac{3}{2}$	1	1	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1

$\hat{S}_e$	$\hat{S}_\nu$	$\hat{S}_d$	$\hat{S}_u$	Θ_{0e}	$\Theta_{0\nu}$	Θ_{0d}	Θ_{0u}	Φ_0	E	$\hat{E}$	$\bar{Y}_R$	Θ_R	$\hat{\Theta}_\phi$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{8}+\mathbf{1}$	$\mathbf{6}^*$	$\mathbf{6}$	$\mathbf{8}+\mathbf{1}$
$\mathbf{8}+\mathbf{1}$	$\mathbf{8}+\mathbf{1}$	$\mathbf{8}+\mathbf{1}$	$\mathbf{8}+\mathbf{1}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
1	1	$\frac{1}{2}$	$\frac{3}{2}$	1	1	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{8}{3}$	$-\frac{2}{3}$	1

have denoted those as $\bar{f}_L$ simply. Such a seesaw-like scenario with the democratic form of S_f has been proposed by Fusaoka and one of the authors (YK) [7] in order to understand the observed fact

$$m_t \sim \Lambda_{weak}, \qquad m_u \sim m_d \sim m_e. \tag{2.5}$$

In the Yukawaon model, when we consider $|\hat{Y}_f| \ll |\Phi_{0f}| \ll |S_f|$ (i.e. $\Lambda \ll \Lambda'$), we obtain a mass matrix for $\bar{f}'_L$ and f'_R,

$$M_f \simeq \hat{Y}_f + \Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}, \tag{2.6}$$

after the block diagonalization of Eq.(2.4). Regrettably, this relation (2.6) is not one we want, because the first term $\hat{Y}_f$ in Eq.(2.6) is independent of the second term $\Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}$. Therefore, in the previous paper [1], we have put some phenomenological assumptions in order to obtain the relation (2.3). Hereafter, for simplicity, we denote Dirac mass matrices M_f as

$$(\hat{Y}_f)_i^{\ j} \simeq (\Phi_{0f})_i^{\ \alpha} (S_f)_\alpha^{\ \beta} (\bar{\Phi}_0)_\beta^{\ j}. \tag{2.7}$$

(ii) The VEV form of $\langle \hat{S}_f \rangle$, which is due to the symmetry breaking U(3)′ → S_3, is given by

$$\langle \hat{S}_f \rangle = v_{Sf} (\mathbf{1} + b_f X_3), \tag{2.8}$$

where $\mathbf{1}$ and X_3 are defined as

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{2.9}$$

The parameters b_f in Eq. (2.8) are typical examples of "family-number-independent" parameters. Our Dirac mass matrices are described only by parameters b_f. However, we will be obliged to take $b_e = 0$ and $b_\nu = 0$ in the lepton sector $f = e$ and $f = \nu$ as seen in Eq.(3.1) later. Therefore, the Dirac mass matrices are described only by b_u and b_d.

(iii) Motivated by the relation given below in (2.13), we assume that the VEV form $\langle \Phi_{0f} \rangle$ are diagonal in the flavor basis in which $\langle S_f \rangle$ take the form (2.8), and are given by

$$\langle \Phi_{0f} \rangle_i{}^\alpha = \frac{1}{\Lambda} \langle \Phi_0 \rangle_{ik} \langle \bar{P}_f \rangle^{k\alpha}. \tag{2.10}$$

Another choice $(\Phi_{0f})_i{}^\alpha = (P_f)_{ik}(\Phi_0)^{k\alpha}$ is also possible. However, in this choice, the R charge assignment given in Table 1 should be modified, so that the phenomenological success becomes badly broken.

(iv) The VEV of a flavon Φ_0, $\langle \Phi_0 \rangle$ was defined by

$$\langle \Phi_0 \rangle = v_0 \, \mathrm{diag}\,(z_1, z_2, z_3) \propto \mathrm{diag}\,(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}), \tag{2.11}$$

where z_i is normalized as $z_1^2 + z_2^2 + z_3^2 = 1$. The $\langle \Phi_0 \rangle$ plays a crucial role in the phenomenological investigation of the Yukawaon model. Such the existence of the VEV matrix $\langle \Phi_0 \rangle$ was suggested by a phenomenological success of the charged lepton mass relation [8]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}, \tag{2.12}$$

which is excellently satisfied by the observed charged lepton masses (pole masses) as $K = (2/3) \times (0.999989 \pm 0.000014)$. If we accept the flavon Φ_0 with its VEV (2.11), the charged lepton relation (2.12) is simply expressed as

$$K = \frac{\mathrm{Tr}[\langle \Phi_0 \rangle \langle \Phi_0]}{(\mathrm{Tr}[\langle \Phi_0 \rangle])^2}. \tag{2.13}$$

However, the purpose of the Yukawaon model is not to understand the relation (2.12). Therefore, in the Yukawaon model, we do not ask the origin of the charged lepton mass spectrum. It is a future task to understand the origin of the mass values (m_e, m_μ, m_τ). In this paper we accept the observed charged lepton mass values as fundamental family-dependent parameters, and we give a unified description of quark and lepton mass matrices.

On the other hand, we know that there exist the CKM mixing in quark sector and the PMNS mixing in lepton sector. Therefore, true regularity in the mass spectra ought to be disturbed by such mixings. The relation (2.12) is a specific case only for the charged leptons. We consider that a fundamental flavor basis in flavor physics is a basis in which the charged lepton mass matrix is diagonal. Moreover, we speculate that all masses and mixings of quarks and leptons might be described by inputting the observed charged lepton mass values. Under such the idea, the Yukawaon model has been introduced and investigated [4].

(v) The VEV form $\langle P_f \rangle$ are defined as

$$\langle P_u \rangle = v_P \,\mathrm{diag}(e^{i\phi_1},\ e^{i\phi_2},\ e^{i\phi_3}), \quad \langle P_d \rangle = v_P \mathbf{1}, \quad \langle P_\nu \rangle = v_P \mathbf{1}, \qquad \langle P_e \rangle = v_P \mathbf{1}. \tag{2.14}$$

As we show $\langle P_f \rangle \langle \bar{P}_f \rangle = \mathbf{1}$ in Sec.3, the special choices (2.13) are obviously ansatzes. (Also see a comment below Eq.(3.5) later.) The parameters (ϕ_1, ϕ_2, ϕ_3) in Eq. (2.14) are typical examples of "family-number-dependent" parameters. However, in Sec.4, we will show that the parameters (ϕ_1, ϕ_2, ϕ_3) are described by the charged lepton masses $(m_e, m_\mu . m_\tau)$ with help of two family-number-independent parameters. The origin of these VEV forms will be discussed in Sec.3.

(vi) A neutrino mass matrix is given as

$$(M_\nu^{Majorana})_{ij} = \langle \hat{Y}_\nu \rangle_i{}^k \langle \bar{Y}_R^{-1} \rangle_{kl} \langle \hat{Y}_\nu^T \rangle^l{}_j, \tag{2.15}$$

by adopting the conventional seesaw mechanism [9]. Here in this paper, differently from the previous paper [1], we assume the following VEV structure of the Y_R (the Majorana mass matrix of the right-handed neutrinos ν_R):

$$\langle \bar{Y}_R \rangle^{ij} = k_R \frac{1}{\Lambda^2} \left[\left(\langle \bar{\Phi}_0 \rangle^{ik} \langle \hat{E} \rangle_k{}^l \langle \hat{Y}_u \rangle_l{}^j + \langle \hat{Y}_u^T \rangle^i{}_k \langle \hat{E}^T \rangle^k{}_l \langle \bar{\Phi}_0 \rangle^{lj} \right) + \xi_R \langle \hat{Y}_\nu^T \rangle^i{}_k \langle \bar{E} \rangle^{kl} \langle \hat{Y}_\nu \rangle_l{}^j \right], \tag{2.16}$$

where $\langle \hat{E} \rangle = \langle \bar{E} \rangle = v_E \mathbf{1}$. The form of the first term in Eq.(2.16), $\bar{\Phi}_0 \hat{Y}_u$, was first introduced in Ref.[10]. The new form (2.16) for $\langle \bar{Y}_R \rangle$ has been adopted in this paper in order for the R charge assignment to be more natural.

Since we deal with mass ratios and mixings only, the common coefficients k_f, v_{Sf}, and so on does not affect the numerical results, so that hereafter we omit such coefficients even it those have dimensions.

3 Examples of superpotentials

In this section, we demonstrate how to derive the VEV relations presented in Sec.2 from superpotentials given bellow. Hereafter, for convenience, we have sometimes dropped the notations "$\langle$" and "$\rangle$".

First, let us discuss a simple case which gives $b_e = b_\nu = 0$ by considering the following superpotential:

$$W_S = \sum_{f=e,\nu} \left\{ \lambda_{1f} [(S_f)_\alpha{}^\beta (P_f)_{\beta k} (\bar{P}_f)^{k\alpha}] + \lambda_{2f} [(S_f)_\alpha{}^\alpha] [(P_f)_{\beta k} (\bar{P}_f)^{k\beta}] \right\}, \tag{3.1}$$

where we have taken R charges of S_f and P_f as

$$R(S_f) + 2R(P_f) = 2 \qquad (f = e, \nu). \tag{3.2}$$

The form (3.1) is newly adopted in this paper. Of course, the potential form is unique as far as we accept the R charges given in Table 1. However, it is an ansatz that we consider the potential form only for lepton sector. Here and hereafter, we take $R(\bar{A}) = R(A)$ for arbitrary flavons A. (Eq.(3.1) is slightly improved from the previous paper [1].) The vacuum condition for the superpotential (3.1) leads to

$$S_f = \mathbf{1}, \quad P_f \bar{P}_f = \mathbf{1}. \quad (f = e, \nu) \tag{3.3}$$

The result $S_f = \mathbf{1}$ means $b_f = 0$, so that

$$\hat{Y}_e \propto \hat{Y}_\nu \propto \Phi_0 \bar{\Phi}_0. \tag{3.4}$$

However, this does not mean that $\hat{Y}_e$ and $\hat{Y}_\nu$ are an identical flavon.

We take a specific VEV form

$$P_f = \mathrm{diag}(e^{i\phi_1^f}, e^{i\phi_2^f}, e^{i\phi_3^f}), \tag{3.5}$$

from the general form $P_f \bar{P}_f = \mathbf{1}$ by assuming $\langle \bar{P}_f \rangle = \langle P_f \rangle^\dagger$ and by assuming that the VEV matrix is diagonal. Since the VEV matrix form of S_e (and also S_ν) is diagonal, the VEV matrix P_e is commutable with S_e, so that the phase parameters ϕ_i^e in P_e cannot play any physical role in $\bar{P}_e(S_e^{-1})P_e$. Therefore, we have simply put $P_f = \mathbf{1}$ for $f = e, \nu$ in Eq.(2.14).

For $f = u, d$, we assume the following superpotential:

$$W_{Pq} = \frac{1}{\Lambda} \left\{ \lambda_{1P} \mathrm{Tr}[P_u \bar{P}_u P_d \bar{P}_d] + \lambda_{2P} \mathrm{Tr}[P_u \bar{P}_u] \mathrm{Tr}[P_d \bar{P}_d] \right\}, \tag{3.6}$$

where we take

$$R(P_u) + R(P_d) = 1. \tag{3.7}$$

The SUSY vacuum condition leads to $P_u \bar{P}_u = \mathbf{1}$ and $P_d \bar{P}_d = \mathbf{1}$.

For the VEV relations (2.10) and (2.16), we consider somewhat tricky prescription: We assume existence of Θ fields which always take VEV values $\langle \Theta \rangle = 0$. For example, in order to obtain the VEV relation (2.10), we assume the following superptential

$$W_0 = \mu_{0f} (\Phi_{0f})_i^{\ \alpha} (\bar{\Theta}_{0f})_\alpha^{\ i} + \lambda_{0f} (\Phi_0)_{ik} (\bar{P}_f)^{k\alpha} (\bar{\Theta}_{0f})_\alpha^{\ i}. \tag{3.8}$$

From $\partial W_0 / \partial \bar{\Theta}_{0f} = 0$, we obtain VEV relation (2.10). On the other hand, a derivative of W_0 with respective to other flavon, for example, Φ_0 leads to $\partial W_0 / \partial \Phi_0 = \lambda_{0f} \bar{P}_f \Theta_{0f} = 0$. However, the result always includes Θ_{0f}, so that the condition does not lead to any new VEV relations. This prescription is very useful when one flavon appears in the different (two or more) superpotentials. (For example, instead of $W = (\mu A + \lambda BC)\Theta$, we may consider $W = (\mu A + \lambda BC)(\mu A + \lambda BC)^\dagger$. However, then we have an R charge constraint $R(A) = R(B) + R(C) = 1$ addition to $R(A) = R(B) + R(C)$. Beside, the SUSY vacuum condition $\partial W / \partial A = 0$ will lead to unwelcome relation if there is another potential term which includes the flavon A.) Of course, such Θ flavon prescription is a big ansatz in the Yukawaon model. we have to search for more reasonable prescription in future.

Similarly, for the VEV relation (2.16), we assume the following new superpotential presented in this paper,

$$W_R = \left\{ \mu_R \bar{Y}_R^{ij} + \frac{\lambda_R}{\Lambda} \left((\bar{\Phi}_0)^{ik} (\hat{E})_k{}^l (\hat{Y}_u)_l{}^j + (\hat{Y}_u^T)^i{}_k (\hat{E}^T)^k{}_l (\bar{\Phi}_0)^{lj} + \xi_R (\hat{Y}_\nu^T)^i{}_k \bar{E}^{kl} (\hat{Y}_\nu)_l{}^j \right) \right\} (\Theta_R)_{ji}, \tag{3.9}$$

together with

$$W_E = \lambda_{E1} [(\hat{E})_i{}^j (E)_{jk} (\bar{E})^{ki}] + \lambda_{E2} [(\hat{E})_i{}^i] [(E)_{jk} (\bar{E})^{kj}], \tag{3.10}$$

where we have taken

$$R(\hat{E}) = R(E) = R(\bar{E}) = \frac{2}{3}. \tag{3.11}$$

4 Relation between (ϕ_1, ϕ_2, ϕ_3) and (m_e, m_μ, m_τ)

In this section, we give a relation which connects the phase parameters (ϕ_1, ϕ_2, ϕ_3) ($\phi_i = \phi_i^u - \phi_i^d$) with the family-number-dependent input parameters (z_1, z_2, z_3). Although the basic idea has already given in Ref.[6], the explicit relation is renewed in the present paper as follows: We consider the following superpotential in this paper.

$$W_\phi = \left\{ \lambda_1 \left[(P_u)_{i\alpha} (\bar{P}_d)^{\alpha j} + (P_d)_{i\alpha} (\bar{P}_u)^{\alpha j} \right] + \lambda_2 \left[(P_\nu)_{i\alpha} (\bar{P}_e)^{\alpha j} + (P_e)_{i\alpha} (\bar{P}_\nu)^{\alpha j} \right] + \lambda_3 (\Phi_0)_{ik} (\bar{\Phi}_0)^{kj} \right\} (\hat{\Theta}_\phi)_j{}^i. \tag{4.1}$$

In order to get Eq.(4.1), it is essential that the VEVs of the flavons satisfy the following R charge relation

$$R(P_u) + R(P_d) = R(P_\nu) + R(P_e) = 2R(\Phi_0), \tag{4.2}$$

as we show in the next section.

The first term in Eq.(4.1) gives a VEV relation

$$\left[(P_u)_{i\alpha} (\bar{P}_d)^{\alpha j} + (P_d)_{i\alpha} (\bar{P}_u)^{\alpha j} \right] \propto \cos \phi_i, \tag{4.3}$$

where $\phi_i = \phi_i^u - \phi_i^d$. On the other hand, VEVs of the second and third terms are proportional to 1 and z_i^2, respectively. Therefore, we obtain a relation

$$\cos \phi_i = a + b z_i^2, \tag{4.4}$$

where the parameters a and b are family-number-independent parameters.

Note that observable parameters in the three phase parameters (ϕ_1, ϕ_2, ϕ_3) are only two. When we denote

$$\phi_1 = \phi_0 + \tilde{\phi}_1, \quad \phi_2 = \phi_0 + \tilde{\phi}_2, \quad \phi_3 = \phi_0, \tag{4.5}$$

the parameter ϕ_0 is not observable. Therefore, we can always choose arbitrary value of ϕ_0, so that the relation (4.4) is satisfied by choosing two family-number-independent parameters a and b suitably. (Note that although the parameter ϕ_0 is not observable in the framework of the standard model (SM), the parameter Φ_0 in the Yukawaon is observable because we consider U(3)×U(3)′ which are gauged. The value ϕ_0 will be confirmed by future experiments.)

Explicitly, we can obtain numerical results as follows: We obtain a relation

$$\cos\phi_1 - b\, z_1^2 = \cos\phi_2 - b\, z_2^2 = \cos\phi_3 - b\, z_3^2, \tag{4.6}$$

by eliminating the parameter a. Then, we can obtain a relation for the parameter ϕ_0:

$$b = \frac{\cos\phi_3 - \cos\phi_1}{z_3^2 - z_1^2} = \frac{\cos\phi_3 - \cos\phi_2}{z_3^2 - z_2^2}. \tag{4.7}$$

Since we have obtained the parameter values [1]

$$(\tilde{\phi}_1, \tilde{\phi}_2) = (-176.05^\circ, -169.91^\circ), \tag{4.8}$$

from fitting of the observed CKM mixing data, we obtain family-number-independent parameters (a, b)

$$(a, b) = (1.71573, -0.790018), \tag{4.9}$$

together with a value $\phi_0 = 33.905^\circ$. Note that the value $\phi_0 = 33.905^\circ$ is observable if we consider U(3) family gauge bosons, although it is not observable in the CKM parameter fitting. We predict the phase parameters (ϕ_1, ϕ_2, ϕ_3) as follows:

$$(\phi_1, \phi_2, \phi_3) = (-142.14^\circ, -136.00^\circ, 33.91^\circ). \tag{4.10}$$

In future, those values will be confirmed by family gauge boson experiments.

5 R-charge assignments

In the Yukawaon model, the R-charge assignment is very crucial from the phenomenological point of view. If we assign unsuitable R charges, we will obtain unwelcome VEV matrix combinations among flavons. If different flavons, say A and B, which have the same transformation property in U(3)×U(3)′, have the same R charge, then those flavons cause unwelcome mixing $A \leftrightarrow B$. Therefore, the R-charge assignments must be carefully done.

In the present Yukawaon model, the number of flavons is still lager than that of VEV relations, so that we cannot uniquely determine R charges of flavons. Therefore, we put the following rules on the R charge assignments for simplicity: (i) flavons A and $\bar{A}$ have the same R charge:

$$R(\bar{A}) = R(A). \tag{5.1}$$

(ii) We consider that R charge is conserved even after mixing between fermions f and F in the seesaw mass matrix (2.4):

$$R(\hat{Y}_f) = R(\Phi_{0f}) = R(S_f) \equiv r_f. \tag{5.2}$$

On the other hand, from the relation (2.10), we obtain

$$R(P_f) = r_f - r_0, \tag{5.3}$$

where $r_0 \equiv R(\Phi_0)$, so that, from the R charge relation (3.2), we obtain

$$r_e = r_\nu = \frac{2}{3}(r_0 + 1). \tag{5.4}$$

For the quark sector, we obtain an R charge relation

$$r_u + r_d = 1 + 2r_0, \tag{5.5}$$

from Eq.(3.7). On the other hand, we obtain

$$r_u + r_d = r_\nu + r_e = \frac{4}{3}(r_0 + 1), \tag{5.6}$$

from the superpotential (4.1). Eqs.(5.4) and (5.5) fix the value of r_0 as

$$r_0 = \frac{1}{2}, \quad \Rightarrow \quad r_e = r_\nu = 1. \tag{5.7}$$

Moreover, the form of $\bar{Y}_R$, Eq.(3.9) requires a relation

$$r_u + r_0 + r_E = 2r_\nu + r_E, \tag{5.8}$$

so that we obtain r_u and r_d as follows:

$$r_u = \frac{3}{2}, \quad r_d = \frac{1}{2}. \tag{5.9}$$

In other words, under the R charge assignment given in Eqs.(5.7) and (5.9), we can uniquely obtain the VEV relations given in Secs.2-4. R charges of whole flavons in the present models have been summarized in Table 1.

6 Numerical results

In spite of the revised formulation of the model, the resulting VEV relations are the same as those in the previous model [1] as far as parameter fitting in the framework of the SM is concerned. (As we have emphasized in Sec.4, the prediction of (ϕ_1, ϕ_2, ϕ_3), Eq.(4.10), is different from one in the previous paper.) Therefore, in the present paper, we quote only the numerical results of the previous paper [1]. For the details, see Ref.[1]. Note that the parameter fitting in

Ref.[1] has been done at $\mu = m_Z$, so that, even for the charged lepton masses inputted, we have used values at $\mu = m_Z$, not pole mass values.

We summarize our mass matrices for the numerical analysis as follows:

$$\hat{Y}_u = \Phi_0 \bar{P}_u (\mathbf{1} + b_u X_3)^{-1} P_u \bar{\Phi}_0, \quad \hat{Y}_d = \Phi_0 (\mathbf{1} + b_d e^{i\beta_d} X_3)^{-1} \bar{\Phi}_0, \tag{6.1}$$

$$M_\nu^{Majorana} = \hat{Y}_\nu \bar{Y}_R^{-1} \hat{Y}_\nu^T, \quad \bar{Y}_R = \bar{\Phi}_0 \hat{Y}_u + \hat{Y}_u^T \bar{\Phi}_0 + \xi_R \hat{Y}_\nu^T \hat{Y}_\nu, \tag{6.2}$$

where $\hat{Y}_e = \Phi_0 \cdot \mathbf{1} \cdot \bar{\Phi}_0$, and $\hat{Y}_\nu = \Phi_0 \cdot \mathbf{1} \cdot \bar{\Phi}_0$. Here, from the view of economy of parameter numbers, by way of trial, we assume that b_u is real, but b_d is complex (we denote b_d as $b_d e^{i\beta_d}$), and $\phi_u \neq 0$, but $\phi_d = 0$.

For convenience of numerical fitting, we re-define all VEV matrices of flavons as dimensionless matrices, i.e. $\bar{P}_u = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$, $\Phi_0 = \text{diag}(z_1, z_2, z_3)$, and so on.

In the present model, we have six family-number-independent parameters. b_u and (b_d, β_d) are fixed by the observed quark mass ratios [12]. the parameters (ϕ_1, ϕ_2) are fixed by the observed CKM mixing parameters. (For convenience, we count the parameters (ϕ_1, ϕ_2) as family-number-independent parameters.) ξ_R is fixed by the neutrino mass squared difference ratio $R_\nu \equiv (m_{\nu 2}^2 - m_{\nu 1}^2)/(m_{\nu 3}^2 - m_{\nu 2}^2)$. Our input parameter values are

$$b_u = -1.011, \ \ b_d = -3.3522, \ \ \beta_d = 17.7^\circ, \ \ (\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ), \ \ \xi_R = 2039.6, \tag{6.3}$$

In order to see which parameter is determined from which observed quantity, we quote Table 2 from Ref.[1].

By these six parameter values (6.3), our predicted results are in excellent agreement with the observed values as seen in Table 3. Note that the PMNS mixing is predicted with no free parameter after the mass ratios and the CKM mixing are fitted.

It is interesting that the model predicts $\delta_{CP}^\ell = -68^\circ$, which shows $\delta_{CP}^\ell \simeq -\delta_{CP}^q$. It is also worthwhile noticing that we obtain a large value 21 meV for the effective Majorana neutrino mass $\langle m \rangle$ in spite of the normal hierarchy for the neutrino mass in our model.

7 Concluding remarks

In conclusion, we have given formulation of a new Yukawaon model on the basis of seesaw type mass matrix model by presenting U(3)×U(3)′ assignments, R charges of flavons, superpotential forms, and so on. In spite of a model with quite few parameters, the model can give a remarkable agreement with the observed quark and lepton mixings and mass ratios. The phenomenological (numerical) results of the model has already been reported in the previous paper [1]. We emphasize that the phenomenological success highly depends on whether we can assign the R charges reasonably and consistently or not. It is in the present paper that the explicit R charge assignment is completed.

The phenomenological success of the present model seems to suggest the following points:
(a) The observed quark and lepton masses and mixings are caused by a common origin.

Table 2: Process for fitting parameters. $N_{parameter}$ and N_{input} denote a number of free parameters in the model and a number of observed values which are used as inputs in order to fix these free parameters, respectively. $\sum N_{...}$ means $\sum N_{parameter}$ or $\sum N_{input}$. [Quoted from Ref.[1].]

Step	Inputs	N_{input}	Parameters	$N_{parameter}$	Predictions
1st	m_c/m_t	1	b_u	1	m_u/m_c
	m_d/m_s, m_s/m_b	2	a_d, β_d	2	m_d/m_u
2nd	$\|V_{us}\|$, $\|V_{cb}\|$	2	(ϕ_1, ϕ_2)	2	$\|V_{ub}\|$, $\|V_{td}\|$, δ^q_{CP}
3rd	R_ν	1	ξ_R	1	$\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, δ^ℓ_{CP}
					2 Majorana phases, $\frac{m_{\nu 1}}{m_{\nu 2}}$, $\frac{m_{\nu 2}}{m_{\nu 3}}$
option	Δm^2_{32}		$m_{\nu 3}$		$(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$, $\langle m \rangle$
$\sum N_{...}$		6		6	

Table 3: Predicted values vs. observed values. [Quoted from [1]].

	$\|V_{us}\|$	$\|V_{cb}\|$	$\|V_{ub}\|$	$\|V_{td}\|$	$\delta^q_{CP}(^\circ)$	r^u_{12}	r^u_{23}	r^d_{12}	r^d_{23}
Predicted	0.2257	0.03996	0.00370	0.00917	81.0	0.061	0.060	0.049	0.027
Observed	0.22536	0.0414	0.00355	0.00886	69.4	0.045	0.060	0.053	0.019
	± 0.00061	± 0.0012	± 0.00015	$^{+0.00033}_{-0.00032}$	± 3.4	$^{+0.013}_{-0.010}$	± 0.005	$^{+0.005}_{-0.003}$	$^{+0.006}_{-0.006}$
	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$	R_ν (10^{-2})	$\delta^\ell_{CP}(^\circ)$	$m_{\nu 1}$ (eV)	$m_{\nu 2}$ (eV)	$m_{\nu 3}$ (eV)	$\langle m \rangle$ (eV)
Predicted	0.8254	0.9967	0.1007	3.118	-68.1	0.038	0.039	0.063	0.021
Observed	0.846	0.999	0.093	3.09	no data	no data	no data	no data	$< \mathrm{O}(10^{-1})$
	± 0.021	$^{+0.001}_{-0.018}$	± 0.008	± 0.15					

(b) Flavor physics should be investigated on a flavor basis in which charged lepton mass matrix is diagonal. (Mass matrix of family gauge bosons is also diagonal in this basis, so that, family gauge bosons will not cause flavor violation in the charged lepton sector [11] .)
(c) Masses and mixings in the quark sector are given by the parameters b_u and b_d in the form of S_f as shown in Eq.(2.8). This mechanism is very interesting.

On the other hand, for the theoretical aspect, the model has still many problems which should be improved in future. For example, in the present paper, we did not discuss explicit scales of Λ and Λ', although we have tacitly assumed that $\langle A_{ij}\rangle \sim \Lambda$, $\langle A_{\alpha\beta}\rangle \sim \Lambda'$ and $\langle A_{i\alpha}\rangle \sim \sqrt{\Lambda\Lambda'}$. The choice is highly correlated in the tininess of neutrino masses. Since we have discussed masses and mixings only, we have neglected the common coefficients and VEV values in the sectors, for example, k_f in Eq.(2.3) v_{Sf} in Eq.(2.8), v_0 in Eq.(2.11), k_R in Eq.(2.16), and so on. Those are our future task.

We believe that the present model can give fruitful suggestions for the study of flavor physics.

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