Unified description of quark and lepton mixing matrices based on a yukawaon model

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Based on a supersymmetric yukawaon model with O(3) family symmetry, possible forms of quark and lepton mixing matrices are systematically investigated under a condition that the up-quark mass matrix form leads to the observed nearly tribimaximal mixing in the lepton sector. Although the previous model could not provide a good fitting of the observed quark mixing, the present model can give a reasonably good fitting not for lepton mixing but also the quark mixing by using a different origin of the CP violation from one in the previous model.

I. INTRODUCTION

One of the current subjects of the particle flavor physics is to understand quark and lepton masses and mixings. The investigation, even if it is phenomenological one, will provide a promising clue to new physics. The so-called "tribimaximal" mixing observed in the neutrino mixing [1, 2] is very suggestive of a fundamental law in the lepton sector. Usually, the observed tribimaximal mixing has been explained by assuming a discrete symmetry [3].

Meanwhile, as a neutrino mass matrix model without assuming any discrete symmetry, there is an unfamiliar model [4, 5], where in a seesaw type neutrino mass matrix $M_\nu = m_D M_R^{-1} m_D^T$, the Dirac neutrino mass matrix m_D is given by $m_D \propto M_e$ (M_e is a charged lepton mass matrix) and the Majorana mass matrix M_R of the right-handed neutrinos is given by

$$M_R \propto M_u^{1/2} M_e + M_e M_u^{1/2}. \tag{1.1}$$

Here $M_u^{1/2}$ is defined by $M_u^{1/2}(M_u^{1/2})^T = M_u$ (M_u is an up-quark mass matrix with a symmetric form $M_u^T = M_u$). The model [5] can lead to a nearly tribimaximal mixing by assuming suitable up-quark mass matrix as we give a short review in the next section. The model has only four parameters: one (ξ_ν) is in the neutrino sector, and one (a_u) is in the up-quark sector, and two ($a_d e^{i\alpha_d}$) in the down-quark sector. (Here, we consider that the charged lepton mass (m_e, m_μ, m_τ) are known parameters, and we do not count these parameters as adjustable parameters.) This model leads to excellent fitting for up-quark mass ratios m_u/m_c and m_c/m_t and neutrino mixing parameters $\sin^2 2\theta_{atm}$, $\tan^2 \theta_{solar}$ and $|U_{e3}|^2$, only by adjusting the two parameters a_u and ξ_ν. On the other hand, for down-quark sector, the fitting is not so excellent, especially, the predicted values of $|V_{ub}|$ and $|V_{td}|$ are somewhat large compared with the observed values as far as we use parameter values which can give reasonable values for the observed down-quark mass ratios.

The purpose of the present paper is to investigate an improved version of the above model and to search systematically for parameter values which can give reasonable quark mass ratios, quark mixing parameters (Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix) and neutrino mixing parameters. In Sec.2, we will show that the four parameter model cannot have reasonable parameter region consistent with four quark mass ratios, three neutrino mixing parameters, and four CKM mixing parameters. In Sec.3, we propose a revised model and give parameter fitting for 11 observables. (In the present model, we do not discuss the observed value $R \equiv \Delta m_{solar}^2/\Delta m_{atm}^2$ for neutrino masses, because we can always have an additional one parameter which inevitably appears in the model and affects only the mass ratios R, but does not affect neutrino mixing and observables in the quark sector.) Finally, Sec.4 is devoted to the summary and discussions.

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II. SUPERSYMMETRIC YUKAWAON MODEL

In this section, we give a short review of a quark and lepton mass matrix model [5] based on the supersymmetric yukawaon model, because, in this paper, we propose a revised version of this model.

In the yukawaon model, we put the following assumption:

(i) We consider that the Yukawa coupling constants are effectively given by

$$Y_f^{eff} = y_f \frac{\langle Y_f \rangle}{\Lambda}, \tag{2.1}$$

where $\langle Y_f \rangle$ ($f = u, d, e, \nu$ and so on) are vacuum expectation values (VEVs) of new scalars Y_f with 3×3 components of O(3) family symmetry and Λ is an energy scale of the effective theory. (For the time being, we assume $\Lambda \sim 10^{14-15}$ GeV.) Therefore, the would-be Yukawa interactions are given by

$$H_Y = \sum_{i,j} \frac{y_u}{\Lambda} u_i^c (Y_u)_{ij} q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} d_i^c (Y_d)_{ij} q_j H_d$$

$$+ \sum_{i,j} \frac{y_\nu}{\Lambda} \ell_i (Y_\nu)_{ij} \nu_j^c H_u + \sum_{i,j} \frac{y_e}{\Lambda} \ell_i (Y_e)_{ij} e_j^c H_d + h.c. + \sum_{i,j} y_R \nu_i^c (Y_R)_{ij} \nu_j^c, \tag{2.2}$$

where q and ℓ are SU(2)$_L$ doublet fields, and f^c ($f = u, d, e, \nu$) are SU(2)$_L$ singlet fields.

(ii) In order to distinguish each Y_f from others, we assume a U(1)$_X$ symmetry (i.e. "sector charge") in addition to the O(3) symmetry, and we have assigned U(1)$_X$ charges as $Q_X(Y_f) = x_f$, $Q_X(f^c) = -x_f$ and $Q_X(Y_R) = 2x_\nu$. (The SU(2)$_L$ doublet fields q, ℓ, H_u and H_d are assigned to sector charges $Q_X = 0$.)

(iii) For the neutrino sector, we assume $Q_X(\nu^c) = Q_X(e^c)$, so that the yukawaon Y_e can also couple to the neutrino sector as $(\ell Y_e \nu^c) H_u$ instead of $(\ell Y_\nu \nu^c) H_u$ in Eq.(2.2). Therefore, we can change the above model into a model without Y_ν. Hereafter, we read Y_ν in Eq.(2.2) as Y_e. Besides, we can have a term

$$\sum_{i,j,k} \frac{y_R'}{\Lambda} \nu_i^c (Y_e)_{ik} (Y_e)_{kj} \nu_j^c, \tag{2.3}$$

in addition to the right-hand side of Eq.(2.2), because $Y_e Y_e$ has the same U(1)$_X$ charge as Y_R, i.e. $Q_X = 2x_e$. Although this term (2.3) leads to an additional neutrino mass term, the term does not affect neutrino mixing [6] as far as the neutrino mass matrix M_ν is real, [1] because of $M_\nu \propto Y_e[(\cdots) + Y_e Y_e]^{-1} Y_e = [Y_e^{-1}(\cdots) Y_e^{-1} + \mathbf{1}]^{-1}$.

(iv) We give a superpotential W which is invariant under O(3) family symmetry and U(1)$_X$ symmetry, and solve supersymmetric (SUSY) vacuum conditions. As a result, we obtain VEV relations among yukawaons.

For example, we have assumed the following superpotential

$$W_e = \lambda_e \text{Tr}[\Phi_e \Phi_e \Theta_e] + \mu_e \text{Tr}[Y_e \Theta_e] + W_\Phi. \tag{2.4}$$

Here we have assumed $Q_X(\Phi_e) = \frac{1}{2} Q_X(Y_e) = -\frac{1}{2} Q_X(\Theta_e)$ and the term W_Φ has been introduced in order to determine a VEV spectrum $\langle \Phi_e \rangle$ completely. Then, from a SUSY vacuum condition

$$\frac{\partial W}{\partial \Theta_e} = \lambda_e \Phi_e \Phi_e + \mu_e Y_e = 0, \tag{2.5}$$

we obtain a VEV relation

$$\langle Y_e \rangle = -\frac{\lambda_e}{\mu_e} \langle \Phi_e \rangle \langle \Phi_e \rangle. \tag{2.6}$$

The VEV value $\langle \Phi_e \rangle$ is derived from the term W_Φ (for example, see Refs.[7–9]). However, for simplicity, in this paper, we use the observed values of the charged lepton masses straightforwardly for the VEV value as given by

$$\langle \Phi_e \rangle_e = \text{diag}(v_1, v_2, v_3) \propto \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}). \tag{2.7}$$

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[1] When $R^T M_1 R = D_1 \equiv$ diagonal, the inverse matrix M_1^{-1} is also diagonalized as $R^T M_1^{-1} R = D_1^{-1}$ by the same orthogonal transformation matrix R; When we take $M = M_1 + m_0 \mathbf{1}$, M is diagonalized as $R^T M R = D_1 + m_0 \mathbf{1}$, so that we can diagonalize M^{-1} as $R^T M^{-1} R = (R^T M R)^{-1} = (D + m_0 \mathbf{1})^{-1}$.

Here, the notation $\langle A \rangle_f$ denotes a form of a VEV matrix $\langle A \rangle$ in the diagonal basis of $\langle Y_f \rangle$ (we refer to it as f basis). The scalar Θ_e does not have a VEV, i.e. $\langle \Theta_e \rangle = 0$. Therefore, terms which include more than two of Θ_e do not play any physical role, so that we do not consider such terms in the present effective theory. [Hereafter, we will denote fields whose VEV values are zero as notations Θ_A $(A = u, d, \cdots)$.]

Next, for the purpose of the comparison of a model in the present paper with the previous one, we give a short review of quark and lepton mass matrix forms given in Ref.[5]. Since we give an explicit superpotential form of the present model in the next section, we do not give an explicit form of the mass matrix in this section. The explicit form of the superpotential for the previous model is given in Ref.[5].

Quark mass matrices, i.e. $\langle Y_u \rangle$ and $\langle Y_d \rangle$, are given as

$$M_u \propto \langle Y_u \rangle \propto \langle \Phi_u \rangle \langle \Phi_u \rangle, \tag{2.8}$$

$$\langle \Phi_u \rangle_e \propto \langle \Phi_e \rangle_e \left(\langle E \rangle_e + a_u \langle X \rangle_e \right) \langle \Phi_e \rangle_e, \tag{2.9}$$

$$M_d \propto \langle Y_d \rangle_e \propto \langle \Phi_e \rangle_e \left(\langle E \rangle_e + a_d e^{i\alpha_d} \langle X \rangle_e \right) \langle \Phi_e \rangle_e, \tag{2.10}$$

respectively. (For convenience, we have changed the definitions of a_u and a_d from those in Ref.[5].) Here, $\langle X \rangle_e$ and $\langle E \rangle_e$ are given by

$$\langle X \rangle_e = X v_X \equiv \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} v_X, \qquad \langle E \rangle_e = \mathbf{1} v_E \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_E. \tag{2.11}$$

(Here, the VEV form $\langle X \rangle_e$ breaks the O(3) flavor symmetry into S$_3$.) Therefore, we obtain quark mass matrices

$$M_u^{1/2} \propto M_e^{1/2} \left(\mathbf{1} + a_u X \right) M_e^{1/2}, \quad M_d \propto M_e^{1/2} \left(\mathbf{1} + a_d e^{i\alpha_d} X \right) M_e^{1/2}, \tag{2.12}$$

on the e basis. Here, we have redefined the coefficients $a_u v_X / v_E$ and $a_d v_X / v_E$ in Eqs.(2.9) and (2.10) as a_u and a_d, respectively. Hereafter, for numerical estimate of a_u and a_d, we use the definition of those in Eq.(2.12). (This quark mass matrix form (2.12) has first been proposed in Ref.[10] as a "democratic universal seesaw mass matrix model".) Note that we have assumed that the O(3) relations are valid only on the e and u bases, so that $\langle Y_e \rangle$ and $\langle Y_u \rangle$ must be real. [The VEV matrix $\langle \Phi_u \rangle$ must satisfy the relation (2.9) on the e basis, while $\langle \Phi_u \rangle$ must also satisfy the relation $\langle Y_u \rangle \propto \langle \Phi_u \rangle_u \langle \Phi_u \rangle_u$ on the u basis. However, for the down-quark sector, such a condition is not required, because $\langle Y_d \rangle$ is given by Eq.(2.9) only on the e basis.]

A case $a_u \simeq -1.79$ can give a reasonable up-quark mass ratios $\sqrt{m_{u1}/m_{u2}} = 0.043$ and $\sqrt{m_{u2}/m_{u3}} = 0.057$, which are in favor of the observed values [11]

$$\sqrt{\frac{m_u}{m_c}} = 0.045^{+0.013}_{-0.010}, \qquad \sqrt{\frac{m_c}{m_t}} = 0.060 \pm 0.005, \tag{2.13}$$

at $\mu = m_Z$.

In this paper, we will carry out parameter-fitting at $\mu = m_Z$, because we interest in the mixing values at $\mu = m_Z$. Exactly speaking, fitting for the mass ratios must be done at $\mu = \Lambda \sim 10^{14-16}$ GeV. However, at present, our model does not intend to give so precise predictions of the quark mass ratios. For example, we know [11] $\sqrt{m_u/m_c} = 0.046^{+0.013}_{-0.012}$ and $\sqrt{m_c/m_t} = 0.051^{+0.002}_{-0.006}$ even at $\mu = 2 \times 10^{16}$ GeV $(\tan\beta = 10)$. Even in $\sqrt{m_c/m_t}$, the discrepancy is smaller than 20%. Besides, the mass values are dependent on the value of $\tan\beta$ in the SUSY model. Therefore, for simplicity, in this paper, we will carry out the parameter-fitting at $\mu = m_Z$.

On the other hand, in the neutrino mass matrix

$$M_\nu \propto \langle Y_e \rangle_e \langle Y_R \rangle_e^{-1} \langle Y_e \rangle_e, \tag{2.14}$$

the Majorana mass matrix of the right-handed neutrinos $\langle Y_R \rangle_e$ is given by

$$\langle Y_R \rangle \propto \langle Y_e \rangle_e \langle P_u \rangle_e \langle \Phi_u \rangle_e + \langle \Phi_u \rangle_e \langle P_u \rangle_e \langle Y_e \rangle_e + \xi_\nu (\langle P_u \rangle_e \langle Y_e \rangle_e \langle \Phi_u \rangle_e + \langle \Phi_u \rangle_e \langle Y_e \rangle_e \langle P_u \rangle_e). \tag{2.15}$$

Here, we have introduced a new field P_u with a VEV

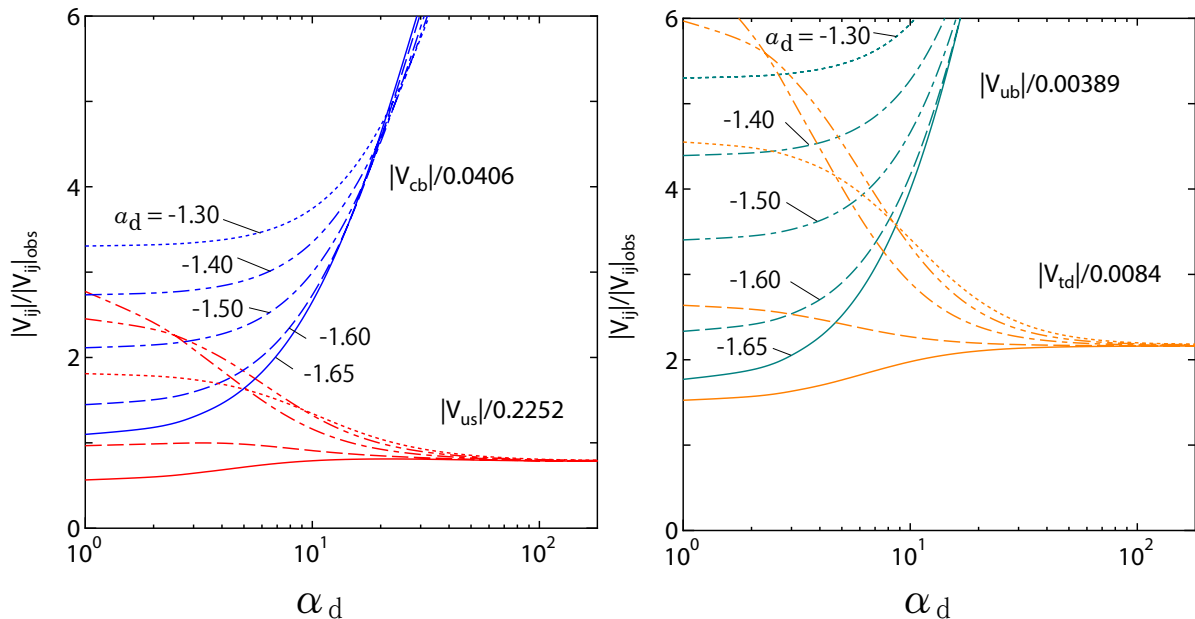$$\langle P_u \rangle_u \propto \mathrm{diag}(+1, -1, +1), \tag{2.16}$$

FIG. 1: $|V_{ij}|/|V_{ij}|_{obs}$ versus a phase parameter α_d. (a) $|V_{us}|/0.2252$ and $|V_{cb}|/0.0406$; (b) $|V_{ub}|/0.00389$ and $|V_{td}|/0.0084$. The parameter a_u in the up-quark sector is fixed at $a_u = -1.79$. The five curves represent $a_d = -1.30, -1.40, -1.50, -1.60$ and -1.65, respectively.

in order to make "effective" eigenvalues of $\langle\Phi_u\rangle_u$ positive, because the the eigenvalues of $\langle\Phi_u\rangle_u = (v_{u1}, v_{u2}, v_{u3})$ give signs $(+,-,+)$ for the parameter value $a_u \sim -1.8$. (The introducing of the field P_u is a phenomenological reason. If the factor (2.16) were absence [i.e. Y_R were given by $\langle Y_R\rangle \propto \langle Y_e\rangle_e\langle\Phi_u\rangle_e + \langle\Phi_u\rangle_e\langle Y_e\rangle_e$], we could not give the observed maximal mixing $\sin^2 2\theta_{atm} \simeq 1$ [2] for any values of the parameters a_u and ξ_ν.) The reason for the ξ_ν-term in Eq.(2.15) is as follows: When we consider a term $Y_e P_u \Phi_u + \Phi_u P_u Y_e$ we must also consider an existence of $P_u Y_e \Phi_u + \Phi_u Y_e P_u$ [5], because they have the same U(1)$_X$ charges. The results at $a_u \simeq -1.78$ are excellently in favor of the observed neutrino oscillation parameters $\sin^2\theta_{atm} = 1.00_{-0.13}$ [2] and $\tan^2 2\theta_{solar} = 0.469^{+0.047}_{-0.041}$ [1] by taking a small value of $|\xi_\nu|$, $\xi_\nu = +0.005$ or $\xi_\nu = -0.0012$.

Thus, the model in Ref.[5] can successfully fit two up-quark mass ratios and three neutrino mixing parameters only by the two parameters a_u and ξ_ν. On the other hand, the model tries to fit six observable quantities (two down-quark mass ratios and four CKM mixing parameters) only by two parameters a_d and α_d. However, the results given in Ref.[5] are not in excellent agreement with the observed values. Especially, the predicted values of $|V_{ub}|$ and $|V_{td}|$ are considerably larger than the observed ones. First, from a systematical parameter search, we show that this is not due to incompleteness of the parameter search, but we cannot obtain plausible values of the CKM mixing parameters even if we abandon the fitting of the down-quark mass ratios.

Considering the success in the up-quark and neutrino sectors, we do not change the model for the up-quark and neutrino sectors. We fix the parameter values as $a_u \sim -1.8$. The observed values of the down-quark masses are as follows [11]

$$\frac{m_d}{m_s} = 0.053^{+0.051}_{-0.029}, \qquad \frac{m_s}{m_b} = 0.019 \pm 0.006, \qquad (2.17)$$

at $\mu = m_Z$. We consider that the mass ratio m_d/m_s may be sensitive due to an unknown effect of a minor change of the model, so that, for the time being, we disregard the fitting of m_d/m_s and concentrate on the fitting of m_s/m_b. Although a parameter value $a_d \sim -16$ can give a reasonable prediction of m_s/m_b, the solution cannot give reasonable predictions of the CKM mixing parameters, we rule out this solution $a_d \sim -16$. We find that there are another solutions of a_d in a range $-a_d = 1.3 - 1.7$, which can roughly give $m_s/m_b = 0.1 - 0.4$. The solutions have a possibility that they can give reasonable values of the CKM mixing parameters. Therefore, we investigate the case with $-a_d = 1.3 - 1.7$ in detail.

The results are shown in Fig.1, where the predicted values $|V_{ij}|$ versus the phase parameter α_d are given in the unit of the observed values [12] $|V_{ij}|_{obs}$

$$|V_{us}|_{obs} = 0.2252 \pm 0.0009, \quad |V_{cb}|_{obs} = 0.0406 \pm 0.0013,$$
$$|V_{ub}|_{obs} = 0.00389 \pm 0.00044, \quad |V_{td}|_{obs} = 0.0084 \pm 0.0006. \qquad (2.18)$$

Here, we have illustrated the behaviors of $|V_{ij}|$ for the range $\alpha_d = 0° - 180°$, because the behaviors for $\alpha_d = 360° - 180°$ are just the same as that for $\alpha_d = 0° - 180°$. As seen in Fig.(a), in order to obtain a reasonable value of $|V_{cb}|$, we cannot chose a value of a_d larger than $\alpha_d \sim 10°$, and also a value of a_d smaller than $-a_d \sim 1.5$. However, from Fig.(b), we can conclude there is no solution for a reasonable value of $|V_{us}|$ for any values of a_d and α_d even at the cost of the fitting of down-quark mass ratios. Therefore, in the next section, we proposed a revised model for quark mass matrices keeping the model for the neutrino sectors.

III. PHENOMENOLOGY OF QUARK MASS MATRICES

In this paper, we put the following assumptions for a phenomenological forms of quark mass matrices M_u ($M_u^{1/2}$) and M_d:

(i) Differently from the previous model [5], we regard that not only $\langle Y_u \rangle$ (also $\langle \Phi_u \rangle$) but also $\langle Y_d \rangle$ are real, i.e. $\alpha_d = 0$ in Eq.(2.10). Instead, we consider that CP violation in the quark sector originates in a phase matrix $\langle P_d \rangle = v_{Pd} P_d \equiv v_{Pd} \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ which does not affect the down-quark mass ratios, but does only the CKM mixing. Namely the quark mass matrices $M_u^{1/2}$ and M_d is given by

$$M_u^{1/2} \propto M_e^{1/2}(\mathbf{1} + a_u X)M_e^{1/2} + \xi_u \left(M_e^{1/2}M_e^{1/2}(\mathbf{1} + a_u X) + (\mathbf{1} + a_u X)M_e^{1/2}M_e^{1/2} \right) + m_{0u}\mathbf{1}, \qquad (3.1)$$

$$M_d \propto P_d \left[M_e^{1/2}(\mathbf{1} + a_d X)M_e^{1/2} + \xi_d \left(M_e^{1/2}M_e^{1/2}(\mathbf{1} + a_d X) + (\mathbf{1} + a_d X)M_e^{1/2}M_e^{1/2} \right) + m_{0d}\mathbf{1} \right] P_d, \qquad (3.2)$$

so that the CKM matrix V is given by $V = R_u^T P_d R_d$, where R_u and R_d are defined by $R_u^T M_u^{1/2} R_u = \mathrm{diag}(+\sqrt{m_u}, -\sqrt{m_c}, +\sqrt{m_t})$ (for $a_u \sim -1.8$) and $R_d^T (P_d^\dagger M_d P_d^\dagger) R_d = \mathrm{diag}(m_d, m_s, m_b)$, respectively.

(ii) Similarly to Eq.(2.15), we assume the ξ_q-terms which originate in the re-ordering of the fields with the same quantum numbers.

(iii) Since only two of the three phase parameters ϕ_1, ϕ_2 and ϕ_3 in the phase matrix $P_d = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ are physically independent parameters. For convenience, we take $\phi_3 = 0$.

(iv) It is better that the parameter number is as few as possible. We consider that the first term is dominant in Eq.(3.1) [and also Eq.(3.2)], and we will consider ξ_q- and m_{0q}-terms as the need arises. As seen later, we can do fitting without ξ_d- and m_{0u}-terms.

Of course, we consider that these relations are derived from SUSY vacuum conditions for a given superpotential W. However, prior to investigating the superpotential form, from a phenomenological point of view, we would like to investigate whether there is a possible parameter region or not in the present model. A yukawaon model for the phenomenological forms (3.1) and (3.2) will be discussed in the next section.

Since the mass spectra $M_u^{1/2}(a_u)$ and $M_d(a_d)$ have the same behavior for the parameter a_q (a_u and a_d), we illustrate the mass spectra versus a_q in the limit of $\xi_q = 0$ in Fig.2. (The mass values in Fig.2 read ($\sqrt{|m_u|}, \sqrt{|m_c|}, \sqrt{|m_t|}$) and ($|m_d|, |m_s|, |m_b|$) for the up- and down-quark sectors, respectively.

In the present model, too, the model for the up-quark sector and neutrino sector is essentially unchanged from the previous model [5] except for the ξ_u-term given in (3.1). For reference, in Fig.3, we illustrate the up-quark mass ratios $\sqrt{m_u/m_c}$ and $\sqrt{m_c/m_t}$ versus a_u and ξ_u. As seen in Fig.3, there are two set of the solution (a_u, ξ_u) [regions (i) and (ii) illustrated in Fig.3] which can give reasonable up-quark mass ratios. However, the region (ii) cannot give reasonable CKM mixing parameters. Hereafter, by taking fitting of neutrino mixing parameters into consideration, too, we will take $a_u = -1.764$ and $\xi_u = 0.0070$ in the region (i). The choice of $(a_u, \xi_u) = (-1.764, 0.0070)$ can give up-quark mass ratios

$$\sqrt{\frac{m_u}{m_c}} = 0.0619, \qquad \sqrt{\frac{m_c}{m_t}} = 0.0559. \qquad (3.3)$$

In model-building of the down-quark sector, we give the down-quark mass ratio m_s/m_b preference rather than m_d/m_s, because it is not so difficult to adjust the ratio m_d/m_s without affecting the CKM parameter fitting as we demonstrate later. In Fig.4, we illustrate behavior of m_s/m_b versus a_d. As seen in Fig.4, there are three regions which can give reasonable mass ratio m_s/m_b. However, the regions (ii) and (iii) cannot give reasonable CKM mixing parameters. (The region (ii) corresponds to a parameter region adopted in the old model [5].) Hereafter, we will show the case (i) (i.e. $a_d \sim -17$) in detail.

Next, we investigate possible parameter regions which can give reasonable CKM mixing parameters. We take $|V_{us}|$, $|V_{ub}|$, $|V_{cb}|$ and $|V_{td}|$ as four independent parameters in the CKM matrix. In Fig.5, we illustrate allowed regions in
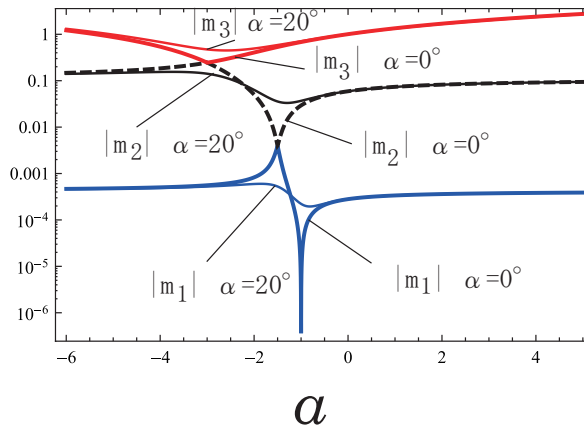
FIG. 2: Eigenvalues $|m_1|, |m_2|, |m_3|$ versus a parameter a in a mass matrix $M = M_e^{1/2}(\mathbf{1}+ae^{i\alpha}X)M_e^{1/2}$ with $\alpha = 0$ (thick curves). For reference, a case with $\alpha = 20°$ is also illustrated by thin curves in the figure. $(|m_1|, |m_2|, |m_3|)$ read $(\sqrt{|m_u|}, \sqrt{|m_c|}, \sqrt{|m_t|})$ and $(|m_d|, |m_s|, |m_b|)$ for up- and down-quark sectors, respectively. Numerical values of the eigenvalues are given in a unit of $(m_e + m_\mu + m_\tau)$.
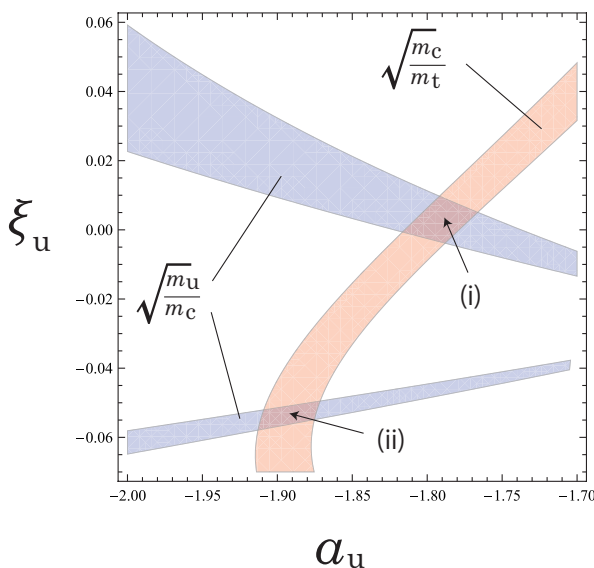

FIG. 3: Allowed region in the a_u - ξ_u plane obtained from the up-quark mass ratios $\sqrt{m_u/m_c}$ and $\sqrt{m_c/m_t}$. The shaded areas are consistent with the observed values given in Eq.(2.13).

the ϕ_1 - ϕ_2 plane obtained from $|V_{ij}|$ with $|V_{ij}|_{obs}$ under $a_u = -1.764$, $\xi_u = 0.0070$ and $a_d = -16.6$ whose values are obtained from global best fit. As seen in Fig.5, the value of $\phi_2 \simeq 180°$ is in favor of the observed CKM mixing parameters. The case with $\phi_2 = 180°$ is also illustrated in Fig.6. It is interesting that $|V_{ij}|$ take their minimum at $\phi_1 \simeq 180°$. From Fig.5 We find that $\phi_1 \simeq \pm16° + 180°$ is in favor of the observed CKM mixing parameters.

In conclusion, our best hit parameters are

$$a_u = -1.764, \quad \xi_u = 0.0070, \quad a_d = -16.6, \quad \phi_1 = 196.0° \ (164.0°), \quad \phi_2 = 181.5° \ (178.5°) \tag{3.4}$$

together with $\xi_d = 0$, and then we obtain the predicted CKM mixing parameters

$$|V_{us}| = 0.2259, \quad |V_{cb}| = 0.04141, \quad |V_{ub}| = 0.00418, \quad |V_{td}| = 0.00854. \tag{3.5}$$

However, the parameter value $a_d = -16.6$ gives considerably small value of m_d/m_s, i.e. $m_d/m_s = 0.00358$. In order to correct this wrong value, we must take m_{0d} with a non-zero value. By taking a value
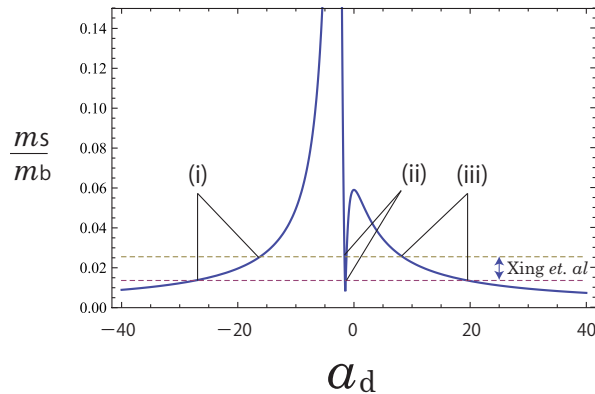
$$m_{0d}/m_0 = -0.0061, \tag{3.6}$$

FIG. 4: a_d dependence of m_s/m_b. The dotted lines show the observed down-quark mass ratio $m_s/m_d = 0.019 + 0.006$ and $0.019 - 0.006$.
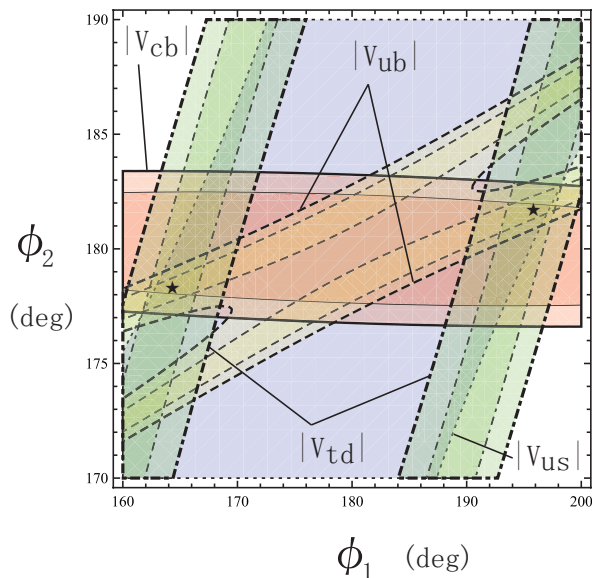

FIG. 5: Allowed regions in the ϕ_1 - ϕ_2 plane obtained from the CKM mixing parameters $|V_{ij}|$. Shaded areas are consistent with the observed values $|V_{ij}|_{obs}$ in Eq.(2.18) . The parameter values of a_u, ξ_u and a_d are chosen as $a_u = -1.764$, $\xi_u = 0.0070$ and $a_d = -16.6$ respectively. The star ($\star$) indicates the best fit points [see Eq.(3.4)].

where $m_0 \equiv m_e + m_\mu + m_\tau$, we obtain the reasonable down-quark mass ratios

$$\frac{m_d}{m_s} = 0.0529, \qquad \frac{m_s}{m_b} = 0.0231, \tag{3.7}$$

without affecting the CKM mixing parameters.

On the other hand, for neutrino mixing parameters, the model is essentially the same as before. In Fig.7, we illustrate ξ_ν dependence of the neutrino mixing parameters. As seen in Fig.7, the model can predict $\sin^2 2\theta_{atm} \simeq 1$ and $|U_{13}|^2 \simeq 0$ independently of ξ_ν. The value of ξ_ν is determined from the observed value $\tan^2 \theta_{solor} = 0.457^{+0.038}_{-0.041}$. The value of ξ_ν and the neutrino mixing parameters are listed in Table I.

IV. SUPERPOTENTIAL

In this section, by taking the phenomenological results with $\xi_d = 0$ and $m_{0u} = 0$ in the previous section into consideration, we discuss a possible form of the superpotential W. We assume the following form for W:
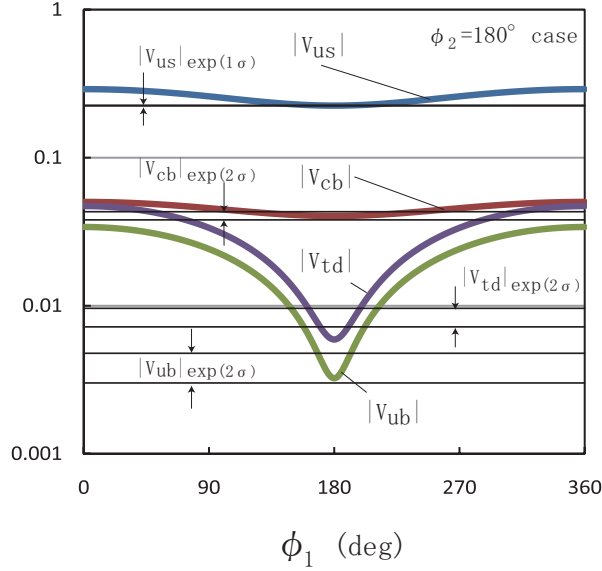
$$W = W_e + W_R + W_u + W_d, \tag{4.1}$$

FIG. 6: ϕ_1 dependence of the CKM mixing parameters $|V_{ij}|$ in the case $\phi_2 = 180°$. In the case of $\phi_2 = 180°$, $\phi_1 = 191°$ and $169°$ are in favor of all the observed $|V_{ij}|_{exp}(2\sigma)$.
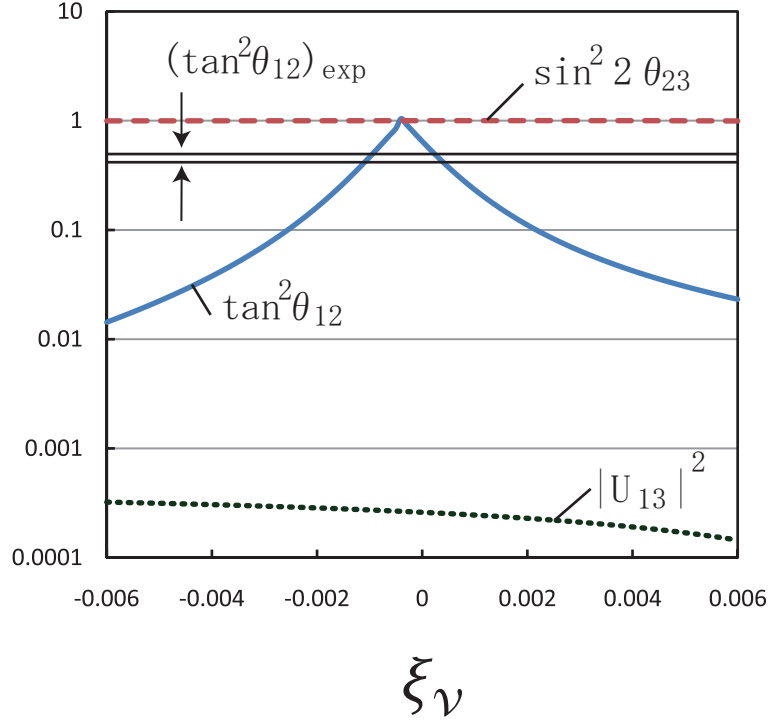

FIG. 7: ξ_ν dependence of the neutrino mixing parameters $\tan^2 \theta_{solar}$ (solid curve), $\sin^2 2\theta_{atm}$ (dashed curve) and $|U_{13}|^2$ (dotted curve). The up-quark mass matrix parameters are chosen as $a_u = -1.764$ and $\xi_u = 0.0070$.

$$W_e = \mu_e \mathrm{Tr}[Y_e \Theta_e] + \lambda_e \mathrm{Tr}[\Phi_e \Phi_e \Theta_e], \tag{4.2}$$

$$W_R = \mu_R \mathrm{Tr}[Y_R \Theta_R] + \frac{\lambda_R}{\Lambda} \left\{ \mathrm{Tr}[(Y_e P_u \Phi_u + \Phi_u P_u Y_e)\Theta_R] + \xi_\nu \mathrm{Tr}[(P_u Y_e \Phi_u + \Phi_u Y_e P_u)\Theta_R] \right\}, \tag{4.3}$$

$$W_u = \mu_u \mathrm{Tr}[Y_u \Theta_u] + \lambda_u \mathrm{Tr}[\Phi_u \Phi_u \Theta_u]$$

ξ_ν	$\tan^2\theta_{solor}$	$\sin^2 2\theta_{atm}$	$|U_{13}|^2$
$+0.00031$	0.457	0.999	2.56×10^{-4}
-0.00102	0.457	0.998	2.74×10^{-4}

TABLE I: Input value of ξ_ν and predicted values of the neutrino mixing parameters in the up-quark mass matrix parameters $a_u = -1.764$ and $\xi_u = 0.0070$ which can give reasonable up-quark mass ratios.

Fields	Y_e	Φ_e	Θ_e	Y_R	Θ_R	Y_u	Φ_u	P_u	Θ_u	Θ'_u	E_u,X_u	Y_d	P_d	Θ_d	E_d,X_d	E_{0d}
Q_X	x_e	$\frac{1}{2}x_e$	$-x_e$	$2x_e$	$-2x_e$	x_u	$\frac{1}{2}x_u$	$x_e-\frac{1}{2}x_u$	$-x_u$	$-\frac{1}{2}x_u$	$\frac{1}{2}x_u-x_e$	x_d	x_P	$-(x_d+2x_P)$	$x_d+2x_P-x_e$	x_d+2x_P
R charge	r	$\frac{1}{2}r$	$2-r$	0	2	0	0	0	2	2	$-r$	0	0	$2-r$	0	r

TABLE II: U(1)$_X$ charges of the yukawaons. For the time being, we assign different charges for the fields E_u and E_d (X_u and X_d) by assuming that those are different fields. For R charges, see text.

$$+\mu'_u\mathrm{Tr}[\Phi_u\Theta'_u] + \frac{\lambda'_u}{\Lambda}\left\{\mathrm{Tr}[\Phi_e S_u\Phi_e\Theta'_u] + \xi_u\mathrm{Tr}[(\Phi_e\Phi_e S_u + S_u\Phi_e\Phi_e)\Theta'_u]\right\}, \tag{4.4}$$

$$W_d = \frac{\lambda_d}{\Lambda}\mathrm{Tr}[P_d Y_d P_d\Theta_d] + \frac{\lambda'_d}{\Lambda}\mathrm{Tr}[\Phi_e S_d\Phi_e\Theta_d] + \mu_{0d}\mathrm{Tr}[E_{0d}\Theta_d], \tag{4.5}$$

where, for convenience, we have denoted linear combinations of fields E_q and X_q as $S_q = E_q + a_q X_q$ ($q = u, d$) and $\langle E_q\rangle = v_{Eq}\mathbf{1}$ and $\langle X_q\rangle = v_{Xq}X$. The relations (2.6), (2.15), (3.1) and (3.2) are obtained from supersymmetric vacuum conditions $\partial W/\partial\Theta_e = 0$, $\partial W/\partial\Theta_R = 0$, $\partial W/\partial\Theta'_u = 0$ and $\partial W/\partial\Theta_d = 0$, respectively. Since we take a supersymmetric vacuum with $\langle\Theta_e\rangle = \langle\Theta_R\rangle = \langle\Theta_u\rangle = \langle\Theta'_u\rangle = \langle\Theta_d\rangle = 0$, SUSY vacuum conditions which are obtained by differentiating with respective to a field except for Θ_A ($A = u, d, \cdots$) does not play an effective role because such a condition always includes $\langle\Theta_A\rangle$. Although further superpotential are needed in order to derive the specific VEV forms (2.7), (2.11) and (2.16), we do not give those superpotential terms explicitly in this paper. For the time being, we ad hoc assume those VEV forms. Especially, for the form of $\langle\Phi_e\rangle$, Eq.(2.7), we will use the charged lepton mass values at $\mu = m_Z$.

In Table II, we give U(1)$_X$ charge assignments which can lead to the superpotential forms (4.2) – (4.5). However, the assignments can allow unwelcome terms in the superpotential, for example, $\mathrm{Tr}[S_u P_u]$, $\mathrm{Tr}[\Phi_u P_u\Theta_e]$, and so on. Such terms can be forbidden by assuming suitable R charge assignments. For example, when we take R-charges as $R(\ell) = 1-r$, $R(e^c) = R(\nu^c) = R(q) = R(u^c) = R(d^c) = 1$ and $R(H_u) = R(H_d) = 0$, we can forbid the terms $\mathrm{Tr}[S_u P_u]$ and $\mathrm{Tr}[\Phi_u P_u\Theta_e]$ by taking R-charges of other fields as given in Table II.

In the phenomenological study in Sec.III, the VEV values of Φ_e play an essential role in evaluating the predicted values. Although it has been tried to build a model [8, 9] which gives VEV spectrum (2.7), it is not clear whether such a model can be applicable or not to the present model straightforwardly. In this paper, we do not give a superpotential form which can lead to the VEV spectrum (2.7). We have assumed the VEV value given by Eq.(2.7), where we have used the values of charged lepton masses at the scale $\mu = m_Z$.

Also, so far, we have not given superpotential forms which lead to VEV matrices $\langle E_q\rangle$, $\langle X_q\rangle$, $\langle P_u\rangle$ and $\langle P_d\rangle$. In general, any Hermitian VEV matrix $\langle A\rangle$ can be obtained from a superpotential

$$W = \lambda_1(\mathrm{Tr}[A])^3 + \lambda_2\mathrm{Tr}[AA]\mathrm{Tr}[A] + \lambda_3\mathrm{Tr}[AAA]. \tag{4.6}$$

[However, we must assign a U(1)$_X$ charge $-3Q_X(A)$ for the coefficients λ_i ($i = 1, 2, 3$).] For example, $(\lambda_1, \lambda_2, \lambda_3) = (1/6, -1/2, 1/3)$ (i.e. $W = \det A$) gives the form $\langle X\rangle$ given in Eq.(2.11). However, this method is not applicable to the form $\langle P_d\rangle$, because the VEV matrix is not Hermitian. In this paper, we have ad hoc assumed these VEV forms.

V.   CONCLUDING REMARKS

In conclusion, we have proposed a phenomenological quark and lepton mass matrices based on a yukawaon model. In Sec.II, we have demonstrated that the previous model [5], in which the CP violation originates only in the complex parameter a_d, cannot give reasonable CKM mixing values even if at the cost of the quark mass ratios. Differently from the previous model, in the present model, CP violating phases are introduced in the phase matrix P_d given in Eq.(3.2). In the up-quark sector, we have considered the ξ_u-term in Eq.(3.1), which originates that another order of the fields, $\Phi_e\Phi_e S_u + S_u\Phi_e\Phi_e$, cannot, in general, forbidden compared with the order of $\Phi_e S_u\Phi_e$ which has the same

U(1)$_X$ charges. The similar situation have been assumed in the neutrino sector, too, i.e. the ξ_ν-term in Eq.(4.3). (The values of ξ_u and ξ_ν are very small.) In contrast to those sectors, in the down-quark sector, we have not considered such a ξ_d-term as well as an additional term $P_d P_d Y_d + Y_d P_d P_d$ corresponding to $P_d Y_d P_d$ in Eq.(4.5). This is a result from the phenomenological study, and the theoretical reason for the absence is unknown at present. Also we note that the phenomenological fit requires the m_{0d} term added to Eq.(3.2), but it does not need an m_{0u} in Eq.(3.1).

Although we have been able to obtain reasonable parameter fitting not only for the observed lepton mixing but also for the observed quark mixing, the model still includes ad hoc assumptions. Those are our future task. We consider that for the next step of the investigation it is important to clarify what parts are problems to get good fitting of the data. Our model-building will proceed step by step.

Acknowledgment

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