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U(3)×[U(1)]^3 Model and Visible Family Gauge Boson Effects

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Abstract

A U(3) family gauge boson model is investigated based on U(3)×[U(1)]^3 gauge symmetry. In this model, of nine U(3) family gauge bosons $A_i^{\ j}$, those with $i = j$ are considerable heavy compared with those with $i \neq j$, so that the model can be released from severe constraints due to the observed K^0-$\bar{K}^0$ mixing, and so on. We speculate that the lightest gauge boson is $A_2^{\ 3}$ and its mass is of $2 - 6$ TeV. Thereby, visible effects of the family gauge bosons are discussed. Especially, an observation of $\mu^- + N \to e^- + N$ versus no observation of $\mu \to e + \gamma$ will be a promising test for the present scenario.

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1 Introduction

It seems to be very attractive to consider families (generations) in quarks and leptons as a family symmetry [1]. However, constraints from the observed pseudo-scalar-anti-pseudo-scalar (P-$\bar{P}$) meson mixings (K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, and so on) are too tight to allow family gauge bosons with lower masses, so that it is usually taken that a scale of the symmetry braking is considerably high. It is taken that it is hard to observe the gauge boson effects even in the LHC era. However, if the family gauge symmetry really exists, it is rather likely that the effects are certainly visible.

In 2009, Sumino has proposed a new family gauge boson model [2]. Quarks and leptons are assigned to triplets of a family symmetry U(3). In the Sumino model, the family gauge boson mass matrix is diagonal on the family basis on which the charged lepton mass matrix M_e is diagonal. As we give a brief review of the Sumino model later, regrettably, the model allows effective current-current interaction with $\Delta N_{fam} = 2$ (N_{fam} is family number), so that we cannot be released from the sever constraint due to P-$\bar{P}$ mixing. (If we take a specific quark mixing, we can suppress the contribution to K^0-$\bar{K}^0$ mixing [3]. However, it is too artificial.)

Recently, an extended model of the Sumino model has been proposed by Yamashita and the author [4]. (Hereafter, we refer the Sumino model and the extended model as Model I and Model II, respectively.) In Model II, the family gauge bosons $A_i^{\ j}$ of U(3) have masses with an inverted mass hierarchy. The lightest family gauge boson $A_3^{\ 3}$ couples only to tau lepton, bottom and top quarks. In Model II, only gauge bosons $A_1^{\ 1}$, $A_2^{\ 2}$ and $A_3^{\ 3}$ can contribute to the P-$\bar{P}$ mixing through quark mixings $U^u \neq \mathbf{1}$ and $U^d \neq \mathbf{1}$. Because of a Cabibbo suppression factor, for example, the dominant contribution to the K^0-$\bar{K}^0$ mixing is brought by the gauge boson ${A_2}^2$ (not the lightest gauge boson ${A_3}^3$). Nevertheless, in Model II, too, it is hard to obtain a lowest gauge boson mass $M(A_3^3) \sim$ a few TeV [5].

Note that in Model II, the P-$\bar{P}$ mixing are caused only by the U(3) gauge bosons ${A_1}^1$, ${A_2}^2$ and ${A_3}^3$. (In Models I and II, there is no U(3)-**6** scalar, so that the gauge boson mixing ${A_i}^j \leftrightarrow {A_j}^i$ is absent.) A simple way to be released from this severe constraint due to the observed P-$\bar{P}$ mixing is to make the gauge bosons ${A_i}^j$ with $i = j$ considerably heavy compared with those with $i \neq j$. In this paper, we attempt to build such a model with heavy ${A_i}^i$, and thereby, we discuss possible gauge boson effects.

First, prior to giving our model, let us give a brief review of a gauge boson model (Model I) proposed by Sumino [2], because Model I is a starting point of our model. The purpose of Model I was to understand why the charged lepton mass relation [6], $m_e + m_\mu + m_\tau = (2/3)(\sqrt{m_e} + \sqrt{m_\tau} + \sqrt{m_\tau})^2$, is well satisfied by the pole masses (not by the running masses). In Model I, charged lepton masses are given by

$$H_{mass} = \bar{e}_L^i \Phi_{i\alpha} \Phi_{\alpha j}^T e_R^j + h.c., \tag{1}$$

where e_i are charged lepton fields, Φ is a scalar of $(\mathbf{3}, \mathbf{3})$ with U(3)×O(3) family symmetries, and H is the Higgs scalar in the standard model. Since the O(3) symmetry is broken at a high energy scale $\mu = \Lambda'$, where we assume $\Lambda' \gg \Lambda$ [U(3) is broken at $\mu = \Lambda$], the U(3) gauge boson masses are effectively given by

$$H_{mass} = \frac{1}{2} g_A^2 \mathrm{Tr}[\Phi \Phi^T A A] = \frac{1}{2} g_A^2 \sum_{i,j} v_i^2 {A_i}^j {A_j}^i, \tag{2}$$

in the limit of $\Lambda/\Lambda' \to 0$, where the vacuum expectation value (VEV) of Φ is given by $\langle \Phi_i^\alpha \rangle = \delta_i^\alpha v_i$. Therefore, the VEV of Φ is given in terms of charged lepton masses m_{ei} as

$$\langle \Phi \rangle = \mathrm{diag}(v_1, v_2, v_3) \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}), \tag{3}$$

and the gauge boson masses are given by $M_{ij} \equiv M(A_i^j) = k_e \sqrt{m_{ei} + m_{ej}}$. A factor $\log m_{ei}$ in the QED correction [7] is completely canceled by a factor $\log M_{ii}$ in the radiative correction due to family gauge boson ${A_i}^i$.[1] It is worthwhile noticing that, in the Sumino model, on account of the cancellation condition (for example, see Eq.(21) later), the family gauge coupling constants g_A is not free from the electroweak gauge coupling constant g_W.

Recently, Yamashita and the author [4] have proposed an extended version (Model II) of the Sumino model. In Model I, in order to realize the cancellation, the charged lepton fields

[1] Note that the charged lepton mass relation is invariant under the transformation $m_{ei} \to m_{ei}(1 + \varepsilon_0 + \varepsilon_i)$ with $\varepsilon_i = 0$ (ε_0 is a family number independent factor). Therefore, it is essential that the factor $\log m_{ei}$ in the QED correction [7] is canceled. Thus, in $\log M_{ii}^2 = \log m_{ei} + \log(2k_e^2)$, the i-independet factor $\log(2k_e^2)$ is not essential in this cancellation.

(e_L, e_R) are assigned to $(\mathbf{3}, \mathbf{3}^*)$ of U(3), so that the model is not anomaly free for U(3) symmetry, and, besides, unwelcome current-current interactions with family number change $\Delta N_{fam} = 2$ appear. In Model II, U(3) assignment of the charged leptons is $(e_L, e_R) = (\mathbf{3}, \mathbf{3})$, and, instead, the cancellation is guaranteed by the family gauge bosons with an inverted mass hierarchy:

$$M_{ij} \equiv M^2(A_i^j) \simeq k \left({m_{ei}}^{-1} + {m_{ej}}^{-1} \right), \tag{4}$$

i.e. by $\log M_{ii}^2 = -\log m_{ei} + \log(2k)$. The inverted mass hierarchy is realized by introducing an additional scalar Ψ of $(\mathbf{3}^*, \mathbf{3})$ of U(3)×U(3)$'$ with a relation $\langle\Psi\rangle\langle\Phi\rangle \propto \mathbf{1}$ [Φ in Model I is reread as $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)$'$ in Model II]. The charged lepton masses are still given by the VEV $\langle\Phi\rangle$ [we reread $\Phi\Phi^T$ in Eq.(1) as $\Phi\bar{\Phi}$], while the gauge boson masses are dominantly given by $\langle\Psi\rangle \sim \Lambda'$ because of $\Lambda' \gg \Lambda$.

In Model II, too, as well as Model I, the family gauge boson mass matrix is diagonal on the diagonal basis of M_e. Because of the U(3) assignment of the charged leptons $(e_L, e_R) = (\mathbf{3}, \mathbf{3})$ in Model II, the family number N_f is exactly conserved in the charged lepton sector. Family number violation appears only in the quark sector, because quark mass matrices M_u and M_d are, in general, not diagonal on the diagonal bases of M_e. The P-$\bar{P}$ mixings are only caused by exchange of family gauge bosons ${A_i}^j$ with $i = j$ and non-vanishing quark family mixing, $U^u \neq \mathbf{1}$ and $U^d \neq \mathbf{1}$. In Model II, for example, K^0-$\bar{K}^0$ mixing is dominantly caused by the gauge boson ${A_2}^2$ with a suppression factor $(U_{21}^{d*} U_{22}^d)^2$ differently from conventional family gauge boson model. Besides, since the gauge bosons are pure vector, ${A_i}^i$ cannot contribute to P-$\bar{P}$ mixing through s-channel, and it can do only through t channel, so that a color suppression factor 1/3 appears in the P-$\bar{P}$ mixing amplitude. Thus, in Model II, we can take a considerably lower mass value of the lightest gauge boson ${A_3}^3$. Nevertheless, in Model II, too, it is hard to obtain the lightest gauge boson ${A_3}^3$ with a mass of a few TeV [5].

A simple way to be released from this sever constraint due to the observed P-$\bar{P}$ mixing is to consider a model with $M_{ii} \gg M_{ij}$. In order to realize such a model, in the present paper, we introduce three U(1) gauge symmetries which mix with the U(3) family symmetry. As a result of the mixing between U(3) and $[\mathrm{U}(1)]^3$, we can make ${A_i}^i$ bosons heavy. Thereby, we discuss visible effects of the family gauge bosons ${A_i}^j$ with $i \neq j$.

2 Model

We consider gauge symmetries U(3)×U(3)$'$×$[\mathrm{U}(1)]^3$, which are broken at $\mu = \Lambda$, $\mu = \Lambda'$ and $\mu = \Lambda''$, respectively. The U(3) symmetry is a family gauge symmetry of quarks and leptons. The present model (Model III) is identical with Model II as far as U(3)×U(3)$'$ are concerned. Correspondingly to the three U(1)s, we assume three scalars of U(3) $\mathbf{3}^*$ scalars (χ_a, χ_b, χ_c), whose VEV are given by

$$\langle\chi_a^i\rangle = \delta_a^i \, x_i, \quad \langle\chi_b^i\rangle = \delta_b^i \, x_i, \quad \langle\chi_c^i\rangle = \delta_c^i \, x_i. \tag{5}$$

As seen in Eq.(5), the gauge bosons B_a, B_b and B_c of gauge symmetries $\mathrm{U}(1)_a$, $\mathrm{U}(1)_b$ and

$U(1)_c$, respectively, mix only with ${A_1}^1$, ${A_2}^2$ and ${A_3}^3$. Therefore, hereafter, we denote $(a,b,c) = (1,2,3) = i$. Scalars in the present model are summarized in Table 1.

Table 1: Scalars in the present model based on U(3)×U(3)′×[U(1)]^3.

Scalars	VEV	U(3)	U(3)′	$U(1)_a$	$U(1)_b$	$U(1)_c$
Φ	$\langle \Phi^\alpha_i \rangle = \delta^\alpha_i \, v_i$	**3**	**3***	0	0	0
Ψ	$\langle \Psi^i_\alpha \rangle = \delta^i_\alpha \, u_i$	**3***	**3**	0	0	0
χ_a	$\langle \chi^i_a \rangle = \delta^i_a \, x_i$	**3***	**1**	g_{Ba}	0	0
χ_b	$\langle \chi^i_b \rangle = \delta^i_b \, x_i$	**3***	**1**	0	g_{Bb}	0
χ_c	$\langle \chi^i_c \rangle = \delta^i_c \, x_i$	**3***	**1**	0	0	g_{Bc}
ϕ^a	$\langle \phi^a \rangle = w_a$	**1**	**1**	$-g_{Ba}$	0	0
ϕ^b	$\langle \phi^b \rangle = w_b$	**1**	**1**	0	$-g_{Bb}$	0
ϕ^c	$\langle \phi^c \rangle = w_c$	**1**	**1**	0	0	$-g_{Bc}$

Since we assume $\Lambda' \gg \Lambda'', \Lambda$, we obtain effective mass terms for U(3) and $[\mathrm{U}(1)]^3$ gauge bosons, A and B,

$$H_{mass} = \frac{g_A^2}{2} \sum_{i<j} (u_i^2 + u_j^2 + x_i^2 + x_j^2) {A_i}^j {A_j}^i$$

$$+ \frac{g_A^2}{2} \sum_i u_i^2 {A_i}^i {A_i}^i + \sum_i g_{Bi}^2 w_i^2 B_i B_i + \sum_i x_i^2 \left(\frac{g_A}{\sqrt{2}} {A_i}^i - g_{Bi} B_i \right)^2 , \tag{6}$$

in the limit of $\Lambda' \gg \Lambda'', \Lambda$. The form (6) is valid only when all the U(3)′ gauge bosons (say, ${C_\alpha}^\beta$) take super heavy masses of the order Λ'. For example, we can suppose U(3)′ **6** scalars in order to make ${C_\alpha}^\beta$ heavy . Then, ${C_\alpha}^\beta \leftrightarrow {C_\beta}^\alpha$ mixing appears. However, since we consider the case $\Lambda' \gg \Lambda'', \Lambda$, the ${C_\alpha}^\beta \leftrightarrow {C_\beta}^\alpha$ mixing does not affect ${A_i}^j$ visibly.

As seen in Table 1, mixings between ${A_i}^j$ and $B_{a,b,c}$ are caused only between ${A_i}^j$ with $i = j$ and $B_{a,b,c}$ with $(a,b,c) = i$. The mass matrix for $({A_i}^i, B_i)$ is given by

$$M = \frac{1}{2} \begin{pmatrix} g_A^2 (u_i^2 + x_i^2) & -\sqrt{2} g_A g_{Bi} x_i^2 \\ -\sqrt{2} g_A g_{Bi} x_i^2 & 2 g_{Bi}^2 (w_i^2 + x_i^2) \end{pmatrix} \tag{7}$$

Hereafter, we drop the index i, because we treat only mixing between those with i component. When we define the mixing angle θ between A and B by $A' = A\cos\theta - B\sin\theta$ and $B' = A\sin\theta + B\cos\theta$, the mixing angle θ is given by

$$\tan 2\theta = \frac{2\sqrt{2} g_A g_B x^2}{2 g_B^2 (w^2 + x^2) - g_A^2 (u^2 + x^2)}, \tag{8}$$

and masses of A' and B' are given by

$$M^2(A') = \frac{1}{2}[2g_B^2(w^2+x^2)+g_A^2(u^2+x^2)] - \frac{1}{2}\sqrt{[2g_B^2(w^2+x^2)-g_A^2(u^2+x^2)]^2+8g_A^2g_B^2x^4}, \quad (9)$$

$$M^2(B') = \frac{1}{2}[2g_B^2(w^2+x^2)+g_A^2(u^2+x^2)] + \frac{1}{2}\sqrt{[2g_B^2(w^2+x^2)-g_A^2(u^2+x^2)]^2+8g_A^2g_B^2x^4}. \quad (10)$$

Since the gauge bosons A_μ and B_μ couple to U(3) currents J^μ and U(1) currents j^μ, respectively, as follows:

$$H_{int} = \frac{g_A}{\sqrt{2}}J^\mu A_\mu + g_B j^\mu B_\mu = \left(\frac{g_A}{\sqrt{2}}\cos\theta\, J^\mu - g_B\sin\theta\, j^\mu\right)A'_\mu + \left(\frac{g_A}{\sqrt{2}}\sin\theta\, J^\mu + g_B\cos\theta\, j^\mu\right)B'_\mu, \quad (11)$$

when we define effective coupling constants of current-current interactions by

$$H^{eff} = G_{AA}\, J^\mu J_\mu + G_{BB}\, j^\mu j_\mu + G_{AB}\, J^\mu j_\mu, \quad (12)$$

the effective coupling constants are given by

$$G_{AA} = \frac{g_A^2}{2}\left(\frac{\cos^2\theta}{M^2(A')}+\frac{\sin^2\theta}{M^2(B')}\right) = \frac{1}{2}\frac{w^2+x^2}{w^2u^2+(w^2+u^2)x^2}, \quad (13)$$

$$G_{BB} = \frac{g_B^2}{2}\left(\frac{\sin^2\theta}{M^2(A')}+\frac{\cos^2\theta}{M^2(B')}\right) = \frac{1}{2}\frac{u^2+x^2}{w^2u^2+(w^2+u^2)x^2}, \quad (14)$$

$$G_{AB} = -\frac{g_Ag_B}{2}2\sin\theta\cos\theta\left(\frac{1}{M^2(B')}-\frac{1}{M^2(A')}\right) = \frac{x^2}{w^2u^2+(w^2+u^2)x^2}. \quad (15)$$

Note that the effective coupling constants G_{AA}, G_{BB} and G_{AB} are only dependent on the values u^2, v^2 and w^2, and they are independent of the values g_A and g_B.

Since quarks and leptons do not have $[\mathrm{U}(1)]^3$ charges as seen in Table 1, the currents j^μ do not contain quark and lepton components. Our interest is only in the magnitude of G_{AA}. If we suppose $x^2 \gg w^2, u^2$, then we obtain

$$G_{AA} \simeq \frac{1}{2(w^2+u^2)} \quad \stackrel{w^2\gg u^2}{\longrightarrow} \quad \frac{1}{2w^2}, \quad (16)$$

Thus, in Model III with $w^2 \gg u^2$, the current-current interactions due to exchange of the family gauge boson $A_i^{\ i}$ can highly be suppressed compared with that of $A_i^{\ j}$ ($i \neq j$), because the former and the latter are approximately given by the effective coupling constants $1/2w_i^2$

and $1/(u_i^2 + u_j^2)$, respectively. This is confirmed from that the mass of ${A_i}^j$ $(i \neq j)$ is given by $M^2 = (g_A^2/2)(u_i^2 + u_j^2)$, while the mass of ${(A')_i}^i$ is approximately given by

$$M^2({(A')_i}^i) \simeq \frac{2g_A^2 g_{Bi}^2}{2g_{Bi}^2 + g_A^2}(w_i^2 + u_i^2), \tag{17}$$

from Eq.(9).

The cancellation mechanism proposed by Sumino [2] is realized by the family gauge bosons ${(A')_i}^i$ in the present model as well as Model II. Therefore, we must require a relation $w_i^2 \propto v_i^{-2} \propto m_{ei}^{-1}$. In Model II, the inverted mass hierarchy has been realized $\langle\Psi\rangle \propto \langle\Phi\rangle^{-1}$, i.e. by a superpotential

$$W = \left(\delta_i^j \mu S + \lambda \Phi_i^\alpha \Psi_\alpha^j\right) \Theta_j^i, \tag{18}$$

based on a SUSY scenario, where S is a U(3)×U(3)′ singlet scalar, and Θ is a scalar with VEV $\langle\Theta\rangle = 0$. (Here, the superpotential (15) has been given by a considerably simplified form in order to show the outline of the model. For the full expression, see in the reference [4].) In order to obtain a relation $w_i^2 \propto v_i^{-2}$, by following the example of Model II, we assume a superpotential

$$W = (\mu\phi^a + \lambda\Psi_\alpha^i E^{\alpha a})\Theta_a, \tag{19}$$

where VEVs of E and Θ take $\langle E\rangle \propto \mathbf{1}$ and $\langle\Theta\rangle = 0$, respectively. A SUSY vacuum condition $\partial W/\partial\Theta = 0$ leads to a VEV relation $w_i^2 \propto u_i^2 \propto v_i^{-2} \propto m_{ei}^{-1}$. However, in order to realize the Sumino mechanism, the relation $M^2({(A')_i}^i) \propto m_{ei}^{-1}$ is required. That is, not only $w_i^2 \propto u_i^2 \propto m_{ei}^{-1}$, but also $g_{B1} = g_{B2} = g_{B3}$ must hold as seen in Eq.(17). Therefore, we assume a permutation symmetry S_3 among the three U(1) symmetries. Then, we can obtain

$$M^2({(A')_i}^i) \propto w_i^2 \propto u_i^2 \propto {m_{ei}}^{-1}. \tag{20}$$

3 Phenomenology of the family gauge bosons

3.1 A tentative mass value of the lightest gauge boson ${A_3}^2$

Let us speculate numerical values of the gauge bosons ${A_i}^j$. As well as in Models I and II, the family gauge coupling constant g_A in Model III is not a free parameter because of the cancellation condition

$$g_A^2 = \frac{3}{2}\zeta\, e^2 = \frac{3}{2}\zeta\, g_W^2 \sin^2\theta_W, \tag{21}$$

where g_W is the weak gauge coupling constant given by $G_F/\sqrt{2} = g_W^2/8M_W^2$, and ζ is a fine tuning parameter. In Models II and III, the parameter ζ is numerically given by $\zeta = 1.752$ [4]. If we suppose that the gauge boson mass of $(A')_2^3$ is considerably low, we will observe a violation of the e-μ universality in the tau decays. From the present observed branching ratios [8] $Br(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau) = (17.41 \pm 0.04)\%$ and $Br(\tau^- \to e^-\bar{\nu}_e\nu_\tau) = (17.83 \pm 0.04)\%$, we obtain

Table 2: Family gauge boson masses $M({A_i}^j)$ $(i \neq j)$ and $M({(A')_i}^i)$. The values of M_{ij} are presented in a unit of TeV. The values with underlines are input values.

M_{23}	M_{31}	M_{12}	M_{33}	M_{22}	M_{11}
<u>2.70</u>	37.7	37.8	48.8	<u>200</u>	2776

the ratio $R_{Br} \equiv Br(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)/Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = 0.97644 \pm 0.00314$. Then, the observed data give a deviation from the e-μ universality [5]

$$\delta \equiv \sqrt{R_{BR} \times (\text{phase space factors})} - 1 = 0.0020 \pm 0.0016. \tag{22}$$

Of course, from the value (22), we cannot conclude that we found a significant difference of the deviation from the e-μ universality. However, we may speculate a possibility of family gauge bosons. We can consider that the deviation in the tau decays originates in exchange of gauge bosons $(A')_3^2$ and $(A')_3^1$ which interact as $\tau \to (A')_3^2 + \mu$ and $\tau \to (A')_3^1 + e$, respectively. The present deviation $\delta = (2.0 \pm 1.6) \times 10^{-2}$ gives a family gauge boson mass of ${A_3}^2$ [5] by assuming an inverted mass hierarchy

$$M_{23} = 2.6^{+3.2}_{-0.7} \text{ TeV}. \tag{23}$$

On the other hand, from the observed K^0-$\bar{K}^0$ mixing and so on, we obtain a constraint [5]

$$M_{22} \gtrsim 10^2 \text{ TeV}. \tag{24}$$

If we take values $M_{23} = 2.6$ TeV and $M_{22} = 200$ TeV, by using relations

$$M_{ij}^2 \propto u_i^2 + u_j^2 \propto m_{ei}^{-1} + m_{ej}^{-1}, \qquad M_{ii}^2 \propto w_i^2 \propto m_{ei}^{-1}, \tag{25}$$

we obtain masses of the family gauge bosons ${(A')_i}^j$ as shown in Table 2. The values in Table 2 are only examples in order to see the relative ratios, and those should not be taken rigidly.

3.2 Visible gauge boson effects

In the present model, since we consider that the family gauge bosons ${(A')_i}^j$ with $i = j$ are sufficiently heavy compared with those with $i \neq j$, those gauge bosons cannot contribute to the P-$\bar{P}$ mixings. (As we previously emphasized, in Models II and III, ${A_i}^j$ cannot contribute to a process with $\Delta N_{fam} = 2$.) Also, in Model III, the family number is exactly conserved in the charged lepton sector, so that the ratidative charged lepton decays $\tau \to \mu + \gamma$ and so on are highly suppressed.[2] In Model II, we have speculated [5] a violation of the e-μ-τ universality in Υ decays, $\Upsilon \to \tau^+\tau^-/\mu^+\mu^-/e^+e^-$, and a direct production of ${A_3}^3$ at LHC. However, in this Model III, it is hard to observe those because of the large values of M_{ii} as seen in Table 2.

[2] However, since, for example, $({A_3}^2)_\mu$ can couple to $\bar{t}\gamma^\mu t$ with up-quark mixing factor $U_{33}^{u*} U_{23}^u$, the decay $\tau \to \mu + \gamma$ is possible through the $t\bar{t}$ loop, although it is considerably suppressed. Such a loop effect will be discussed elsewhere.

Nevertheless, in the present model, we can see rather fruitful phenomenology, since we do not have any constraint on the masses of ${A_i}^j$ $(i \neq j)$ from the observed P-$\bar{P}$ mixings, In Model III, the family gauge bosons interact with quarks and leptons as follows:

$$\mathcal{H}_{fam} = g_F \left[(\bar{e}_i \gamma_\mu e_j) + (\bar{\nu}_i \gamma_\mu \nu_j) + U^{*d}_{ik} U^d_{jl} (\bar{d}_k \gamma_\mu d_l) + U^{*u}_{ik} U^u_{jl} (\bar{u}_k \gamma_\mu u_l) \right] ({A_i}^j)^\mu , \tag{26}$$

where U^u and U^d are quark mixing matrices. We can expect the following observations:

(i) *Deviation from the e-μ-τ universality in tau decays*: So far, we have used an input value estimated from the deviation from e-μ-τ universality in tau decays. Although the value $M_{23} = 2 - 6$ TeV is a tentative value, we may expect that the deviation will soon become more accurate.

(ii) *Lepton number violating rare decays of B meson decays*: For lepton-flavor violating rare decays, bottom meson decays $B^+ \to K^+ \mu^- \tau^+$ and $B^0 \to K^+ \mu^- \tau^+$ become soon within our reach (we expect $Rr \sim 10^{-6}$ as seen in Fig.1 in Ref.[4]). Regrettably, since $M_{12} \sim 40$ TeV, the rare decays $K^+ \to \pi^+ \mu^+ e^-$ and $K_L \to \pi^0 \mu^\pm e^\mp$ are invisible as seen in Fig.1 in Ref.[4]).

(iii) *μ-e conversion*: Most sensitive test for our scenario is to observe the so-called μ-e conversion. (For a review of the μ-e conversion and more detailed calculations, for example, see Ref.[9] and Ref.[10], respectively.) At present, we do not know values of $|U^{q*}_{11} U^q_{21}|$ $(q = u, d)$. Therefore, it is not practical, at this stage, to estimate a μ-e conversion rate strictly. Instead, we roughly estimate a μ-e conversion rate in the quark level as follows:

$$R_q \equiv \frac{\sigma(\mu^- + q \to e^- + q)}{\sigma(\mu^- + u \to \nu_\mu + d)} \simeq \left(\frac{|U^{q*}_{11} U^q_{21}|}{|V_{ud}|} \frac{2\sqrt{2} g_A^2}{g_W^2} \frac{M_W^2}{M_{12}^2} \right)^2 , \tag{27}$$

where $q = u$ and $q = d$, $g_A^2/g_W^2 = (3/2)\zeta \sin^2\theta_W = 0.586$ from Sumino's cancellation relation (21). It is likely that $|U^u_{21}|^2 \ll |U^d_{21}|^2$. Then, we may regard the ratios R_q as $R_u \ll R_d$, so that we can neglect contribution to nucleon from R_u compared with that from R_d. When we suppose $|U^{d*}_{11} U^d_{21}|/|V_{ud}| \sim 10^{-1}$, we can roughly estimate values of R_d for the input values of M_{32} which are corresponding to the values (23) speculated from τ decays. The results are listed in Table 3. Present experimental limit is, for instance for Au, $R(A_u) \equiv \sigma(\mu^- + Au \to e^- + Au)/\sigma(\mu \text{ capture}) < 7 \times 10^{-13}$ [11]. The estimated values in Table 3 soon become within reach of our observation, even for the case of $M_{12} \sim 80$ TeV. If we observe $\mu^- + N \to e^- + N$ without observation of $\mu^- \to e^- + \gamma$, then, it will highly support our family gauge boson scenario.

(iv) *Direct production of the lightest gauge boson ${A_3}^2$*: The direct production $p + p \to s + X \to b + {A_2}^3 + X$ (and $p + p \to b + X \to s + {A_3}^2 + X$) is also expected, although the cross section of b ($\bar{b}$) production with a large energy-momentum is quit small. If the production of ${A_3}^2$ is realized, the gauge boson will decay into $b + \bar{s}$, $\tau^- + \mu^+$ and $\nu_\tau + \bar{\nu}_\mu$ with branching fractions 6/9, 2/9 and 1/9, respectively. The decay mode ${A_3}^2 \to \tau^- + \mu^+$ will be distinctive. However, if $M_{23} > 3$ TeV, the observation at the present LHC will almost be hopeless. The value (23) is critical for observing of ${A_2}^3$.

(v) *Radiative decay of hadrons*: A decay mode $b \to s + \gamma$ is also important to see ${A_3}^2$ gauge boson effects. The decay is, in fact, allowed through a $t\bar{t}$ loop, ${A_3}^2 \to t\bar{t} \to \gamma$. Recently, an importance of a rare decay mode $\bar{B} \to X_s + \gamma$ has been pointed out by Buras *et al.* [12]. In our model, a value of the parameter $|U^{u*}_{33} U^u_{23}|$ is unknown (although we assumed that it is small in the previous item (iii)), we cannot estimate it at present. Radiative decays of hadrons are not peculiar in this model, and those can be caused even in the standard model. We do not discuss such radiative decays of hadrons in this paper.

Table 3: Rough estimate of μ-e conversion rate in d quark. R_d is defined by Eq.(24). Values of the gauge boson masses M_{ij} are presented in a unit of TeV. The input values M_{32} correspond to the values Eq.(20) speculated from τ decays.

M_{23}	M_{12}	R_d
1.9	26.6	1.7×10^{-13}
2.6	36.4	4.8×10^{-14}
5.8	81.2	1.9×10^{-15}

4 Summary

In conclusion, we have investigated a gauged U(3)×[U(1)]^3 model in order to make the U(3) family gauge bosons ${A_i}^j$ with $i = j$ heavy compared with those with $i \neq j$. Thereby, we have speculated that the lightest gauge boson is ${A_3}^2$ with a mass $2 - 6$ TeV (a tentative value determined by τ decays) [and $M({A_3}^1) \sim M({A_2}^1) \sim 30 - 80$ TeV]. Possible ${A_3}^2$ bosons effects will be observed in a deviation from e-μ universality in τ decays, and in rare decays of B mesons, $B^+ \to K^+ \mu^- \tau^+$ and $B^0 \to K^0 \mu^- \tau^+$. Possible ${A_2}^1$ bosons effects will be observed in μ-e conversion experiments, $\mu^- + N \to e^- + N$, sensitively. Also, we may expect to observe a direct production of ${A_3}^2$ at LHC as $p + p \to b + X \to s + {A_3}^2 + X$.

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