Family Gauge Boson Model with Inverted Mass Hierarchy and Sumino's Cancellation Mechanism

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Abstract

An interesting family gauge boson (FGB) model (Model A) has been proposed by Sumino. The model can give FGBs with a considerably low energy scale in spite of the sever constraints form the observed K^0-$\bar{K}^0$ mixing and so on. An essential idea in Model A is in the so-called Sumino cancellation mechanism between QED and FGB diagrams. But Model A is not anomaly free and, besides, it causes effective interactions with $\Delta N_{\rm family} = 2$. In order to avoid these problems, a revised Sumino model with an inverted mass hierarchy (Model B) has proposed, but, in this time, it cannot satisfy the Sumino cancellation mechanism exactly. In this paper, we propose a revised version of Model B, where the model still keeps anomaly free, but it can exactly satisfy the Sumino mechanism. An effect of the revised model will be confirmed by observations $K^+ \to \pi^+ e^- \mu^+$ and $\mu^- N \to e^- N$.

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1 Introduction:

The most challenging subject in particle physics is to understand the origin of flavors. There is an attractive idea that the flavor physics is understood from the point of view of a family symmetry. Then, it is natural to consider that the symmetry is gauged, because if we consider a global symmetry, we will have unwelcome massless Nambu-Gloldstone scalars. So, we may expect existence of family gauge bosons (FGBs). However, in the conventional FGB models, it is considered that a scale of the family symmetry breaking is extremely high, because of constraints from the observed P^0-$\bar{P}^0$ mixings ($P = K, D, B, B_s$). Therefore, the observation of FGBs was not a realistic topic.

Against such a conventional view, in 2009, Sumino [2] gave a realistic role to FGBs. Thereby, Sumino has speculated that a scale of the FGBs is of an order of 10^3 TeV.

Prior to a review of the Sumino FGB model, let us give a brief review of a charged lepton mass relation [3],

$$K \equiv \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\tau} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}. \tag{1.1}$$

The relation is satisfied by the pole masses [i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$], but not so well satisfied by the running masses [i.e. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$].

The formula (1.1) was first derived by the author in 1982 prior to observations of the precise tau lepton mass value in 1992 [4]. He has derived the formula (1.1) by assuming a U(3) family symmetry [3, 5]. Therefore, the formula (1.1) should be satisfied only by running masses without QED correction. In other words, the relation (1.1) can never be satisfied by pole masses. This was an utter mystery.

The QED correction is given as [6]:

$$m_{e_i}(\mu) = m_{e_i}^{pole}\left[1 - \frac{\alpha_{em}(\mu)}{\pi}\left(1 + \frac{3}{4}\log\frac{\mu^2}{m_{e_i}^2(\mu)}\right)\right]. \tag{1.2}$$

However, note that if the family-number dependent factor $\log m_{e_i}^2$ in Eq.(1.2) is absent, then the running masses $m_{e_i}(\mu)$ will also satisfy the formula (1.1). Noticing this fact, Sumino has proposed a U(3) family gauge model [2], where a factor $\log m_{e_i}^2$ in the QED correction for the charged lepton mass m_{e_i} $(i = 1, 2, 3)$ is canceled by a factor $\log M_{ii}^2$ in a corresponding diagram due to the FGBs. Here, Sumino has introduced a scalar Φ which is $(\mathbf{3}, \mathbf{3})$ of U(3)×O(3), whose symmetry breaking scales are Λ and Λ', respectively $(\Lambda \ll \Lambda')$. The vacuum expectation value (VEV) $\langle\Phi\rangle$ is given by

$$\langle\Phi_i^{\ \alpha}\rangle = [\mathrm{diag}(v_{e1}, v_{e2}, v_{e3})]_i^{\ \alpha}, \tag{1.3}$$

where i and α are indexes of U(3) and O(3), respectively. (Hereafter, since we consider Φ of $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)$'$ in an extended Sumino model later, we have denoted the components of Φ as $\Phi_i^{\ \alpha}$, not $\Phi_{i\alpha}$.) Then, the charged lepton mass matrix M_e in the Sumino model is given by

$$(M_e)_i^{\ j} \propto \langle\Phi_i^{\ \alpha}\rangle\langle\bar{\Phi}_\alpha^{\ j}\rangle \propto \delta_i^j v_{ei}^2, \qquad i.e. \;\; m_{ei} = k_e v_{ei}^2. \tag{1.4}$$

On the other hand, the scalar Φ also generates masses of FGBs $A_i^{\ j}$ as

$$M_{ij}^2 = \frac{1}{2}g_F^2\left\{\left(\langle\Phi\rangle_i^{\ \alpha}\langle\Phi^\dagger\rangle_\alpha^{\ i} + \langle\bar{\Phi}^\dagger\rangle_i^{\ \alpha}\langle\bar{\Phi}\rangle_\alpha^{\ i}\right) + (i \to j)\right\} \propto v_{ei}^2 + v_{ej}^2 \propto m_{ei} + m_{ej}, \tag{1.5}$$

in the limit of $\Lambda' \gg \Lambda$. Sumino's idea is as follows: The factor $\log m_{ei}$ in (1.2) is canceled by a factor $\log M_{ii}$ in the radiative FGB diagram by taking a suitable relation between the electromagnetic gauge coupling constant e and the family gauge coupling constant g_F and by noticing $\log M_{ii}^2 = \log(k_e m_{ei}) = \log m_{ei} + \log k_e$. This cancellation works only for the term $\log m_{ei}$ in Eq.(1.2), so that the running mass $m_{ei}(\mu)$ still has the energy scale dependence.

Note that not only the Sumino model gives a possible explanation for well-satisfied relation (1.1). but also the model gives a FGB model with a considerably low energy scale. (Sumino has speculated that the scale is of the order of 10^3 TeV [2].) In the Sumino model, the FGB mass matrix is diagonal in the diagonal basis of the charged lepton mass matrix, so that family-number violation does not occur in the lepton sector. Family-number violation occurs only via the quark mixing. In the limit of no quark mixing, the family-number violation is also forbidden.

Therefore, contribution of FGBs to the P^0-$\bar{P}^0$ mixing can be considerably reduced compared with the conventional FGB modes.

However, in the Sumino model (we call it Model A), in order to get a minus sign for the purpose of the cancellation, leptons $(\nu_i, e_i^-)_L$ and $(\nu_i, e_i^-)_R$ are assigned to $\mathbf{3}$ and $\mathbf{3}^*$ of U(3) family symmetry, respectively. Therefore, the Sumino model is not anomaly free. In order to avoid this anomaly problem, we may add some additional heavy leptons. As a result, the model becomes somewhat complicated. Besides, the model inevitably causes effective interaction with $\Delta N_F = 2$ (N_F is a family number).

In order to evade this problem, Yamashita and the author [9] have proposed an extended model (we call it Model B) with anomaly free: $(\nu_i, e_i^-)_L$ and $(\nu_i, e_i^-)_R$ are assigned to $\mathbf{3}$ and $\mathbf{3}$ of U(3) family symmetry, respectively and, instead, they consider that the FGB masses have inverted masses

$$M_{ij}^2 = K_e(m_{ei}^{-1} + m_{ej}^{-1}), \tag{1.6}$$

in order to obtain a minus sign for the Sumino cancellation mechanism, i.e. $\log M_{ii}^2 = -\log m_{ei} + \log(2K_e))$. Here, they have introduced additional scalar Ψ which belongs to $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)′ and which has a VEV

$$\langle \Psi_i{}^\alpha \rangle = [\mathrm{diag}(v_{F1}, v_{F2}, v_{F3})]_i{}^\alpha . \tag{1.7}$$

They have proposed a superpotential which leads to a VEV relation

$$\langle \Psi_i{}^\alpha \rangle \langle \bar{\Psi}_\alpha{}^k \rangle \langle \Phi_k{}^\beta \rangle \langle \bar{\Phi}_\beta{}^j \rangle = k_\Psi k_\Phi \delta_i^j, \quad \text{i.e.} \quad (v_{Fi})^2 = k_\Psi k_\Phi (v_{ei})^{-2}. \tag{1.8}$$

For convenience, let us show the superpotential as a simple form

$$W_{\Phi\Psi}^{eff} = \frac{\lambda_1}{\Lambda^2} \left[(\Psi\bar{\Psi}\Phi\bar{\Phi})_i{}^j (\Theta)_j{}^i \right] + \frac{\lambda_2}{\Lambda^2} \left[(\Psi\bar{\Psi}\Phi\bar{\Phi})_i{}^i \right] \left[(\Theta)_j{}^j \right], \tag{1.9}$$

where we have assumed that the flavon Θ always takes $\langle\Theta\rangle = 0$. (The expression (1.9) is not complete form of a superpotential for the related flavons. We need more subsidiary flavons for theoretical consistency. The form (1.9) is nothing but a provisional expression. For the full expression for (1.8), see Ref.[9].)

Since they assume that $\langle\Psi\rangle \gg \langle\Phi\rangle$, they can neglect contributions to FGB masses from $\langle\Phi\rangle$, so that they obtain the inverse mass hierarchy of M_{ij} (1.6).

However, we have to note that only the loop diagram $e_i \to e_i + A_i{}^i \to e_i$ can contribute to $m_{ei}(\mu)$ in Model A, while, in Model B, another loop diagrams $e_i \to e_j + A_i{}^j \to e_i$ with $j \neq i$ too can contribute to $m_{ei}(\mu)$ in addition to $e_i \to e_i + A_i{}^i \to e_i$. This means that the original cancellation scenario $\delta_i \equiv \log m_{ei}^2 - \xi \log M_{ii}^2 = const$ is exchanged by

$$\delta_i \equiv \log m_{ei}^2 + \xi \sum_{j=1,2,3} \log M_{ij}^2 = \log m_{ei}^2 + \xi \log S_i, \tag{1.10}$$

where

$$S_i \equiv M_{i1}^2 M_{i2}^2 M_{i3}^2 = K_e^3 \left(\frac{1}{m_{ei}} + \frac{1}{m_{e1}}\right)\left(\frac{1}{m_{ei}} + \frac{1}{m_{e2}}\right)\left(\frac{1}{m_{ei}} + \frac{1}{m_{e3}}\right), \tag{1.11}$$

and ξ is a family-number independent factor. From (1.10), the ratios S_1/S_2 and S_2/S_3 are given by $S_1/S_2 \simeq (m_\mu/m_e)^2$ and $S_2/S_3 \simeq m_\tau/m_\mu$, so that the Sumino's cancellation mechanism holds only approximately in Model B. Of course, the cancellation mechanism can be practically satisfied by adjusting a fine tuning parameter [9]. However, the cancellation mechanism is not so beautiful compared with the original one (Model A).

The purpose of the present paper is to propose a revised version of the FGB model (Model B) with inverted hierarchical FGB masses (i.e. with anomaly free), and to discuss possible new effects of the family gauge bosons. The purpose is not to derive the relation (1.1) itself. For the derivation of (1.1), for example, see a U(9) model by Sumino [7], a $\Sigma(81)$ model by Ma [8], and so on.

2 Basic idea:

In a new model (Model C), we change the relation (1.8) in Model B into

$$\langle \Psi_i^{\ \alpha}\rangle\langle\bar{\Psi}_\alpha^{\ k}\rangle \left(\langle\Phi_k^{\ \beta}\rangle\langle\bar{\Phi}_\beta^{\ j}\rangle + \delta_k^{\ j}\langle(\Phi_0)\rangle\langle(\bar{\Phi}_0)\rangle\right) = k_\Psi k_\Phi \delta_i^j, \tag{2.1}$$

where we have assumed R charges $R(\Phi_0) = R(\Phi)$. Here. we have introduced a new flavon (Φ_0) with $(\mathbf{1},\mathbf{1})$ of U(3)×U(3)$'$ whose VEV is given by

$$\langle\Phi_0\rangle = v_0. \tag{2.2}$$

A crucial assumption in Model C is that we put an ad hoc VEV relation

$$\frac{(v_{F0})^2}{(v_{Fi})^2} = \frac{(v_{ei})^2 + (v_0)^2}{(v_0)^2}, \tag{2.3}$$

where v_{F0} is defined by

$$(v_{F0})^2 \equiv (v_{F1})^2 + (v_{F2})^2 + (v_{F3})^2. \tag{2.4}$$

This relation (2.3), for example, can be obtained from the following superpotential

$$W = \left[(\Theta)_i^{\ j}(\Psi\bar{\Psi})_j^{\ k}(\Phi\bar{\Phi})_k^{\ i}\right] + \left(\left[((\Theta)_i^{\ j}(\Psi\bar{\Psi})_j^{\ i}\right] - \left[(\Theta)_i^{\ i}\right]\left[(\Psi\bar{\Psi})_j^{\ j}\right]\right)(\Phi_0\bar{\Phi}_0). \tag{2.5}$$

However, the form (2.5) is not a general form. The form (2.5) is nothing but an ad hoc assumption. (The last term $\mathrm{Tr}[\Theta]\mathrm{Tr}[\Psi\bar{\Psi}]\Phi_0\bar{\Phi}_0$ has been added without any theoretical reason.) In this paper, we do not discuss the origin of the form (2.5) or (2.3).

Hereafter, we denote the relation (2.3) as

$$\frac{(v_{F0})^2}{(v_{Fi})^2} = \frac{m_{ei} + m_0}{m_0}, \tag{2.6}$$

where we put $m_{ei} = k_e(v_{ei})^2$ and $m_0 = k_e(v_0)^2$. Since we can express the left-hand side in Eq.(2.6) as

$$\frac{(v_{F0})^2}{(v_{Fi})^2} = \frac{(v_{Fi})^2 + (v_{Fj})^2 + (v_{Fk})^2}{(v_{Fi})^2} = 1 + \frac{(v_{Fj})^2 + (v_{Fk})^2}{(v_{Fi})^2}, \tag{2.7}$$

where (i, j, k) denotes cyclic permutation of $(1, 2, 3)$, we obtain a relation

$$\frac{m_{ei}}{m_0} = \frac{(v_{Fj})^2 + (v_{Fk})^2}{(v_{Fi})^2}. \tag{2.8}$$

On the other hand, the factor S_i is given by

$$S_i = M_{i1}^2 M_{i2}^2 M_{i3}^2 = 2(g_F^2)^3 (v_{Fi})^2 [(v_{Fi})^2 + (v_{Fj})^2][(v_{Fi})^2 + (v_{Fk})^2] = A \frac{(v_{Fi})^2}{(v_{Fj})^2 + (v_{Fk})^2}, \tag{2.9}$$

where $A \equiv 2(g_F^2)^3[(v_{F1})^2+(v_{F2})^2][(v_{F2})^2+(v_{F3})^2][(v_3^F)^2+(v_1^F)^2]$ is a family-number independent constant. Thus, from Eq(2.8), we can express S_i as

$$S_i = A\,\frac{m_0}{m_{ei}}. \tag{2.10}$$

Therefore, we get $\log S_i = -\log m_{ei} + \log(A\, m_0)$, and we can achieve a complete Sumino cancellation mechanism.

3 Possible effects in the new scenario

Note that the value m_0 in Eq.(2.6) is not free under the observed values (m_{e1}, m_{e2}, m_{e3}). By using the relation (2.6), we obtain the following equation for m_0

$$1 = \frac{1}{(v_{F0})^2}[(v_{F1})^2 + (v_{F2})^2) + (v_{F3})^2] = \frac{m_0}{m_{e1} + m_0} + \frac{m_0}{m_{e2} + m_0} + \frac{m_0}{m_{e3} + m_0}. \tag{3.1}$$

Then, Eq.(3.1) has only a positive solution $m_0 = 7.6219$ MeV, so that we can regard as $m_{e1} \ll m_0 \ll m_{e2} \ll m_{e3}$. We can approximate Eq.(3.1) as

$$\frac{1}{1 + (m_{e1}/m_0)} + \frac{m_0}{m_{e2}} + \frac{m_0}{m_{e3}} \simeq 1, \tag{3.2}$$

i.e.

$$\frac{m_0}{m_{e2}} + \frac{m_0}{m_{e3}} \simeq \frac{m_{e1}}{m_0} \quad \Rightarrow \quad (m_0)^2 \simeq m_{e1} m_{e2}. \tag{3.3}$$

Therefore, from (2.6) and (3.3), we obtain approximate relations

$$(v_{F1})^2 \simeq (v_{F0})^2, \quad (v_{F2})^2 \simeq \left(\frac{\sqrt{m_{e1} m_{e2}}}{m_{e2}} \right) (v_{F0})^2, \quad (v_{F3})^2 \simeq \left(\frac{\sqrt{m_{e1} m_{e2}}}{m_{e3}} \right)^2 (v_{F0})^2. \tag{3.4}$$

Effect due to $m_0 \neq 0$ appears in values of FGB masses M_{ij}: As well as in Model B, the relative mass ratios of the FGBs are given by

$$M_{33} : M_{32} : M_{22} : M_{31} : M_{21} : M_{11} = 1; \sqrt{\frac{a^2+1}{2}} : a : \sqrt{\frac{b^2+1}{2}} : \sqrt{\frac{b^2+a^2}{2}} : b, \tag{3.5}$$

where $a \equiv M_{22}/M_{33}$ and $b \equiv M_{11}/M_{33}$, but the parameters a and b in Model C are given by

$$a \equiv \frac{M_{22}}{M_{33.}} = \frac{v_{F2}}{v_{F3}} = \left(\frac{m_{e3}+m_0}{m_{e2}+m_0} \right)^{1/2}, \qquad b \equiv \frac{M_{11}}{M_{33}} = \frac{v_{F1}}{v_{F3}} = \left(\frac{m_{e3}+m_0}{m_{e1}+m_0} \right)^{1/2}. \tag{3.6}$$

(The values in Model B are obtained by putting $m_0 = 0$ in Eq.(3.6).) Let us show numerical values a and b in the present model (Model C) without approximation (3.3):

$$\begin{array}{ll} a^C = 3.97347, & b^C = 15.0691, \\ (a^B = 4.10081, & b^B = 58.9674), \end{array} \tag{3.7}$$

Here, for the sake of comparison, we have also shown values a and b in Model B as a^B and b^B, respectively. The value a^C is almost same as the value a^B, while the value b^C is considerably smaller than the value b^B, $b^C \sim \frac{1}{4} b^B$.

Lower bounds of M_{ij} are constrained by the observed P^0-$\bar{P}^0$ mixing ($P = K, D, B, B_s$). The numerical results in Model B have been given in Ref.[1]. (However, take notice that Model B in the present paper corresponds to "Model A_1" in Ref.[1].) As seen in Eq.(3.5), the values M_{33}, M_{32} and M_{22} are independent on the parameter b, so that those values will almost be unchanged under the change of the value c_0. On the other hand, in a model with an inverse FGB masses, the constraint from the P^0-$\bar{P}^0$ mixing is almost determined by the value M_{22} (and M_{33} for B_s^0-$\bar{B}_s^0$ mixing) [1], so that the values of M_{22}, M_{23} and M_{33} are almost unchanged in Model C. (Exactly speaking, the constraint is fixed by a form of effective mass $\tilde{M}_{ij} \equiv M_{ij}/(g_F/\sqrt{2})$.) Therefore, we approximately put

$$\tilde{M}_{22}^C \equiv \frac{M_{22}^C}{g_F^C/\sqrt{2}} \simeq \tilde{M}_{22}^B \equiv \frac{M_{22}^B}{g_F^B/\sqrt{2}}. \tag{3.8}$$

In Model B [9], a cancellation condition $(g_F/\sqrt{2})^2 = (3/2)\zeta e^2$ has taken, where ζ is a fine tuning factor because the Sumino's cancellation is not complete, and the value $\zeta = 1.752$ was taken. In

Model C, the Sumino's cancellation holds exactly, so that we have to put $\zeta = 1$. Therefore, the value g_F^C/g_F^B is given by

$$\frac{g_F^C}{g_F^B} = \frac{1}{\sqrt{\zeta}} = \frac{1}{\sqrt{1.752}}. \tag{3.9}$$

Then. we can estimate values of M_{ij}^C in Model C from the numerical results M_{ij}^C in Model C (in Ref.[1]). The results are given Table 1.

For the sake of comparison, correspondingly to Ref.[1], we have added the case of $n = 2$ in addition to the case $n = 1$, where n is defined by

$$M_{ij}^2 = K_e(m_{ei}^{-n} + m_{ej}^{-n}). \tag{3.10}$$

(The extension to $n = 2$ is also possible, although it needs somewhat complicated framework among flavons as stated in Ref. [1].) For Case $n = 2$, we obtain parameter values $a^C = 16.7763$ and $b^C = 241.447$, ($a^B = 16.8167$ and $b^B = 3477.15$), correspondingly to Eq.(3.7). Also, we use $(g_F^{n=1}/\sqrt{2})^B = 0.4339$ and $(g_F^{n=2}/\sqrt{2})^B = 0.3068$.

Table 1 Lower bound of M_{ij} [TeV] from the observed P^0-$\bar{P}^0$ mixing

Model	n	M_{11}	M_{12}	M_{13}	M_{22}	M_{23}	M_{33}
Model C	$n = 1$	4.38×10^2	3.20×10^2	3.10×10^2	115	84.1	29.0
Model B	$n = 1$	2.20×10^3	1.56×10^3	1.55×10^3	153	111	37.2
Model C	$n = 2$	1.18×10^3	0.839×10^3	0.837×10^3	82.1	58.2	4.90
Model B	$n = 2$	2.25×10^4	1.59×10^4	1.59×10^4	109	77.1	6.47

As seen in Table 1, we can obtain somewhat lower mass values compared with the previous values (in Model B) only for M_{11}, M_{12} and M_{13}. However, the FGBs ${A_1}^1$, ${A_2}^1$ and ${A_3}^1$ have masses of 10^{2-3} TeV scale, it is usually not easy to observe the effects due to $m_0 \neq 0$. For example, let us see expect rare decays $K^+ \to \pi^+\mu^+e^-$ via ${A_1}^2$ and $B^+ \to \pi^+\tau^+e^-$ via ${A_1}^3$, which are proportional to $(\tilde{M}_{ij})^{-4} = (g_F/M_{ij})^4$. From Table 1 and Eq.(3.9), we obtain

$$\left(\frac{\tilde{M}_{12}^B}{\tilde{M}_{12}^C}\right)^4 \simeq 1.84 \times 10^2, \quad \left(\frac{\tilde{M}_{13}^B}{\tilde{M}_{13}^C}\right)^4 \simeq 2.04 \times 10^2, \tag{3.11}$$

for $n = 1$. The values (3.11)are still insufficient to observe such rare decays. However, if we consider such observations for the case $n = 2$, we obtain

$$\left(\frac{\tilde{M}_{12}^B}{\tilde{M}_{12}^C}\right)^4 \simeq 4.20 \times 10^4, \quad \left(\frac{\tilde{M}_{13}^B}{\tilde{M}_{13}^C}\right)^4 \simeq 4.24 \times 10^4, \tag{3.12}$$

Since the FGB prediction in Model B was $Br(K^+ \to \pi^+ e^- \mu^+) \simeq 2.3 \times 10^{-16}$ [9], we can predict

$$Br(K^+ \to \pi^+ e^- \mu^+) \simeq 0.97 \times 10^{-11}, \tag{3.13}$$

in Model C. Since the experiments have reported $Br(K^+ \to \pi^+ e^- \mu^+) < 1.3 \times 10^{-11}$, the observation is promising in the near future. We also expect observation of $B^+ \to \pi^+ \tau^+ e^-$ in the near future. (Of course the values M_{ij} in Table 1 are lower bound of FGB masses from the observation of P^0-$\bar{P}^0$ mixing, so that it is not likely that the actual masses are coincidentally the same as the lower bounds from the P^0-$\bar{P}^0$ mixing .) We hope further investigation of rare decays.

Besides, the case $n = 2$ predicts the lightest FGB mass as $M_{33} = 4.9$ TeV. This value is within reach of the 14 TeV LHC experiments, i.e. $p + p \to {A_3}^3 + X \to \tau^+ \tau^- + X$.

Only the most certain visible effect will appear the so-called μ-e conversion experiments

$$R(N) \equiv \frac{\sigma(\mu^- N \to e^- N)}{\sigma(\mu \text{ capture})}. \tag{3.14}$$

For example, the COMET experiment [11] aims for a goal $R \sim 10^{-17}$. On the other hand, we roughly estimate $R(\text{Al})$ as

$$R(\text{Al}) \simeq 0.85 \times 10^{-16} \frac{1}{n^2} \left(\frac{10^3 [\text{TeV}]}{M_{12} [\text{TeV}]} \right)^4, \tag{3.15}$$

in Models B and C. (For μ-e conversion induced by exchange of the FGB ${A_2}^1$, for example, see Ref.[10].) Since the previous value of M_{12} in the case $n = 1$ was 1.56×10^3 TeV, so that the value gives $R(\text{Al}) \sim 1.44 \times 10^{-17}$, the observation of μ-e conversion was critical in Model B. On the other hand, in Model C, the revised value (3.11) in the case $n = 1$ can give $R(\text{Al}) \sim 2.7 \times 10^{-15}$. Therefore, the observation of μ-e conversion due to FGB is promising in the experiments [11, 12], even if we take into consideration that the mass values in Table 1 are nothing but lower bounds constrained from the observed P^0-$\bar{P}^0$ mixing.

4 Concluding remarks

In conclusion, we have proposed a revised FGB model of Model B [9] with an inverted FGB mass hierarchy. The model is anomaly free as well as Model B, while the Sumino cancellation mechanism exactly holds as well as the original Sumino model (Model A). Therefore, we have obtained a FGB model, where all merits in the original Sumino model are retained, but all problems in the original model disappear.

The purpose of the present paper is to improve the formulation of the Model B. (The improvement of Model A has been already done by Model B.) As a result, we have obtained considerably low mass scales of the FGBs ${A_2}^1$ and ${A_3}^1$ compared with Model B. If we suppose that the nature chooses not always a simple case, the case $n = 2$ is very interesting from the phenomenological point of view, for example, $K^+ \to \pi^+\mu^+e^-$ and so on. (However, we need somewhat complicated model-building for the case $n = 2$.) On the other hand, if we suppose that the nature choose the simplest case $n = 1$, visible effect which is distinguished from Model B is an observation in the $\mu^- N \to e^- N$ experiments. The observation s within our reach.

Although we have obtained interesting results in the revised model (Model C), a basic problem which should be solved still remain in the Sumino cancellation mechanism itself. In the Sumino cancellation mechanism, a specific relation between the family gauge coupling constant g_F and QED coupling constant e is required. Besides, in Model C, a specific value of m_0 is required as stated in Eq.(3.1). Especially, the biggest ad hoc assumption in the present model is the relation (2.3) [i.e. the superpotential form (2.5)]. At present, there is no idea for such a special form (2.5). These problems are a future task to us.

We are convinced that the basic idea for a realistic FGB model by Sumino should be taken seriously. We hope that FGBs become more realistic and more familiar to us.

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