MISC-2012-18

Can Mass of the Lightest Family Gauge Boson be an Order of TeV?

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Abstract

The observed sign of a deviation from the e-μ universality in tau decays suggests family gauge bosons with an inverted mass hierarchy. Constraints from the observed K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixing are re-considerated in this case, and it is speculated that a mass of the lightest gauge boson A_3^3 which couples with only the third generation quarks and leptons can be $M_{33} \sim 1$ TeV, because of its inverted mass hierarchy.

1. Introduction

We know three generations of quarks and leptons. It seems to be natural to regard those as triplets of a family symmetry SU(3) [1] or U(3). However, so far, one has considered that, even if the family gauge symmetry exists, it is impossible to observe such gauge boson effects, because we know a severe constraint from the observed K^0-$\bar{K}^0$ mixing [2] and results from Z' search [3] at the Tevatron. Nevertheless, it is interesting to consider a possibility that a family gauge symmetry really exists and the family gauge bosons are visible at a lower energy scale. In this paper, we pay attention to deviations from the e-μ-τ universality, i.e. in the tau decays and in the upsilon decays. And, we will give a reconsideration of the constraints from the K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixings.

From the present observed branching ratios [2] $Br(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = (17.41 \pm 0.04)\%$ and $Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = (17.83 \pm 0.04)\%$, we obtain the ratio $R_{Br} \equiv Br(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)/Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = 0.97644 \pm 0.00314$. For convenience, we define parameters δ_μ and δ_e which are measures of a deviation from the e-μ universality as follows:

$$R_{amp} \equiv \frac{1+\delta_\mu}{1+\delta_e} = \sqrt{R_{Br} \frac{f(m_e/m_\tau)}{f(m_\mu/m_\tau)}} = 1.0020 \pm 0.0016, \tag{1}$$

where $f(x)$ is known as the phase space function and it is given by $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 12x^4 \log x^2$. Then, the result (1) gives

$$\delta \equiv \delta_\mu - \delta_e = 0.0020 \pm 0.0016. \tag{2}$$

[The values of the deviation parameters δ_μ and δ_e depend on types of the gauge boson interactions, i.e. $(V-A)$, pure V, and so on. In Sec.3, we will discuss corrections for the parameters δ_μ and δ_e which have been defined by Eq.(1).]

Of course, from the value (2), we cannot conclude that we found a significant difference of the deviation from the e-μ universality. However, we may speculate a possibility of family gauge bosons. We can consider that the deviation in the tau decays originates in exchange of gauge bosons A_3^2 and A_3^1 which interact as $\tau \to A_3^2 + \mu$ and $\tau \to A_3^1 + e$, respectively, as shown in Fig.1.

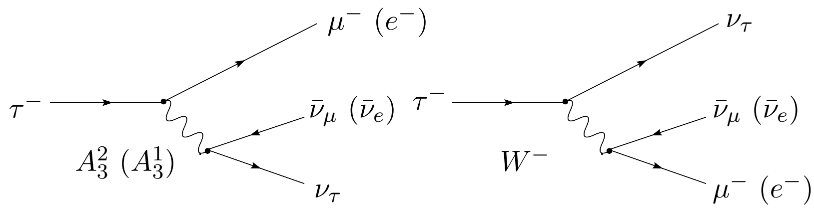

Figure 1: Deviation from e-μ universality in tau decays

Here, let us notice that the observed ratio defined by Eq.(1) shows $R_{amp} > 1$, i.e. $\delta_\mu > \delta_e$. Since the deviations are considered as $\delta_i \sim g_F^2/M_{3i}^2$ $(i = 1, 2)$, this suggests that the mass of A_3^1 is larger than that of A_3^2, i.e. $M_{31}^2 > M_{32}^2$, where $M_{ij} \equiv m(A_i^j)$. This suggests that the deviation (1) is caused by family gauge bosons with an inverted mass hierarchy. (If the gauge boson masses take a normal mass hierarchy, we will obtain $\delta_\tau \simeq 0$ because the gauge boson A_3^3 will take the highest mass.) If it is true, the phenomenological aspect for family gauge bosons will be changed drastically: (i) A family gauge boson with the highest mass is A_1^1, so that it is in favor of a relaxation of the severe constraint from the observed K^0-$\bar{K}^0$ mixing. (ii) The lightest family gauge boson A_3^3 interacts with only quarks and leptons of the third generation, so that the lightest gauge boson search has to be done by $X \to \tau^+\tau^-$, not by $X \to e^+e^-$. (The constraint from $Z' \to \tau^+\tau^-$ search at the Tevatron [4] cannot be apply to this A_3^3 search, because the production rate of A_3^3 is much smaller than that of the conventional Z' boson.) (iii) A large deviation from the μ-τ universality may be seen in the upsilon decays. We consider that it is important to investigate such a possibility phenomenologically.

Such a family gauge symmetry model with an inverted mass hierarchy has recently been proposed by Yamashita and the author [5]. In the present paper, we investigate the possibility on the basis of this model, because mass ratios M_{ij}/M_{33} and gauge coupling constant g_F are fixed in the model as we give a brief review in the next section.

In Sec.3, we estimate the lightest gauge boson mass M_{33} from the tau decay data (1) and also from the upsilon decay data. Regrettably, at present, we cannot obtain a conclusive value of M_{33} because of the large errors. Usually, a sever constraint is obtained from the observed K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixing data. In Sec.4, we discuss the K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixings which are caused though quark-family mixings $U_d \neq \mathbf{1}$ and $U_u \neq \mathbf{1}$. Although the constraint become mild, it is still sever if $(U_d)_{21}$ and $(U_u)_{21}$ are sizable. In Sec.5, we discuss a minimum reversion of the model which gives $M_{ii}^2 = k m_{ei}^2$. Sec.6 is devoted to concluding remarks.

2. Family gauge boson model with an inverted mass hierarchy

The family gauge boson model with an inverted mass hierarchy has been proposed stimulated by the Sumino model [6]. However, the Sumino model has the following problems: (i) The model is not anomaly free because the charged leptons are assigned as $(e_L, e_R) = (\mathbf{3}, \mathbf{3}^*)$ of a U(3)$_{fam}$ gauge symmetry (this assignment is inevitable in order to the so-called Sumino's cancellation mechanism [6]); (ii) Effective current-current interactions with $\Delta N_{fam} = 2$ appear because of the $(e_L, e_R) = (\mathbf{3}, \mathbf{3}^*)$ assignment; (iii) The Sumino's cancellation mechanism cannot be applied to a SUSY model, because the vertex type diagram does not work in a SUSY model.

Therefore, in order to evade the above problems, in the revised model [5], we assign the $U(3)_{fam}$ quantum numbers as $(e_L, e_R) = (\mathbf{3}, \mathbf{3})$, so that the model is anomaly free, and the $\Delta N_{fam} = 2$ interactions do not appear at tree level. On the other hand, in order to realize the cancellation mechanism, we must consider that masses M_{ij} of the gauge bosons A_i^j are given as follows:

$$m^2(A_i^j) \equiv M_{ij}^2 = k\left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}}\right), \tag{3}$$

differently from those in the Sumino model, $M_{ij}^2 = k(m_{ei} + m_{ej})$, where m_{ei} are charged lepton masses. (Note that $\log M_{ii}^2 = +\log m_{ei} + \log 2k$ in the Sumino model, while $\log M_{ii}^2 = -\log m_{ei} + \log 2k$ in our model).

As well as the Sumino model, the family gauge coupling constant g_F in our model is not a free parameter because the cancellation mechanism:

$$g_F^2 = \frac{3}{2}\zeta\ e^2 = \frac{3}{2}\zeta\ g_W^2 \sin^2\theta_W, \tag{4}$$

where g_W is the weak gauge coupling constant given by $G_F/\sqrt{2} = g_W^2/8M_W^2$, and ζ is a fine tuning parameter. In our model, the parameter ζ is numerically given by $\zeta = 1.752$ ($\zeta \simeq 7/4$) [5]. (Hereafter, in numerical estimates of g_F, we will use input values $\zeta = 7/4$ and $\sin^2\theta_W = 0.223$.) Only a free parameter in the model is the magnitude of M_{33} because the ratios M_{ij}/M_{33} are fixed by the relation (3): $M_{33} : M_{23} : M_{22} : M_{13} : M_{12} : M_{11} = 1 : 2.98 : 4.10 : 41.70 : 41.80 : 58.97$.

The family gauge boson interactions are given by

$$H_{fam} = g_F(\bar{e}^i\gamma_\mu e_j)(A^\mu)_j^i, \tag{5}$$

because the U(3) triplet assignment for charged leptons is given by $(e_L, e_R) = (\mathbf{3}, \mathbf{3})$ which gives anomaly free configuration. Note that the interaction type is pure vector differently from that in the Sumino model, in which the currents have been given by $(V \pm A)$. (For example, a decay $B_s^0 \to \tau^- + \mu^+$ via an exchange of family gauge boson A_3^2 is forbidden.)

Note that the family gauge bosons are in the mass-eigenstates on the flavor basis in which the charged lepton mass matrix is diagonal. In this model, a lepton number violating process never occurs at the tree level of the current-current interaction in the charged leptons. As we discuss in Sec.4, since quarks are not in the mass-eigenstates on the diagonal basis of the charged lepton mass matrix, family number changing interactions appear in the quark-quark and quark-lepton interactions. For example, the K^0-$\bar{K}^0$ mixing is cased only through the quark mixings. The μ-e conversion $\mu^- + N \to e^- + N$ is also caused through the quark mixings.

3. Mass of the lightest gauge boson

First, on the basis of the model with the gauge boson masses (3), we investigate a possible deviation from the e-μ universality in the tau decays, because the processes are pure leptonic, so that they are not affected by quark family mixing. (Although the estimate was already discussed in Ref.[5], the purpose was only to estimate an order of the energy scale roughly, and the relation (5) was not used.) In the present model, the deviation from the e-μ universality is characterized by the parameters

$$\delta_i^0 = \frac{g_F^2/M_{3i}^2}{g_W^2/8M_W^2}, \tag{6}$$

where $i = 1, 2$ (i.e. $i = e, \mu$) in the tau decays. Since $(M_{32}/M_{31})^2 = 0.00508$ from the relation (3), we neglect the contribution δ_e^0 compared with the contribution δ_μ^0 hereafter. Since the interactions (5) with the family gauge bosons are pure vector, our parameter δ_μ^0 does not directly mean the observed δ_μ. The effective four Fermi interaction for $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ is given by

$$H^{eff} = \frac{G_F}{\sqrt{2}} \left\{ [\bar{\mu}\gamma_\rho(1-\gamma_5)\nu_\mu][\bar{\nu}_\tau \gamma^\rho (1-\gamma_5)\tau] + \delta_\mu^0 (\bar{\nu}_{L\tau}\gamma_\rho \nu_{L\mu})(\bar{\mu}\gamma^\rho \tau) \right\}, \tag{7}$$

where we have dropped the term $(\bar{\nu}_{R\tau}\gamma_\rho \nu_{R\mu})$ because ν_R have large Majorana masses. By using Fierz transformation, we can express Eq.(7) as

$$H^{eff} = 4\frac{G_F}{\sqrt{2}} \left\{ \left(1 + \frac{1}{4}\delta_\mu^0\right) (\bar{\mu}_L \gamma_\rho \nu_{L\mu})(\bar{\nu}_{L\tau}\gamma^\rho \tau_L) - \frac{1}{2}\delta_\mu^0 (\bar{\mu}_R \nu_{L\mu})(\bar{\nu}_{L\tau}\tau_R) \right\}. \tag{8}$$

Therefore, the observed δ_μ is related to our parameter δ_μ^0 as follows:

$$\delta_\mu = \frac{1}{2}\left(1 - 2x_\mu \frac{g(x_\mu)}{f(x_\mu)}\right)\delta_\mu^0, \tag{9}$$

where $g(x) = 1 + 9x^2 - 9x^4 - x^6 + 6x^2(1+x^2)\log x^2$ and $x_\mu = m_\mu/m_\tau$. Here, we have neglected higher terms of δ_μ^0. (For more details, for example, see Ref.[7].) The predicted value of δ_μ is illustrated in Fig.2. Here, we have illustrated a curve of δ_μ versus M_{33} (not δ_μ versus M_{32}), because our interest is in the lightest gauge boson mass M_{33}. Present data $\delta \equiv \delta_\mu - \delta_e = (2.0 \pm 1.6) \times 10^{-2}$ give the lightest family gauge boson mass

$$M_{33} = 0.87^{+1.07}_{-0.22} \text{ TeV}. \tag{10}$$

However, at present, the numerical result (10) should not be taken rigidly, because, for example, if we change the input value δ from the center value $\delta = 0.0020$ to $\delta = (0.0020 - 1.25\sigma(\delta))$ ($\sigma(\delta) = 0.0016$), the predicted value of M_{33} will become $M_{33} \to \infty$.

At present we have another data of deviations from the e-μ-τ, i.e. the deviations from the e-μ-τ universality, i.e. data of upsilon decays $\Upsilon(1S) \to \ell^+\ell^-$ ($\ell = e, \mu, \tau$). For the time being, we neglect family mixing among quark families. Then, the $b\bar{b}$ sector couples only to the family gauge boson A_3^3 in addition to the standard gauge bosons (photon and Z boson) as seen in Fig.3. Present experimental data [2] $Br(\Upsilon(1S) \to \tau^+\tau^-) = (2.60 \pm 0.10)\%$, $Br(\Upsilon(1S) \to \mu^+\mu^-) =$

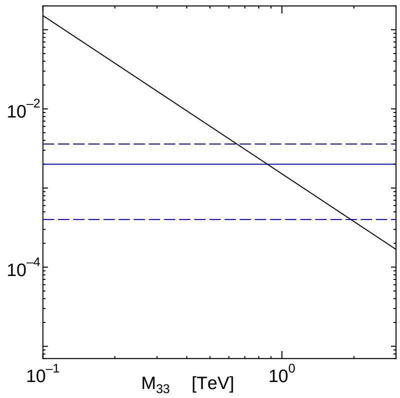

Figure 2: Deviation from e-μ universality in the tau decay versus the lightest family gauge boson mass M_{33}. The horizontal lines denote the present experimental value $\delta \equiv \delta_\mu - \delta_e = (2.0 \pm 1.6) \times 10^{-2}$.

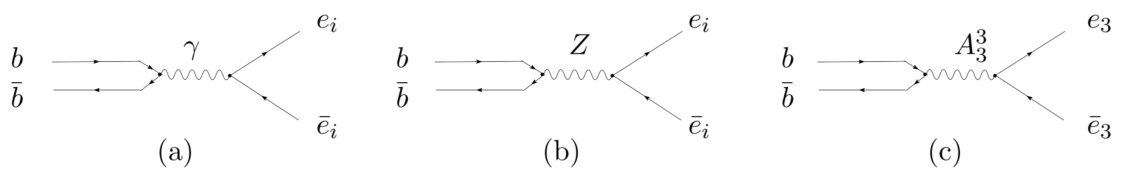

Figure 3: Deviation from e-μ-τ universality in upsilon decay

$(2.48 \pm 0.05)\%$, and $Br(\Upsilon(1S) \to e^+e^-) = (2.38 \pm 0.11)\%$ gives $R_{Br} \equiv Br(\Upsilon \to \tau^+\tau^-)/Br(\Upsilon \to \mu^+\mu^-) = 1.048 \pm 0.046$, which leads to

$$R_{amp} = 1 + \delta_{\tau/\mu} = 1.028 \pm 0.022, \tag{11}$$

where R_{amp} has been defined by $R_{amp} \equiv \sqrt{R_{Br}/R_{kine}}$,

$$R_{kine}^{\tau/\mu} = \frac{1 + 2\frac{m_\tau^2}{M^2}}{1 + 2\frac{m_\mu^2}{M^2}} \sqrt{\frac{1 - 4\frac{m_\tau^2}{M^2}}{1 - 4\frac{m_\mu^2}{M^2}}}. \tag{12}$$

Also, we obtain $R_{amp}^{\mu/e} = 1 + \delta_{\mu/e} = 1.021 \pm 0.051$. However, hereafter, we will not utilize the data on $Br(\Upsilon(1S) \to e^+e^-)$ because of its large error. Since the contributions from photon, Z boson, and A_3^3 boson, are characterized by $1/q^2$, $1/(q^2 - M_Z^2)$ and $1/(q^2 - M_{33}^2)$ with $q^2 = M_\Upsilon^2$, respectively, the sign of the deviation δ_τ has to be negative considering naively, while the observed result (11) has denoted that it is positive. Therefore, we assume that quark fields are assigned as $(q_L, q_R) \sim (\mathbf{3}^*, \mathbf{3}^*)$ of the U(3) family symmetry, differently from that in the charged lepton sector, $(e_L, e_R) \sim (\mathbf{3}, \mathbf{3})$. [The model is still anomaly free in spite of this modification, differently from the Sumino model with $(\mathbf{3}, \mathbf{3}^*)$.] Since we can neglect the Z boson contribution compared with the photon contribution, the deviation parameter δ_τ is given

$$\delta_\tau = \frac{g_F^2}{e^2/3} \frac{M_\Upsilon^2}{M_{33}^2}, \tag{13}$$

where the factor 1/3 has originated in the electric charge of b quark. The predicted value of δ_τ versus M_{33} is illustrated in Fig.3. The observed deviation $\delta_\tau = 0.028 \pm 0.022$ gives $M_{33} = (112^{+130}_{-26})$ GeV. This value is considerably small compared with the result (10) from the tau decay data. However, the upper bound of M_{33} is sensitive to the input value of δ_τ. If we take a slightly smaller value of δ_τ, $\delta_\tau = \delta_\tau^{center} - 1.3\sigma(\delta_\tau)$, the experimental upper value of M_{33} will become infinity.

Although we cannot obtain a conclusive value of M_{33} after all, it should be noted that those results show that the determination of M_{33} is within our reach. We hope to obtain more accurate data on the deviation from e-μ-τ universality in near future.

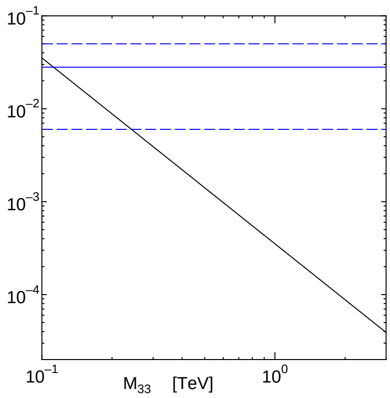

Figure 4: Deviation δ_τ from the μ-τ universality in the upsilon decays versus the lightest family gauge boson mass M_{33}. The horizontal lines denote the present experimental value $\delta_\tau = 0.028 \pm 0.022$.

4. Family-number violating processes due to quark mixing

So far, we have not discussed family mixing in the quark sectors. In the present model, the family number is defined by a flavor basis in which the charged lepton mass matrix M_e is diagonal, while, in general, quark mass matrices M_u and M_d are not diagonal in this basis. When we denote quarks in the mass eigenstates as $u = (u, c, t)$ and $d = (d, s, b)$, and those in the family eigenstates as $u^0 = (u_1^0, u_2^0, u_3^0)$ and $d^0 = (d_1^0, d_2^0, d_3^0)$, the family mixing matrices are defined as $q_L^0 = U_L^q q_L$ (and also $L \to R$) $(q = u, d)$. Quark mass matrices M_q are diagonalized as $(U_L^q)^\dagger M_q U_R^q = D_q$, and the quark mixing matrix V_{CKM} [8] is given by $V_{CKM} = (U_L^u)^\dagger U_L^d$. [Since $(\nu_i, e_i^-)_L$ are doublets in SU(2)$_L$, we can regard the eigenstates of the family symmetry as the eigenstates of weak interactions.] Since we know $V_{CKM} \neq \mathbf{1}$, we cannot take the mixing matrices U_L^u and U_L^d as $U_L^u = \mathbf{1}$ and $U_L^d = \mathbf{1}$ simultaneously. Under this definition of the mixing matrices, the family gauge bosons interact with quarks as follows:

$$H_{fam} = g_F \sum_{q=u,d} (\bar{q^0}^i \gamma_\mu q_j^0)(A^\mu)_i^j = g_F \sum_{q=u,d} (A_\mu)_i^j \left[(U_L^{q*})_{ik} (U_L^q)_{jl} (\bar{q}_{Lk}\gamma^\mu q_{Ll}) + (L \to R) \right]. \quad (14)$$

In the investigation of the upsilon decays in Sec.3, we may consider that b-s mixing (i.e. U_{31}^d and U_{32}^d) is highly suppressed, considering the observed CKM mixing $|V_{ub}| \sim 10^{-3}$ and $|V_{cb}| \sim 10^{-2}$. If the mixing is sizable, we would observe a decay $\Upsilon \to \mu^\pm \tau^\mp$ (the data [2] show $Br(\Upsilon \to \mu^\pm \tau^\mp) < 6.0 \times 10^{-6}$). We can consider that the estimate in Eq.(11) with neglecting the b-s-d mixing is reasonable.

The greatest interest to us is whether we can take a lower value of M_{33} without contradicting the constraint from the observed K^0-$\bar{K}^0$ mixing. The K^0-$\bar{K}^0$ mixing is caused by A_1^1, A_2^2 and A_3^3 exchanges only when the down-quark mixing $U_{L/R}^d \neq \mathbf{1}$ exists:

$$H^{eff} = g_F^2 \left[\frac{1}{M_{33}^2} (U_{31}^{d*} U_{32}^d)^2 + \frac{1}{M_{22}^2} (U_{21}^{d*} U_{22}^d)^2 + \frac{1}{M_{11}^2} (U_{11}^{d*} U_{12}^d)^2 \right] (\bar{s}\gamma_\mu d)(\bar{s}\gamma^\mu d) + h.c., \quad (15)$$

where, for simplicity, we have taken $U_L^d = U_R^d$. If we assume the vacuum-insertion approximation, we obtain

$$\Delta m_K^{fam} = \left[(U_{31}^{d*} U_{32}^d)^2 + (U_{21}^{d*} U_{22}^d)^2 \times 5.95 \times 10^{-2} + (U_{11}^{d*} U_{12}^d)^2 \times 2.88 \times 10^{-4} \right]$$

$$\times 0.7738 \times 10^{-12} / M_{33}^2 \ [\text{TeV}], \quad (16)$$

where the value of M_{33} is taken in a unit of TeV, and we have used values $f_K = 0.1561$ GeV, $m_s(0.5\text{GeV}) = 0.513$ GeV and $m_d(0.5\text{GeV}) = 0.0259$ GeV. On the other hand, the observed value [2] is $\Delta m_K = (4.484 \pm 0.006) \times 10^{-18}$ TeV, and the standard model has a share of $\Delta m_K \sim 2 \times 10^{-18}$ TeV (for example, see [9], and for recent work, for instance, see the second one in Refs.[1]). If we consider $U^d \simeq V_{CKM}$ on trial, Eq.(16) takes the value

$$\Delta m_K^{fam} = [0.949375 \times 10^{-19} + 2.21129 \times 10^{-15} + 1.07122 \times 10^{-17}] / M_{33}^2 \ [\text{TeV}]. \quad (17)$$

The dominant term is the A_2^2 exchange term (the second term). In order to evade this constraint, we have two options: (a) $M_{33} > 10^2$ TeV, or (b) $|U_{21}^d| < 10^{-2}$. (The case (b), $|U_{21}^d| \simeq 0$, does not mean $|U_{12}^d| \simeq 0$. We can build a model with $|U_{21}^d| \simeq 0$ keeping $|U_{12}^d| \simeq |V_{us}|$.) However, even if we assume $U_{21}^d = 0$, the third term (the A_1^1 exchange term) is still in trouble. We have to consider $M_{33} > 10$ TeV for a case $|U_{12}^d| \simeq |V_{us}|$. If we want $M_{33} \simeq 1$ TeV, we must consider $|U_{12}^d| < 10^{-1}$.

The most easy way to evade the constraint from K^0-$\bar{K}^0$ is to assume $U^d \simeq \mathbf{1}$. Then, the constraint from the K^0-$\bar{K}^0$ mixing disappears. However, instead of $U^d = \mathbf{1}$, we must consider $U^u = V_{CKM}^\dagger$. The, we will meet a similar problem on the observed D^0-$\bar{D}^0$ mixing: The D^0-$\bar{D}^0$ mixing gives

$$\Delta m_D^{fam} = \left[(U_{31}^{u*} U_{32}^u)^2 + (U_{21}^{u*} U_{22}^u)^2 \times 5.95 \times 10^{-2} + (U_{11}^{u*} U_{12}^u)^2 \times 2.88 \times 10^{-4} \right]$$

$$\times 0.98974 \times 10^{-11}/M_{33}^2 \text{ [TeV]}, \tag{18}$$

where we have used (center values) $f_D = 0.2067$ GeV, $m_c(m_c) = 1.275$ GeV and $m_u(2\text{GeV}) = 0.0023$ GeV. If we take $U^u = V_{CKM}^\dagger$, the value (18) is given as

$$\Delta m_D^{fam} = [4.3287 \times 10^{-19} + 2.8337 \times 10^{-14} + 1.37722 \times 10^{-16}]/M_{33}^2 \text{ [TeV]}. \tag{19}$$

On the other hand, the present observed value [2] is $\Delta m_D^{obs} = (8.38^{+2.8}_{-2.9}) \times 10^{-18}$ TeV. The second term in Eq.(19) is again incompatible with an idea $M_{33} \sim 1$ TeV, so that we must assume $|U_{21}^u| < 10^{-2}$.

On the other hand, the model can clear the constraints from the observed values [2] of $\Delta m_{B_d}^{obs}$ and $\Delta m_{B_s}^{obs}$, even for the case $U^d = V_{CKM}$.

Concerning the magnitude of U_{21}^d, we are interested in the μ-e conversion. The effective Hamiltonian is given by

$$H_{\mu\to e}^{eff} = \frac{g_F^2}{M_{21}^2}\left[(U_{21}^{u*}U_{11}^u)(\bar{u}\gamma_\rho u) + (U_{21}^{d*}U_{11}^d)(\bar{d}\gamma_\rho d)\right](\bar{e}\gamma^\rho \mu). \tag{20}$$

Estimate of $Br(\mu^- + (A,Z) \to e^- + (A,Z))$ has to be done carefully, because it depends on a structure of the nucleus. Since the purpose of the present paper is to discuss the deviations from the e-μ-τ universality, here, let us roughly give order estimation of the μ-e conversion. (A more careful estimate will be given elsewhere.) The rate $Br(\mu^- + (A,Z) \to e^- + (A,Z))$ is characterized by a factor

$$\left(\frac{g_F^2/M_{21}^2}{g_W^2/8M_W^2}\right)^2 |U_{21}^{q*}U_{11}^q|^2 = |U_{21}^{q*}U_{11}^q|^2 \times \frac{3.00 \times 10^{-10}}{(M_{33}[\text{TeV}])^4}. \tag{21}$$

If we consider too small value of M_{33}, then, even we suppose $|U_{21}^u| < 10^{-2}$, the μ-e conversion should already be found in the past experiments (for a review, for example, see Ref.[10]). We are again interested in a case $M_{33} \sim 1$ TeV when $|U_{21}^d|$ is small, but it takes a non-zero value.

5. Minor change of the model

As seen in the previous section, if we want a model with $M_{33} \sim 1$ TeV, we must suppose $|U_{12}^u| < 10^{-1}$ and $|U_{12}^d| < 10^{-1}$ in addition to the assumption $|U_{21}^u| < 10^{-2}$ and $|U_{21}^d| < 10^{-2}$. However, since we know the observed values $|V_{12}| \simeq |V_{21}| \simeq 0.22$, it is substantially hard to build such a model. One of a way which evades the D^0-$\bar{D}^0$ and D^0-$\bar{D}^0$ problems is to consider that M_{11} and M_{22} are much larger than those given by the relation (3).

The relation (3) is derived as follows [5]: We assume a scalar Φ_i^α which is $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)′ families and whose VEV $\langle\Phi\rangle$ gives the charged lepton mass matrix M_e as $(M_e)_{ij} = k_e\langle\bar{\Phi}_\alpha^i\rangle\langle\Phi_j^\alpha\rangle$ and $\langle\Phi_i^\alpha\rangle \propto \delta_{i\alpha}\sqrt{m_{ei}}$. We also consider another scalar Ψ_i^α whose VEV dominantly gives family gauge boson masses (we have been assume $|\langle\Psi\rangle| \gg |\langle\Phi\rangle|$) and satisfies a relation $\langle\Psi\rangle\langle\Phi\rangle \propto \mathbf{1}$. If we give a minor change of the model in which the scalar Ψ is $(\mathbf{6}^*, 1)$ and the VEV satisfies $\langle\Phi_i^\alpha\rangle\langle\Phi^{ij}\rangle\langle\Phi_j^{T\beta}\rangle = k\delta^{\alpha\beta}$, we can obtain a revised mass relation

$$m^2(A_i^j) \equiv M_{ij}^2 = k\left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}}\right)^2, \tag{22}$$

Then, $M_{33}^2 : M_{22}^2 : M_{11}^2$ is given by $1 : 2.828 \times 10^2 : 1.209 \times 10^7$. Then, the third term problem is cleared, although for the second term we have still to take $U_{21}^q \simeq 0$.

However, in this revised model (Model II), the cancellation condition of the factor $\log m_{ei}^2$ is given by

$$\varepsilon_i = \log \frac{m_i^2}{m_3^2} + \zeta_I \log \left(\frac{M_{i1}^2 M_{i2}^2 M_{i3}^2}{M_{31}^2 M_{32}^2 M_{33}^2} \right)_I = \log \frac{m_i^2}{m_3^2} + \zeta_{II} \log \left(\frac{M_{i1}^2 M_{i2}^2 M_{i3}^2}{M_{31}^2 M_{32}^2 M_{33}^2} \right)_{II}, \tag{23}$$

with $\varepsilon_i = 0$. Here, in the second term, only m_{ei}-dependent part is extracted. Here and hereafter, we denote with a suffix "II" for quantities in Model II, and with a suffix "I" for those in the original model (Model I). Since the gauge boson masses satisfy the relation

$$\left(\frac{M_{ij}}{M_{33}} \right)_{II}^2 = \left(\frac{M_{ij}}{M_{33}} \right)_I^4, \tag{24}$$

the ζ parameter defined by Eq.(4) satisfies

$$\zeta_{II} = \frac{1}{2} \zeta_I, \tag{25}$$

By these modifications for g_F^2 and M_{ij}/M_{33}, we obtain a revised value of M_{33},

$$(M_{33})_{II} = 0.206^{+1.07}_{-0.22} \text{ TeV}, \tag{26}$$

from the observed deviation (2) in the tau decays, and $(M_{33})_{II} = 79^{+92}_{-18}$ GeV from the observed deviation (11) in the $\Upsilon(1S)$ decays. Such small values of M_{33} cannot be ruled out from the current lower bound [4] by the $X \to \tau^+\tau^-$ search at the Tevatron, because the production rate of A_3^3 is much smaller than that of the conventional Z' boson. It is also attractive that two values of $(M_{33})_{II}$ roughly agree.

However, these results $M_{33} \sim 10^{-1}$ TeV are too low contrary to our expectation. If we adopt such low values, the third term in Eq.(17) [and also in Eq.(19)] will become incompatible with the observed value of Δm_K [and Δm_D]. Even in Model II, we again speculate $M_{33} \sim 1$ TeV, which is not ruled out from the result (26). Then, we can clear the constraints from the observed values Δm_K and Δm_D by assuming $|U_{21}^d| < 10^{-2}$ and $|U_{21}^u| < 10^{-2}$.

6. Concluding remarks

In conclusion, as far as we notice the data on the deviations from the e-μ-τ universality, it looks that the data suggest family gauge bosons with an inverted mass hierarchy. Although we cannot obtain a conclusive value of M_{33} after all, we would like to emphasize that those results show that the determination of M_{33} is within our reach.

In the present model, the gauge bosons A_i^j are in the mass-eigenstates on the family basis in which the charged lepton mass matrix is diagonal, while the quarks are, in general, not in the mass-eigenstates, so that family-mixings $U^u \neq \mathbf{1}$ and $U^d \neq \mathbf{1}$ appear. We have also investigated K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixings. Even by taking the observed values of Δm_K and Δm_D into consideration, we find that we can take $M_{33} \sim 1$ TeV as the lightest family gauge boson mass, if we adopt a specific model with $|U^d_{21}| < 10^{-2}$ and $|U^u_{21}| < 10^{-2}$ (but with $|U^d_{12}| \simeq |U^u_{12}| \simeq |V_{us}|$).

We expect a direct search for A^3_3, for example, at the LHC. For the details of the direct search for the lightest family gauge boson A^3_3 at the LHC, we shall report elsewhere. Very recently, an interesting decay mode via the family changing neutral gauge boson has been pointed out [11]. Thus, a family gauge boson model with the inverted mass hierarchy will offer to us fruitful new physics in TeV and sub-TeV regions experimentally and theoretically. Further studies are our future tasks.

Acknowledgments

A part of the present work was done at NTU, Taiwan. The author would like to thank members of the particle physics group at NTU and NTSC for the helpful discussions and their cordial hospitality, and, especially, X.G. He for his interests in this work and inviting the author to NTU. The author also thank T. Yamashita for his valuable and helpful conversations. He also thank Y. Kuno and J. Sato for helpful comments on the μ-e conversion. This work is supported by JSPS (No. 21540266).

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