Neutrino Mass Matrix Model with a Double Seesaw Form

Yoshio Koide\textsuperscript{a} and Hiroyuki Nishiura\textsuperscript{b}

\textsuperscript{a} Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
E-mail address: koide@het.phys.sci.osaka-u.ac.jp

\textsuperscript{b} Faculty of Information Science and Technology, Osaka Institute of Technology, Hirakata, Osaka 573-0196, Japan
E-mail address: nishiura@is.oit.ac.jp

Abstract

Within the framework of the so-called yukawaon model, which has been proposed for the purpose of a unified description of the lepton mixing matrix $U_{PMNS}$ and the quark mixing matrix $V_{CKM}$, a neutrino mass matrix model with a double seesaw form $M_\nu = k_\nu (m_D M_R^{-1} m_D^T)^2$ is proposed. The model has only two adjustable parameters for the PMNS mixing and neutrino mass ratios. (Other parameters are fixed from the observed quark and charged lepton mass ratios and CKM mixing.) The model can give reasonable values $\sin^2 \theta_{12} \approx 0.85$ and $\sin^2 \theta_{23} \approx 1$ and $\sin^2 \theta_{13} \approx 0.09$ together with $R_\nu \equiv \Delta m^2_{21}/\Delta m^2_{32} \sim 0.03$. Our prediction of the effective neutrino mass $\langle m \rangle$ in the neutrinoless double beta decay takes a sizable value $\langle m \rangle \approx 0.0034$ eV.

PCAC numbers: 11.30.Hv, 12.15.Ff, 14.60.Pq, 12.60.-i,

1 Introduction

Most particle physicists think that the mass spectra and mixing patterns of quarks and leptons will be understood by a unified description. As one of such models, “yukawaon” model [1, 2, 3, 4] has been proposed. The model, which is a kind of “flavon” model [5], has the following characteristics: (i) Yukawa coupling constants $Y_f$ ($f = u, d, e, \cdots$) in the standard model are understood as vacuum expectation values (VEVs) of scalars (“yukawaon”) with $3 \times 3$ components, i.e. by $y_f \langle Y_f \rangle / \Lambda$, where $\Lambda$ is an energy scale of the effective theory. (ii) The hierarchical structures of the effective Yukawa coupling constants originate only in a fundamental VEV matrix $\langle \Phi_0 \rangle$, whose hierarchical structure is ad hoc assumed and whose VEVs are fixed by the observed charged lepton masses. (Of course, the goal in the yukawaon model is to understand the VEVs of the fundamental scalar $\Phi_0$ itself on the basis of a dynamical consideration.) In the yukawaon model, in principle, there are no family-number-dependent parameters except for $\langle \Phi_0 \rangle$. (Regrettably, in order to obtain reasonable values of quark mixing matrices [6] $V_{CKM}$, at present, we need a phase matrix $P_u$ (or $P_d$) with two phase parameters [3, 4]. The final goal of our model is also to remove such family dependent parameters.) (iii) Relations among VEV matrices are obtained from SUSY vacuum conditions for a given superpotential under family
symmetries and $R$ charges assumed. (Since we use the observed charged lepton mass values as the input values, it is a characteristic in the yukawaon model that adjustable parameters are quite few.)

In a series of yukawaon models [2, 3, 4], mass matrices of quarks and leptons, $(M_u, M_d)$ and $(M_\nu, M_e)$, have been given as follows:

\begin{align}
M_e &= k_e \Phi_0 (1 + a_e X_3) \Phi_0, \\
M_\nu &= M_D M_R^{-1} M_D^T, \\
M_D &= M_e, \\
M_R &= k_R (\hat{M}_u M_e + M_e \hat{M}_u) + \cdots, \\
P_u M_u P_u &= k_u' \hat{M}_u \hat{M}_u, \\
\hat{M}_u &= k_u \Phi_0 (1 + a_u X_3) \Phi_0, \\
M_d &= k_d \{ \Phi_0 (1 + a_d X_3) \Phi_0 + m_0^2 1 \}.
\end{align}

Here, for convenience, we have denote VEV relations, which are obtained from SUSY vacuum conditions, by using the conventional notations for the quark mass matrices $(M_u, M_d)$ and so on instead of those for VEV matrices $(\langle Y_u \rangle, \langle Y_d \rangle)$ and so on. In Eq. (1.1), the matrices $\Phi_0, X_3, 1$ and $P_u$ have structures given by

\begin{align}
\Phi_0 &= \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix}, \\
X_3 &= \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \\
1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},
\end{align}

and $P_u = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1)$, respectively. The coefficients $a_f$ play an essential role in obtaining the mass ratios and mixings, while the family-number independent coefficients $k_f$ and $k'_u$ do not. The values of $(x_1, x_2, x_3)$ with $x_1^2 + x_2^2 + x_3^2 = 1$ are fixed by the observed charged lepton mass values under the given value of $a_e$.

Although the model can give successful predictions, the model has regrettably failed to obtain the observed value [8] $\sin^2 2\theta_{13} \sim 0.09$. Therefore, from the phenomenological point of view, the authors have recently proposed a new yukawaon model [7] by changing a structure of the Dirac neutrino mass matrix $M_D$:

\begin{align}
M_\nu &= M_D M_R^{-1} M_D^T, \\
M_D &= k_D \Phi_0^T (1 + a_D X_2) \Phi_0, \\
M_R &= k_R \left( \hat{M}_u M_e + M_e \hat{M}_u + \xi_0^\nu M_D M_D \right),
\end{align}

2
where $X_2$ is given by

$$X_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$  \hfill (1.6)

The model with the parameter values $a_e = 7.5$, $a_D = 9.01$, $(a_u, \alpha_u) = (-1.35, -7.6^\circ)$ and $\xi_0^\nu = -0.78$ can give reasonable fitting $\sin^2 2\theta_{12} = 0.860$, $\sin^2 2\theta_{23} = 0.969$, $\sin^2 2\theta_{13} = 0.0711$, and $R_\nu = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = 0.0324$ together with reasonable up-quark mass ratios. More explicitly speaking, the values of $(a_u, \alpha_u)$ have been fixed by the observed up-quark mass ratios. The value of $\xi_0^\nu$ has been fixed by the observed value of $R_\nu$. (The term $\xi_0^\nu$ plays as if it is a constant term $\xi^\nu_1$ in the neutrino mass matrix $M_\nu = M_D M_R^{-1} M_D^T$, so that the value of $\xi_0^\nu$ is used only to adjust the value of $R_\nu$ without changing other predicted values except for $\sin^2 2\theta_{13}$.) However, the existence of the $\xi_0^\nu$ term is somewhat artificial. Besides, the non-vanishing value of $\xi_0^\nu$ slightly lowers a predicted value of $\sin^2 2\theta_{13}$. For example, if we take $\xi_0^\nu = 0$ keeping the values for the other parameters, we can obtain $\sin^2 2\theta_{13} = 0.086$, while we obtain an unwelcome value $R_\nu = 0.091$ at the same time.

Now, we notice that the value of $(R_\nu)^2 \sim (10^{-1})^2$ at $\xi_0^\nu = 0$ is an order of the observed value of $R_\nu$. This suggests a possibility that if we modify the neutrino sector as follows:

$$M_\nu = k_\nu \tilde{M}_\nu \tilde{M}_\nu,$$  \hfill (1.7)

$$\tilde{M}_\nu = M_D M_R^{-1} M_D^T,$$  \hfill (1.8)

$$M_R = k_R (\tilde{M}_u M_e + M_e \tilde{M}_u),$$  \hfill (1.9)

we can obtain reasonable solutions for all lepton mixing parameters. Note that there is no longer the $\xi_0^\nu$-term. In this model, the parameters in the neutrino sector are only $a_e$, $a_D$ and $a_u$. As we denote later, the parameters $a_e$ and $a_u$ are fixed from the observed CKM mixing and quark mass ratios, so that we have only a parameter $a_D$ (complex) in the PMNS mixing and neutrino mass ratios.

In the next section, we give a model for quark and lepton mixings and their mass ratios on the basis of a revised yukawaon model. In Sec.3, as far as parameter fitting for observed values is concerned, since we change only the neutrino sector, we discuss only the PMNS mixing and neutrino mass ratios. The parameter values in the down-quark sector are effectively unchanged, so that we can obtain the same predictions for the down-quark mass ratios and CKM matrix parameters without changing the successful results in the previous paper [7].

2 Model

We assume that a would-be Yukawa interaction is given as follows:

$$W_Y = \frac{y_e}{\Lambda} e_i^c Y_e^{ij} j^c_i H_d + \frac{y_\nu}{\Lambda} (\ell_i H_u) \bar{Y}_\nu^{ij} (\ell_j H_u) + \frac{y_d}{\Lambda} d_i^c Y_d^{ij} q^c_j H_d + \frac{y_u}{\Lambda} u_i^c Y_u^{ij} q^c_j H_u,$$  \hfill (2.1)
where $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$ are SU(2)$_L$ doublets. Note that in Eq.(2.1) there are no SU(2)$_L$ singlet neutrinos. We have straightforwardly defined the neutrino mass matrix $M_\nu$ by the second term in Eq.(2.1). Although we denoted in Eq.(1.3) as if the matrix $M_D$ is a Dirac neutrino mass matrix, the matrix $M_D$ does not have a meaning of the Dirac mass matrix [see Eq.(2.3) later]. Under the definition of $\bar{Y}_f \ (Y')$ in Eq.(2.1), the quark mixing matrix (the Cabibbo-Kobayashi-Maskawa matrix [6] and the lepton mixing matrix (the Pontecorvo-Maki-Nakagawa-Sakata matrix [9]) are given by the Kobayashi-Maskawa matrix [6] and the lepton mixing mixing matrix (the Pontecorvo-Maki-Nakagawa-Sakata matrix [9]) are given by $V_{CKM} = U_d^T U_u$ and $U_{PMNS} = U_\nu^T U_\nu$, respectively, where $U_f$ are defined by $U_f^T M_f U_f = D_f^2$ ($D_f$ are diagonal). (Here and hereafter, sometimes, we denote $\bar{Y}_f$ and $Y_q$ as $Y_f$ for simplicity. In order to distinguish each yukawa from others, we assume that $Y_f$ have different $R$ charges from each other together with $R$ charge conservation. (Of course, the $R$ charge conservation is broken at the energy scale $\Lambda'$.)

We assume the following superpotential for yukawas:

$$W_e = \left\{ \mu_e Y_e^{ij} + \lambda_e (\bar{\Phi}_0)^{ia} \left( E''_{ij}^{\alpha \beta} + \frac{\alpha_e}{\lambda^2} X_{ak} E^{kl} X^T_{ij \beta} \right) (\bar{\Phi}_0^{T \beta}) \right\} \Theta_{ji}^e, \quad (2.2)$$

$$W_\nu = \left[ \mu_\nu Y_\nu^{ij} + \lambda_\nu (\bar{Y}_\nu)^{ia} E''_{ij}^{\alpha \beta} \right] \Theta_{ji}^\nu + \left[ \mu_\nu Y_\nu^{ai} + \lambda_\nu (\bar{Y}_\nu)^{ak} Y_k^{\beta} \right] \Theta_{ja}^\nu, \quad (2.3)$$

$$W_D = \left[ \lambda_D (\bar{\Phi}_0)^{ak} \left( E^{ji} + \frac{\lambda_D}{\lambda} X_{kl} X^T_{ij \beta} \right) \right] \bar{\Phi}_0^{T \beta}, \quad (2.4)$$

$$W_R = \lambda_R \left[ \bar{Y}_R^{ia} \bar{\Phi}_0^{R \alpha} + \mu_R (\bar{Y}_R^{T \beta})^{ji} \right] (\Theta_R^T)_{ij}^R$$

$$+ \left[ (\bar{E}'_R)^{ij} E^{\beta j} + \bar{Y}_R^{T \beta} (\bar{E}'_R)^{T \beta} \right] + \lambda_R \left( \bar{Y}_e^{\beta k} Y_k^{\alpha} E^{ij} + \bar{Y}_u^{\beta k} Y_k^{\alpha} E^{ij} \right) \Theta_{ji}^R, \quad (2.5)$$

$$W_u = \left[ \mu_u Y_{ij} + \lambda_u (\bar{\Phi}_0)^{iu} \right] \bar{Y}_u^{kl} \Theta_{ji}^u$$

$$+ \left[ \lambda_u Y_{uk} Y_k^{ji} + \lambda_u (\bar{\Phi}_0)^{iu} \left( E''_{uj}^{\alpha \beta} + \frac{\alpha_u}{\lambda^2} X_{ak} E^{kl} X^T_{ij \beta} \right) (\bar{\Phi}_0^{T \beta}) \right] \Theta_{ji}^u, \quad (2.6)$$

$$W_d = \left[ \lambda_d \bar{Y}_d^{ik} \left( Y_{kl} + \frac{\alpha_d}{\lambda^2} P_{km} E^{mn} P_{nl} \right) \bar{P}^{ij} + \lambda_d (\bar{\Phi}_0)^{iu} \left( E''_{uj}^{\alpha \beta} + \frac{\alpha_d}{\lambda^2} X_{ak} E^{kl} X^T_{ij \beta} \right) (\bar{\Phi}_0^{T \beta}) \right] \Theta_{ji}^d. \quad (2.7)$$

Here, we have assumed family symmetries $U(3) \times U(3)'$.

From Eqs(2.2) and (2.3), we obtain $R(E'') - R(E) = R(E) - R(E'') = 2R(X)$, i.e. $R(E'') + R(E) = R(E) + R(E) = R(\bar{P}_d) + R(\bar{P}_d) = 1$, and
we can write the following superpotential:

\[
W_{E,P} = \frac{\lambda_1}{\Lambda} \text{Tr}[\bar{E} E \bar{P}_d P_d] + \frac{\lambda_2}{\Lambda} \text{Tr}[\bar{E} E \bar{P}_d].
\]  

(2.8)

From the superpotential (2.8), we obtain \( \langle E \rangle \langle \bar{E} \rangle \propto 1 \) and \( \langle P_d \rangle \langle \bar{P}_d \rangle \propto 1 \). We assume specific solutions from those solutions for \( \langle E \rangle \) and \( \langle P \rangle \):

\[
\frac{1}{v_E} \langle E \rangle = \frac{1}{v_E} \langle \bar{E} \rangle = 1,
\]

(2.9)

\[
\frac{1}{v_P} (P_d)^\dagger = \frac{1}{v_P} \langle \bar{P}_d \rangle = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1),
\]

(2.10)

as the explicit forms of \( \langle E \rangle \), \( \langle \bar{E} \rangle \) and \( \langle P_d \rangle \). We assume similar superpotential forms for \( (E'', \bar{E}'') \) and \( (E', \bar{E}') \).

The term \( P_d \bar{E} P_d \) in Eq.(2.7) is introduced in order to adjust the down-quark mass ratio \( m_d/m_s \) as seen in the next section. If \( R(\bar{E}) = R(\bar{Y}_e) \), such a \( \bar{E} \) term also appears in Eq.(2.2). However, we obtain \( R(Y^d) = R(\bar{E}) + 2R(P_d) \) from Eq.(2.7), and \( R(\bar{Y}_e) = R(Y^d) + 2R(\bar{P}_d) \) from Eqs.(2.2) and (2.7), we see \( R(\bar{Y}_e) = R(\bar{E}) + 2 \), i.e. we obtain \( R(\bar{Y}_e) \neq R(\bar{E}) \).

Since we assume that all \( \Theta \) fields take \( \langle \Theta \rangle = 0 \), the superpotential terms (2.2) - (2.7) lead to relations

\[
\langle \bar{Y}_e \rangle = k_e \langle \bar{\Phi}_0 \rangle (1 + a_eXX^T) \langle \bar{\Phi}_0^T \rangle,
\]

(2.11)

\[
\langle \bar{Y}_\nu \rangle = k_\nu \langle \bar{\Phi}^\nu \rangle \langle \bar{\Phi}^\nu \rangle,
\]

(2.12)

\[
\langle \bar{Y}^u \rangle = k'_\nu \langle \bar{\Phi}_0 \rangle \langle \bar{\Phi}_0 \rangle = k''_\nu \langle \bar{\Phi}_0 \rangle (\langle \bar{\Phi}_0 \rangle)^{-1} \langle \bar{\Phi}_0 \rangle,
\]

(2.13)

\[
\langle \bar{Y}_D \rangle = k_D \langle \bar{\Phi}_0^T \rangle (1 + a_DXX^T) \langle \bar{\Phi}_0 \rangle,
\]

(2.14)

\[
\langle \bar{Y}_R \rangle = k_R \left( \langle \bar{Y}_e \rangle \langle \bar{Y}^u \rangle + \langle \bar{Y}^u \rangle \langle \bar{Y}_e \rangle \right),
\]

(2.15)

\[
\langle Y^u \rangle = k_u \langle \bar{\Phi}^u \rangle \langle \bar{\Phi}^u \rangle,
\]

(2.16)

\[
\langle \bar{Y}^u \rangle = k'_u \langle \bar{\Phi}_0 \rangle (1 + a_uXX^T) \langle \bar{\Phi}_0^T \rangle,
\]

(2.17)

\[
\langle \bar{P}_d \rangle \langle Y^d \rangle \langle \bar{\Phi}_0 \rangle (1 + a_dXX^T) \langle \bar{\Phi}_0^T \rangle + \xi_0^d 1,
\]

(2.18)

where \( X \) has phenomenologically been introduced in the previous model [7] and it has the form

\[
X = \frac{1}{2} \begin{pmatrix}
1 & 1 & 0 \\
1 & 1 & 0 \\
1 & 1 & 0
\end{pmatrix}.
\]

(2.19)
The form (2.19) leads to

\[ XX^T = \frac{3}{2}X_3, \quad X^TX = \frac{3}{2}X_2, \tag{2.20} \]

together with \( XX = X \), where \( X_3 \) and \( X_2 \) is defined by Eqs. (1.2) and (1.6), respectively, and, for simplicity. For convenience, we have already put \( \langle E \rangle \) as 1, and so on.

3 Parameter fitting

We again summarize our mass matrix model as follows:

\[
\bar{Y}_e = k_e \Phi_0 (1 + a_e X_3) \Phi_0^T, \tag{3.1}
\]

\[
\bar{Y}_\nu = k_\nu \bar{Y}_\nu, \tag{3.2}
\]

\[
\bar{Y}_\nu = k''_\nu \bar{Y}_D \bar{Y}^{-1}D, \tag{3.3}
\]

\[
\bar{Y}_D = k_D \Phi_0^T (1 + a_D e^{i \alpha_D} X_2) \Phi_0, \tag{3.4}
\]

\[
\bar{Y}_R = k_R \left( \bar{Y}_e \hat{Y}^u + \hat{Y}^u \bar{Y}_e \right), \tag{3.5}
\]

\[
Y^u = k_u \hat{Y}^u, \tag{3.6}
\]

\[
\hat{Y}^u = k'_u \Phi_0 (1 + a_u e^{i \alpha_u} X_3) \Phi_0^T, \tag{3.7}
\]

\[
\bar{P}_d Y^d \bar{P}_d = k_d \Phi_0 (1 + a_d X_3) \Phi_0^T + \xi_0^d 1, \tag{3.8}
\]

where, for convenience, we have dropped the notations \( \langle \rangle \) and \( \langle \rangle \). In numerical calculations, we use dimensionless expressions \( \Phi_0 = \text{diag}(x_1, x_2, x_3) \) (with \( x_1^2 + x_2^2 + x_3^2 = 1 \)), \( \bar{P}_d = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1) \), and \( E = \text{diag}(1, 1, 1) \). The parameters are re-refined by Eqs.(3.1)-(3.8). In Eqs.(3.4) and (3.7), since we assume that the parameters \( a_e \) and \( a_d \) are real, while \( a_u \) and \( a_D \) are complex, we have denoted \( a_u \) and \( a_D \) as \( a_u e^{i \alpha_u} \) and \( a_D e^{i \alpha_D} \), respectively.

In this model, we have 2 parameters \((a_D, \alpha_D)\) for neutrino sector, 4 parameters \(a_D, \xi_0^d\) and \((\phi_1, \phi_2)\) for down-quark mass ratios and \( V_{CKM} \), and 3 parameters \(a_e, (a_u, \alpha_u)\) for charged lepton mass ratios and up-quark mass ratios. Especially, it is worthwhile noticing that the neutrino mass ratios and \( U_{PMNS} \) are described only two parameters after \( a_e \) and \( (a_u, \alpha_u) \) have been fixed from the observed CKM mixing and up-quark mass ratios. Since we do effectively not change the mass matrix structures except for \( Y_\nu \) from the previous paper [7], we can use the same parameter values as given in the previous study:

\[
a_e = 7.5, \quad (a_u, \alpha_u) = (-1.35, 7.6^\circ), \tag{3.9}
\]

in the present model, too. Therefore, as far as PMNS mixing and neutrino mass ratios are concerned, we have only two free parameters \((a_D, \alpha_D)\) in the present neutrino mass matrix model.
At present, the observed values [10] are as follows:

\[ \sin^2 \theta_{12} = 0.857 \pm 0.024, \quad \sin^2 \theta_{23} > 0.95, \quad \sin^2 \theta_{13} = 0.098 \pm 0.013, \quad (3.10) \]

\[ R_\nu \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{(7.50 \pm 0.20) \times 10^{-5} \text{eV}^2}{(2.32^{+0.12}_{-0.08}) \times 10^{-3} \text{eV}^2} = (3.23^{+0.14}_{-0.19}) \times 10^{-2}. \quad (3.11) \]

Since the parameters \((a_D, \alpha_D)\) are sensitive to the observables \(\sin^2 \theta_{12}\) and \(R_\nu\), we use the observed values \(\sin^2 \theta_{12}\) and \(R_\nu\) in order to fix our parameter values \((a_D, \alpha_D)\). In Fig.1, we illustrate an allowed parameter region of \((a_D, \alpha_D)\) for the observed values [10] \(\sin^2 \theta_{12}^{\text{obs}} = 0.875 \pm 0.024\) and \(R_\nu^{\text{obs}} \times 10 = 0.324^{+0.014}_{-0.019}\). As seen in Fig.1, the observed values uniquely fix the parameter values \((a_D, \alpha_D)\) as

\[ (a_D, \alpha_D) = (8.7, 12^\circ). \quad (3.12) \]

Figure 1: Allowed parameter region in \((a_D, \alpha_D)\) plane. The solid and dashed lines indicate the border and center curves of the allowed region which are obtained from the observe values \(\sin^2 \theta_{12}^{\text{obs}} = 0.875 \pm 0.024\) and \(R_\nu^{\text{obs}} \times 10 = 0.324^{+0.014}_{-0.019}\), respectively.

For reference, in Fig.2, we illustrate behaviors of \(\sin^2 2\theta_{12}\) and \(R_\nu\) versus \(a_D\) for the case of \(a_D = 8.7\). The choice \(\alpha_D = 12^\circ\) gives excellent fittings to the observed values of \(\sin^2 2\theta_{12}\) and \(R_\nu\) simultaneously:

\[ \sin^2 2\theta_{12} = 0.8544, \quad R_\nu = 0.0331. \quad (3.13) \]

Then, we obtain our predictions for \(\sin^2 2\theta_{23}\) and \(\sin^2 2\theta_{13}\) using (3.12) as follows:

\[ \sin^2 2\theta_{23} = 0.9962, \quad \sin^2 2\theta_{13} = 0.0907, \quad (3.14) \]

which are in excellent agreement with the observed values in given in Eq.(3.10).
Figure 2: Lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared ratio $R_\nu$ versus the phase parameter $\alpha_D$ for $a_D = 8.7$. The horizontal lines denote observed values $\sin^2 2\theta_{12}^{\rm obs} = 0.875 \pm 0.024$, $\sin^2 2\theta_{13}^{\rm obs} \times 10 = 0.98 \pm 0.13$ and $R_\nu^{\rm obs} \times 10 = 0.324^{+0.014}_{-0.019}$. Our predicted value for $\sin^2 2\theta_{23}$ is well satisfied the obtained experimental bound $\sin^2 2\theta_{23}^{\rm obs} > 0.95$.

The fixing of the parameters $(a_D, \alpha_D)$, Eq.(3.12), make the prediction of the $CP$ violating phase parameter in the lepton sector possible too:

$$\delta_{CP}^\ell = 127^\circ \quad (J^\ell = 2.74 \times 10^{-2}).$$

We can also predict neutrino masses:

$$m_{\nu 1} = 0.00061 \text{eV}, \quad m_{\nu 2} = 0.00899 \text{eV}, \quad m_{\nu 3} = 0.05011 \text{eV},$$

by using the input value [11] $\Delta m^2_{32} = 0.00243 \text{eV}^2$. We also predict the effective Majorana neutrino mass [12] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = |m_1 U_{c1}^2 + m_2 U_{c2}^2 + m_3 U_{c3}^2| = 0.0034 \text{eV}.$$ (3.17)

This value is a magnitude which is within our reach to observe in a future neutrinoless double beta decay experiments.

Finally, we list the predicted values of the CKM mixing parameters and down-quark mass ratios, although they are essentially the same as those in the previous model [7]:

$$|V_{us}| = 0.2271, \quad |V_{cb}| = 0.0394, \quad |V_{ub}| = 0.00347, \quad |V_{td}| = 0.00780,$$

$$\delta_{CP}^q = 59.6^\circ \quad (J^q = 2.6 \times 10^{-5}),$$

$$r_{12}^u = \sqrt{\frac{m_d}{m_s}} = 0.00465, \quad r_{23}^u = \sqrt{\frac{m_d}{m_b}} = 0.0614.$$ (3.20)
Here, we have used $a_d = 25$, $\xi_0^d = 0.0115$, and $(\phi_1, \phi_2) = (177.0^\circ, 197.4^\circ)$. The observed values are as follows: $|V_{us}| = 0.2252 \pm 0.0009$, $|V_{cb}| = 0.0409 \pm 0.0011$, $|V_{ub}| = 0.00415 \pm 0.0009$, $|V_{td}| = 0.0084 \pm 0.0006$, $J = (2.96^{+0.20}_{-0.16}) \times 10^{-5}$ \cite{10}, and $r_{12}^d = 0.045^{+0.013}_{-0.010}$, $r_{23}^u = 0.060 \pm 0.005$, $r_{12}^d = 0.053^{+0.005}_{-0.003}$, $r_{23}^d = 0.019 \pm 0.006$ \cite{13}.

4 Concluding remarks

In conclusion, we have proposed a new neutrino mass matrix form within the framework of the yukawaoon model, in which we have only two adjustable parameters, $(a_D, \alpha_D)$, for PMNS mixing and neutrino mass ratios. We obtain reasonable results for PMNS mixing and neutrino mass ratios as shown in Eqs.(3.13) - (3.17) for the parameter values $(a_D, \alpha_D) = (8.7, 12^\circ)$. As seen in Fig.2, it is worthwhile noticing that only when we choose a reasonable value of $R_\nu \simeq 0.033$, we obtain a reasonable value of $\sin^2 2\theta_{13} \simeq 0.09$. Also, we would like to emphasize that our prediction gives a sizable value of $\langle m \rangle \simeq 0.0034$ eV in spite of the normal mass hierarchy model (in spite of $m_{\nu_1} \simeq 0.0006$ eV, $m_{\nu_2} \simeq 0.00899$ eV and $m_{\nu_3} \simeq 0.05011$ eV).

Of course, we have also obtained reasonable results for CKM mixing and quark mass ratios as same as those in the previous paper \cite{7}.

The present model gives successful results from the phenomenological point of view. However, we still have some of theoretical problems. One of the major problems is why only $Y_D$ takes the mass matrix form with $X_2$ (not $X_3$). In Ref.[4], the form $X_3$ has been understood by a symmetry breakdown $U(3) \times U(3)' \rightarrow U(3) \times S_3$. However, for the form $X_2$, the model is still in a phenomenological level. We have been able to remove the unnatural term $[\xi_0^d$ term in Eq.(1.5)], but we still have $\xi_0^d$ term in the $Y_d$ sector. These problems are our future tasks.

References


