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Deviations from the e-μ-τ Universality and Family Gauge Bosons with a Visible Energy Scale

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Abstract

From the observed deviations from the e-μ-τ universality, an existence of family gauge bosons with a visible energy scale is speculated. The data suggest that family gauge bosons have an inverted mass hierarchy and the lightest gauge boson mass is $M_{33} \sim 1$ TeV, although the data, at present, have too large errors to obtain a conclusive result. Consistency of such a speculation with other phenomenon is discussed.

1. Introduction

We know three generations of quarks and leptons. It seems to be natural to regard those as triplets of a family symmetry SU(3) or U(3). However, so far, one has considered that, even if the family gauge symmetry exists, it is impossible to observe such gauge boson effects, because we know a severe constraint from the observed K^0-$\bar{K}^0$ mixing [1] and results from Z' search [2] at the Tevatron. Nevertheless, it is interesting to consider a possibility that a family gauge symmetry really exists and the family gauge bosons are visible at a lower energy scale. In this paper, we pay attention to deviations from the e-μ-τ universality, i.e. in the tau decays and in the upsilon decays.

From the present observed branching ratios [1] $Br(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = (17.41 \pm 0.04)\%$ and $Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = (17.83 \pm 0.04)\%$, we obtain the ratio $R_{Br} \equiv Br(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)/Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = 0.97644 \pm 0.00314$. For convenience, we define parameters δ_μ and δ_e which are measures of a deviation from the e-μ universality as follows:

$$R_{amp} \equiv \frac{1+\delta_\mu}{1+\delta_e} = \sqrt{R_{Br} \frac{f(m_e/m_\tau)}{f(m_\mu/m_\tau)}} = 1.0020 \pm 0.0016, \tag{1}$$

where $f(x)$ is known as the phase space function and it is given by $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 12x^4 \log x^2$. Then, the result (1) gives

$$\delta \equiv \delta_\mu - \delta_e = 0.0020 \pm 0.0016. \tag{2}$$

[The values of the deviation parameters δ_μ and δ_e depend on types of the gauge boson interactions, i.e. $(V - A)$, pure V, and so on. We will discuss corrections for the parameters δ_μ and δ_e which have been defined by Eq.(1).]

On the other hand, present experimental data [1] $Br(\Upsilon(1S) \to \tau^+\tau^-) = (2.60 \pm 0.10)\%$, $Br(\Upsilon(1S) \to \mu^+\mu^-) = (2.48 \pm 0.05)\%$, and $Br(\Upsilon(1S) \to e^+e^-) = (2.38 \pm 0.11)\%$ gives $R_{Br} \equiv Br(\Upsilon \to \tau^+\tau^-)/Br(\Upsilon \to \mu^+\mu^-) = 1.048 \pm 0.046$, which leads to

$$R_{amp} = 1 + \delta_{\tau/\mu} = 1.028 \pm 0.022, \tag{3}$$

where R_{amp} has been defined by $R_{amp} \equiv \sqrt{R_{Br}/R_{kine}}$,

$$R_{kine} = \frac{1 + 2\frac{m_\tau^2}{M^2}}{1 + 2\frac{m_\mu^2}{M^2}} \sqrt{\frac{1 - 4\frac{m_\tau^2}{M^2}}{1 - 4\frac{m_\mu^2}{M^2}}}. \tag{4}$$

Also, we obtain $R_{amp} = 1 + \delta_{\mu/e} = 1.021 \pm 0.051$. However, hereafter, we will not utilize the data on $Br(\Upsilon(1S) \to e^+e^-)$ because of its large error.

Of course, from those values (1) and (3) of the deviation parameters, we cannot conclude that we found a significant difference of the deviations from the e-μ-τ universality. However, we may speculate a possibility of family gauge bosons. We can consider that the deviation in the tau decays originates in exchange of gauge bosons A_3^2 and A_3^1 which interact as $\tau \to A_3^2 + \mu$ and $\tau \to A_3^1 + e$, respectively, as shown in Fig.1. Also, we can consider that the deviation (3) in the upsilon decays originates in an exchange of A_3^3 as shown in Fig.2.

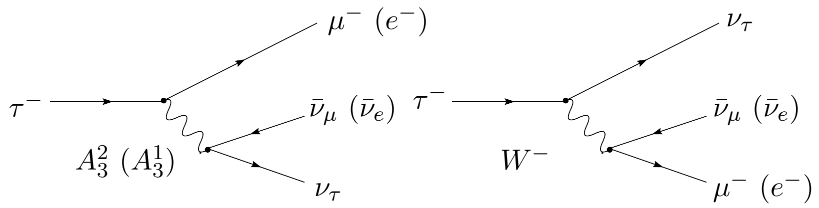

Figure 1: Deviation from e-μ universality in tau decays

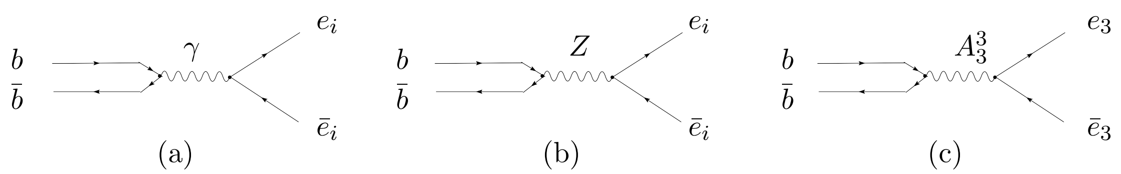

Figure 2: Deviation from e-μ-τ universality in upsilon decay

Here, let us notice that the observed ratio defined by Eq.(1) shows $R_{amp} > 1$, i.e. $\delta_\mu > \delta_e$. Since the deviations are considered as $\delta_i \sim g_F^2/M_{3i}^2$ $(i = 1, 2)$, this suggests that the mass of A_3^1 is larger than that of A_3^2, i.e. $M_{32}^2 < M_{31}^2$, where $M_{ij} \equiv m(A_i^j)$. This suggests that the deviation (1) is caused by family gauge bosons with an inverted mass hierarchy. The idea that family gauge bosons have an inverted mass hierarchy has some advantages in the phenomenological aspect: (i) It is in favor of the large deviation which has been seen in the upsilon decays as seen in Eq.(3). (If the gauge boson masses take a normal mass hierarchy, we will obtain $\delta_\tau \simeq 0$ because the gauge boson A_3^3 will take the highest mass.) (ii) A family gauge boson with the highest mass is A_1^1, so that it is in favor of a relaxation of the severe constraint from the observed

K^0-$\bar{K}^0$ mixing. (iii) The lightest family gauge boson A_3^3 interacts with only quarks and leptons of the third generation, so that the lightest gauge boson search has to be done by $X \to \tau^+\tau^-$, not by $X \to e^+e^-$. (The constraint from $Z' \to \tau^+\tau^-$ search at the Tevatron [3] cannot be apply to this A_3^3 search, because the production rate of A_3^3 is much smaller than that of the conventional Z' boson.)

Such a family gauge symmetry model with an inverted mass hierarchy has recently proposed by Yamashita and the author [4]. In the model, masses M_{ij} of the gauge bosons A_i^j are given as follows:

$$m^2(A_i^j) \equiv M_{ij}^2 = k\left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}}\right), \tag{5}$$

where m_{ei} are charged lepton masses. In the next section, we give a brief review of the family gauge boson model with an inverted mass hierarchy.

2. Family gauge boson model with an inverted mass hierarchy

The model was proposed, stimulated by the Sumino mechanism [5]. Therefore, first, let us give a brief review of the Sumino mechanism. Sumino has seriously taken why the charged lepton mass formula [6] $K \equiv (m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$ is so remarkably satisfied with the pole masses: $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$, while if we take the running masses, the ratio becomes $K(\mu) = (2/3) \times (1.00189 \pm 0.000002)$, for example, at $\mu = m_Z$. The deviation comes from the QED radiative correction [7]

$$\delta m_{ei} = -\frac{\alpha_{em}(\mu)}{\pi} m_{ei} \left(1 + \frac{3}{4} \log \frac{\mu^2}{m_{ei}^2}\right). \tag{6}$$

Note that if we can make $\varepsilon_i = 0$ in the transformation $m_{ei} \to m_{ei}(1 + \varepsilon_0 + \varepsilon_i)$, where ε_i is family number dependent term [i.e. $\log m_{ei}^2$ in Eq.(6)] and ε_0 is family number independent term, then the value of the ratio K can be invariant under this transformation. Therefore, Sumino has proposed an idea that the factor $\log m_{ei}^2$ in Eq.(6) is canceled by a contribution from family gauge bosons. In order to work the Sumino mechanism correctly, the following conditions are essential: (i) The left- and right-handed charged leptons (e_L, e_R) have to be assigned to $(\mathbf{3}, \mathbf{3}^*)$ of the U(3) family symmetry, respectively. (ii) Masses of the gauge bosons are given by $M_{ij}^2 = k(m_{ei} + m_{ej})$. Thus, the factor $\alpha_{em} \log m_{ei}^2$ due to the photon is canceled by a part of $-\alpha_F \log M_{ii}^2 = -\alpha_F(\frac{1}{2} \log m_{ei}^2 + \log k)$ due to the family gauge bosons, where $\alpha_F = g_F^2/4\pi$. However, the Sumino model has the following problems: (i) The model is not anomaly free because of the $(\mathbf{3}, \mathbf{3}^*)$ assignment; (ii) Effective current-current interactions with $\Delta N_f = 2$ appear; (iii) The cancellation mechanism cannot be applied to a SUSY model, because the vertex type diagram does not work in a SUSY model.

For the purpose of evading such problems in the Sumino model, an alternative model [4] with the inverted mass hierarchy (5) was proposed. In the model, the charged lepton fields are assigned to $(e_L, e_R) \sim (\mathbf{3}, \mathbf{3})$ of the U(3) family symmetry. Therefore, the factor $\alpha_{em} \log m_{ei}^2$ due to the photon is canceled by $+\alpha_F \log M_{ii}^2 = +\alpha_F(-\frac{1}{2} \log m_{ei}^2 + \log k)$ due to the family gauge bosons. In the present model as well as the Sumino model, the family gauge coupling constant

g_F is not a free parameter, because of the cancellation condition between QED radiative diagram and the family gauge boson contribution. The cancellation condition is different from that in the Sumino model:

$$g_F^2 = \frac{3}{2}\zeta\ e^2 = \frac{3}{2}\zeta\ g_W^2 \sin^2\theta_W, \tag{7}$$

where g_W is the weak gauge coupling constant given by $G_F/\sqrt{2} = g_W^2/8M_W^2$, and ζ is a fine tuning parameter which is numerically given by $\zeta = 1.752$ ($\zeta \simeq 7/4$). (Hereafter, in numerical estimates of g_F, we will use input values $\zeta = 7/4$ and $\sin^2\theta_W = 0.223$.) Only a free parameter in the model is the magnitude of M_{33} because the ratios M_{ij}/M_{33} are fixed by the relation (5): $M_{33} : M_{23} : M_{22} : M_{13} : M_{12} : M_{11} = 1 : 2.98 : 4.10 : 41.70 : 41.80 : 58.97$. The family gauge boson interactions are given by

$$H_{fam} = g_F(\bar{e}^i\gamma_\mu e_j)(A^\mu)_j^i. \tag{8}$$

Note that the interaction type is pure vector differently from that in the Sumino model.

Note that the family gauge bosons are in the mass-eigenstates on the flavor basis in which the charged lepton mass matrix is diagonal. In this model, a lepton number violating process never occurs at the tree level of the current-current interaction in the charged leptons. (As we discuss in Sec.4, since quarks are not in the mass-eigenstates on the diagonal basis of the charged lepton mass matrix, family number changing interactions appear in the quark-quark and quark-lepton interactions. For example, the K^0-$\bar{K}^0$ mixing is cased only through the quark mixings. The μ-e conversion $\mu^- + N \to e^- + N$ is also caused through the quark mixings.) In this paper, we pay attention to deviations from the e-μ-τ universality although they are family number conserving processes.

3. Mass of the lightest gauge boson

First, on the basis of the model with the gauge boson masses (5), we investigate a possible deviation from the e-μ universality in the tau decays, because the processes are pure leptonic, so that they are not effected by quark family mixing. (Although the estimate was already discussed in Ref.[4], the purpose was only to estimate an order of the energy scale roughly, and the relation (7) was not used.) In the present model, the deviation from the e-μ universality is characterized by the parameters

$$\delta_i^0 = \frac{g_F^2/M_{3i}^2}{g_W^2/8M_W^2}, \tag{9}$$

where $i = 1, 2$ (i.e. $i = e, \mu$) in the tau decays. Since $(M_{32}/M_{31})^2 = 0.00508$ from the relation (5), we neglect the contribution δ_e^0 compared with the contribution δ_μ^0 hereafter. Since the interactions (8) with the family gauge bosons are pure vector, our parameter δ_μ^0 does not directly mean the observed δ_μ. The effective four Fermi interaction for $\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$ is given by

$$H^{eff} = \frac{G_F}{\sqrt{2}}\left\{[\bar{\mu}\gamma_\rho(1-\gamma_5)\nu_\mu][\bar{\nu}_\tau\gamma^\rho(1-\gamma_5)\tau] + \delta_\mu^0(\bar{\nu}_{L\tau}\gamma_\rho\nu_{L\mu})(\bar{\mu}\gamma^\rho\tau)\right\}, \tag{10}$$

where we have dropped the term $(\bar{\nu}_{R\tau}\gamma_\rho \nu_{R\mu})$ because ν_R have large Majorana masses. By using Fierz transformation, we can express Eq.(10) as

$$H^{eff} = 4\frac{G_F}{\sqrt{2}} \left\{ \left(1+\frac{1}{4}\delta^0_\mu\right) (\bar{\mu}_L \gamma_\rho \nu_{L\mu})(\bar{\nu}_{L\tau}\gamma^\rho \tau_L) - \frac{1}{2}\delta^0_\mu (\bar{\mu}_R \nu_{L\mu})(\bar{\nu}_{L\tau}\tau_R) \right\}. \tag{11}$$

Therefore, the observed δ_μ is related to our parameter δ^0_μ as follows:

$$\delta_\mu = \frac{1}{2}\left(1 - 2x_\mu \frac{g(x_\mu)}{f(x_\mu)}\right) \delta^0_\mu, \tag{12}$$

where $g(x) = 1 + 9x^2 - 9x^4 - x^6 + 6x^2(1+x^2)\log x^2$, $x_\mu = m_\mu/m_\tau$, and we have neglected higher terms of δ^0_μ. (For more details, for example, see Ref.[8].) The predicted value of δ_μ is illustrated in Fig.1. Here, we have illustrated a curve of δ_μ versus M_{33} (not δ_μ versus M_{32}), because our interest is in the lightest gauge boson mass M_{33}. Present data $\delta \equiv \delta_\mu - \delta_e = (2.0 \pm 1.6) \times 10^{-2}$ give the lightest family gauge boson mass

$$M_{33} = 0.87^{+1.07}_{-0.22} \text{ TeV}. \tag{13}$$

However, at present, the numerical result (13) should not be taken rigidly, because, for example, if we change the input value δ from the center value $\delta = 0.0020$ to $\delta = (0.0020 - 1.25\sigma(\delta))$ $(\sigma(\delta) = 0.0016)$, the predicted value of M_{33} will become $M_{33} \to \infty$.

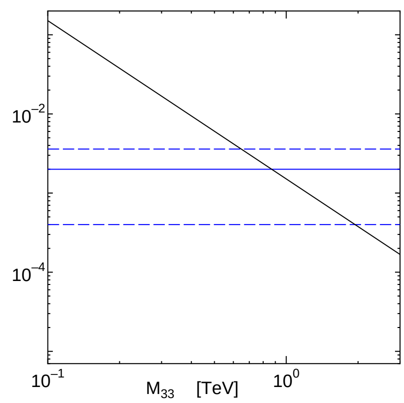

Figure 3: Deviation from e-μ universality in the tau decay versus the lightest family gauge boson mass M_{33}. The horizontal lines denote the present experimental value $\delta \equiv \delta_\mu - \delta_e = (2.0 \pm 1.6) \times 10^{-2}$.

Next, let us investigate deviations from the e-μ-τ universality in the upsilon decays $\Upsilon(1S) \to \ell^+\ell^-$ $(\ell = e, \mu, \tau)$. For the time being, we neglect family mixing among quark families. Then, the $b\bar{b}$ sector couples only to the family gauge boson A^3_3 in addition to the standard gauge bosons (photon and Z boson). Since the contributions from photon, Z boson, and A_3^3 boson, are characterized by $1/q^2$, $1/(q^2 - M_Z^2)$ and $1/(q^2 - M_{33}^2)$ with $q^2 = M_\Upsilon^2$, respectively, the sign of the deviation δ_τ has to be negative considering naively, while the experimental result (3) has denoted that it is positive. Therefore, we assume that quark fields are assigned as $(q_L, q_R) \sim (\mathbf{3}^*, \mathbf{3}^*)$ of the U(3) family symmetry, differently from that in the charged lepton sector, $(e_L, e_R) \sim (\mathbf{3}, \mathbf{3})$. [The model is still anomaly free in spite of this modification, differently from the Sumino model with $(\mathbf{3}, \mathbf{3}^*)$.] Since we can neglect the Z boson contribution compared with the photon contribution, the deviation parameter δ_τ is given

$$\delta_\tau = \frac{g_F^2}{e^2/3} \frac{M_\Upsilon^2}{M_{33}^2}, \tag{14}$$

where the factor 1/3 has originated in the electric charge of b quark. The predicted value of δ_τ versus M_{33} is illustrated in Fig.2. The observed deviation $\delta_\tau = 0.028 \pm 0.022$ gives $M_{33} = (112^{+130}_{-26})$ GeV. This value is considerably small compared with the result (13) from the tau decay data. However, the upper bound of M_{33} is sensitive to the input value of δ_τ. If we take a slightly smaller value of δ_τ, $\delta_\tau = \delta_\tau^{center} - 1.3\sigma(\delta_\tau)$, the experimental upper value of M_{33} will become infinity.

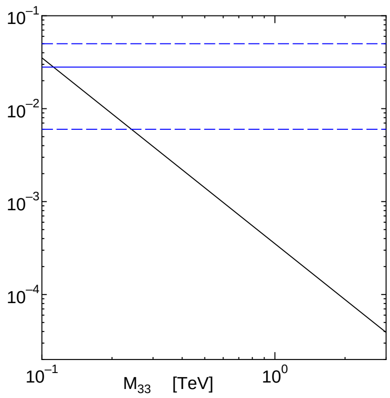

Figure 4: Deviation δ_τ from the μ-τ universality in the upsilon decays versus the lightest family gauge boson mass M_{33}. The horizontal lines denote the present experimental value $\delta_\tau = 0.028 \pm 0.022$.

4. Family-number violating processes due to quark mixing

So far, we have not discussed family mixing in the quark sectors. In the present model, the family number is defined by a flavor basis in which the charged lepton mass matrix M_e is diagonal, while, in general, quark mass matrices M_u and M_d are not diagonal in this basis. When we denote quarks in the mass eigenstates as $u = (u, c, t)$ and $d = (d, s, b)$, and those in the family eigenstates as $u^0 = (u_1^0, u_2^0, u_3^0)$ and $d^0 = (d_1^0, d_2^0, d_3^0)$, the family mixing matrices are defined as $q^0_L = U^q_L q_L$ (and also $L \to R$) ($q = u, d$). Quark mass matrices M_q are diagonalized as $(U^q_L)^\dagger M_q U^q_R = D_q$, and the quark mixing matrix V_{CKM} [9] is given by $V_{CKM} = (U^u_L)^\dagger U^d_L$. [Since $(\nu_i, e^-_i)_L$ are doublets in SU(2)$_L$, we can regard the eigenstates of the family symmetry as the eigenstates of weak interactions.] Since we know $V_{CKM} \neq \mathbf{1}$, we cannot take the mixing matrices U^u_L and U^d_L as $U^u_L = \mathbf{1}$ and $U^d_L = \mathbf{1}$ simultaneously. Under this definition of the mixing matrices, the family gauge bosons interact with quarks as follows:

$$H_{fam} = g_F \sum_{q=u,d} (\bar{q^0}^i \gamma_\mu q^0_j)(A^\mu)^j_i = g_F \sum_{q=u,d} (A_\mu)^j_i \left[(U^{q*}_L)_{ik} (U^q_L)_{jl} (\bar{q}_{Lk} \gamma^\mu q_{Ll}) + (L \to R) \right]. \quad (15)$$

We may consider that b-s mixing (i.e. U^d_{31} and U^d_{32}) is highly suppressed, considering the observed CKM mixing $|V_{ub}| \sim 10^{-3}$ and $|V_{cb}| \sim 10^{-2}$. If the mixing is sizable, we would observe a decay $\Upsilon \to \mu^\pm \tau^\mp$ (the data [1] show $Br(\Upsilon \to \mu^\pm \tau^\mp) < 6.0 \times 10^{-6}$). We consider that the estimate in Eq.(14) with neglecting the b-s-d mixing is reasonable.

The greatest interest to us is whether we can take a lower value of M_{33} without contradicting the constraint from the observed K^0-$\bar{K}^0$ mixing. The K^0-$\bar{K}^0$ mixing is caused by A^1_1, A^2_2 and A^3_3 exchanges only when the down-quark mixing $U^d_{L/R} \neq \mathbf{1}$ exists:

$$H^{eff} = g_F^2 \left[\frac{1}{M^2_{33}} (U^{d*}_{31} U^d_{32})^2 + \frac{1}{M^2_{22}} (U^{d*}_{21} U^d_{22})^2 + \frac{1}{M^2_{11}} (U^{d*}_{11} U^d_{12})^2 \right] (\bar{s}\gamma_\mu d)(\bar{s}\gamma^\mu d) + h.c., \quad (16)$$

where, for simplicity, we have taken $U^d_L = U^d_R$. If we assume the vacuum-insertion approximation, we obtain

$$\Delta m^{fam}_K = \left[(U^{d*}_{31} U^d_{32})^2 + (U^{d*}_{21} U^d_{22})^2 \times 5.95 \times 10^{-2} + (U^{d*}_{11} U^d_{12})^2 \times 2.88 \times 10^{-4} \right] \times \frac{1.291 \times 10^{-11}}{M^2_{33}} \text{ TeV}, \quad (17)$$

where the value of M_{33} is taken in a unit of TeV. On the other hand, the observed value [1] is $\Delta m_K = (4.484 \pm 0.006) \times 10^{-18}$ TeV, and the standard model has a share of $\Delta m_K \sim 2 \times 10^{-18}$ TeV (for example, see [10]). If we consider $U^d \simeq V_{CKM}$, the dominant term is the A^2_2 exchange term (the second term). (Obviously, the first term is negligibly small.) This term gives a contribution of $3.7 \times 10^{-14}/M^2_{33}$ to Δm_K, so that, in order to evade this constraint, we have two options: (a) $M_{33} >$ a few 10^2 TeV, or (b) $|U^d_{21}|^2 < 10^{-5}$. (The case (b), $|U^d_{21}| \simeq 0$, does not mean $|U^d_{12}| \simeq 0$. We can build a model with $|U^d_{21}| \simeq 0$ keeping $|U^d_{12}| \simeq |V_{us}|$.) However, even if we assume $U^d_{21} = 0$, the third term (the A^1_1 exchange term) is still in trouble. We have to consider $M_{33} > 10$ TeV for a case $|U^d_{12}| \simeq |V_{us}|$. If we want $M_{33} \simeq 1$ TeV, we must consider $|U^d_{12}| < 10^{-2}$.

The most easy way to evade the constraint from K^0-$\bar{K}^0$ is to assume $U^d \simeq \mathbf{1}$. However, then, we are obliged to consider $U^u \simeq V^\dagger_{CKM}$, but we will meet a similar problem in the explanation of the observed D^0-$\bar{D}^0$ mixing, although the constraint is somewhat mild compared with that from the K^0-$\bar{K}^0$ mixing. However, the numerical value given in Eq.(17) is dependent on the evaluation method. In obtaining the numerical result (17), we have used the vacuum-insertion approximation, $\langle 0|(\bar{s}\gamma_5 d)|\bar{K}^0\rangle = m_K^2 f_K/(m_s+m_d)$. The result is highly sensitive to the quark mass values (m_s+m_d) used. [The result (17) has been obtained by using the running quark mass values [1] $m_s = 101$ MeV and $m_d = 4.92$ MeV at $\mu = 2$ GeV. The input values give $m_K^2/(m_s+m_d)^2 = 22.1$. If we take the constituent quark masses as $m_K/(m_s+m_d) \simeq 1$, the predicted value will be reduced by 10^{-1}.] We might consider this trouble optimistically as far as the third term is concerned.

Concerning the magnitude of U^d_{21}, we are interested in the μ-e conversion. The effective Hamiltonian is given by

$$H^{eff}_{\mu\to e} = \frac{g_F^2}{M_{21}^2}\left[(U^{u*}_{21}U^u_{11})(\bar{u}\gamma_\rho u) + (U^{d*}_{21}U^d_{11})(\bar{d}\gamma_\rho d)\right](\bar{e}\gamma^\rho\mu). \tag{18}$$

Estimate of $Br(\mu^- + (A,Z) \to e^- + (A,Z))$ has to be done carefully, because it depends on a structure of the nucleus. Since the purpose of the present paper is to discuss the deviations from the e-μ-τ universality, here, let us roughly give order estimation of the μ-e conversion. (A more careful estimate will be given elsewhere.) The rate $Br(\mu^- + (A,Z) \to e^- + (A,Z))$ is characterized by a factor

$$\left(\frac{g_F^2/M_{21}^2}{g_W^2/8M_W^2}\right)^2 |U^{q*}_{21}U^q_{11}|^2 = |U^{q*}_{21}U^q_{11}|^2 \times \frac{3.00\times 10^{-10}}{(M_{33}[\mathrm{TeV}])^4}. \tag{19}$$

If we consider too small value of M_{33}, then, in such a case, the μ-e conversion should already be found in the past experiments (for a review, for example, see Ref.[11]). We are again interested in a case $M_{33} \sim 1$ TeV when $|U^d_{21}|$ is small, but it takes a non-zero value.

5. Concluding remarks

If we consider the constraint from the K^0-$\bar{K}^0$ mixing seriously, we can choose an alternative model in which the gauge boson masses are given by a modified equation, Eq.(5) with $m_{ei} \to m_{ei}^2$. (Such a model with the modified mass relation will be done by a similar way in Ref.[4].) Then, $M_{33}^2 : M_{22}^2 : M_{11}^2$ is given by $1 : 2.828\times 10^2 : 1.209\times 10^7$. Then, the third term problem is cleared, although for the second term we have still to take $U^q_{21} \simeq 0$. However, in this revised model (Model II), the cancellation condition of the factor $\log m_{ei}^2$, Eq.(7), requires the fine turning parameter value ζ as $\zeta = 0.5195$ instead of $\zeta = 1.752$ in the previous model (Model I). (Hereafter, we denote with a suffix "II" for quantities in Model II, and with a suffix "I" for those in Model I, e.g. as $\zeta^{II} = 0.5195$ versus $\zeta^I = 1.752$.) Therefore, the family gauge coupling constant is changed as $(g_F^2)^{II} = (g_F^2)^I\kappa$, where $\kappa = \zeta^{II}/\zeta^I = 0.2965$. By these modifications for g_F^2 and M_{ij}/M_{33}, we obtain revised results for M_{33}: $M_{33}^{II} = 119^{+146}_{-20}$ GeV from the observed deviation (2) in the tau decays, and $M_{33}^{II} = 61^{+71}_{-15}$ GeV from the observed deviation (3) in the $\Upsilon(1S)$ decays. However, these results are too low contrary to our expectation. As we noticed, the upper value of M_{33} is highly sensitive to the error value. (Such a small value of M_{33} cannot be ruled out from the current lower bound [3] by the $X \to \tau^+\tau^-$ search at the Tevatron, because the production rate of A_3^3 is much smaller than that of the conventional Z' boson. It is also attractive that two values of M_{33}^{II} roughly agree.) Considering the present whole data, we think that the data are still in favor of the value $M_{33}^{II} \sim 0.5 - 1$ TeV in Model II too.

In conclusion, as far as we notice the data on the deviations from the e-μ-τ universality, it looks that there is the lightest family gauge boson with the mass $M_{33} \sim 1$ TeV. We are eager for more accurate data on the deviations from the e-μ-τ universality, because those are now within our reach. Also, we expect a direct search for A_3^3, for example, at the LHC. For the details of the direct search for the lightest family gauge boson A_3^3 at the LHC, we shall report elsewhere.

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