Charged Lepton Mass Spectrum and a Scalar Potential Model

Yoshio Koide

Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
E-mail address: koide@het.phys.sci.osaka-u.ac.jp

Abstract

Stimulated by a recent work by Sumino, on the basis of a model in which effective Yukawa coupling constants are described by vacuum expectation values of a scalar Φ with 3×3 components, a possible form of the scalar potential $V(\Phi)$ which can well describe the observed charged lepton mass spectrum is investigated without referring to a specific flavor symmetry model. A general relation among eigenvalues of $\langle \Phi \rangle$ is derived without referring to an explicit form of $V(\Phi)$.

1 Introduction

In the standard model, mass spectra of quarks and leptons are originated in the Yukawa coupling constants which are fundamental constants in the theory. On the other hand, so for, many phenomenological relations in the observed mass spectra and mixings have been known. Therefore, it is unlikely that there are so many fundamental constants in the nature. Then, one may consider that the Yukawa coupling constants are apparent constants in an effective theory. Previously, Froggatt and Neilsen [1] have proposed a model in which masses and mixings are explained by a multiplicative structure $(\langle \phi \rangle / \Lambda)^n$ of a vacuum expectation value (VEV) $\langle \phi \rangle$ of a scalar ϕ. However, in this paper, we will try to understand the hierarchical structures of the flavors not by U(1) flavor charges like a Froggatt-Neilsen model, but by a VEV structure $\langle \Phi \rangle$ of a scalar Φ with 3×3 components. For example, in order to explain a charged lepton mass relation [2]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}, \tag{1.1}$$

the author [3] has previously proposed a simple form of the scalar potential $V(\Phi)$ under an assumption that the charged lepton masses m_{ei} are given by $m_{ei} \propto v_i^2$, where (v_1, v_2, v_3) are eigenvalues of $\langle \Phi \rangle$. The relation (1.1) is well satisfied with the order of 10^{-5} for the observed pole masses [4], i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$. However, in conventional mass matrix models, "masses" mean not "pole masses", but "running masses". The formula (1.1) is, at most, valid with the order of 10^{-3} for the running masses.

Recently, a possible solution of this problem has been proposed by Sumino [5, 6]: He considers that a flavor symmetry is gauged, and radiative corrections due to photon to the charged lepton masses are exactly canceled by those due to flavor gauge bosons. Therefore, as far as the charged lepton masses are concerned, the running mass values are exactly equal

to the pole mass values. (Hereafter, we will refer to this mechanism as Sumino's mechanism.) Moreover, as a byproduct of his model, he has obtained a charged lepton mass relation [6]

$$m_\mu^{3/2} + \sqrt{m_e m_\mu m_\tau} = \sqrt{m_e m_\tau}(\sqrt{m_e} + \sqrt{m_\tau}). \tag{1.2}$$

(We also refer to the relation (1.2) as Sumino's relation on the charged lepton masses.) Then, we can completely determine the charged lepton mass spectrum by using two relations (1.1) and (1.2) simultaneously. The predicted value of the electron mass m_e is in fairly good agreement with the observed one.

In the present paper, stimulated by the Sumino relation (1.2), we investigate a possible scalar potential form of Φ. However, we do not refer to the Sumino mechanism, so that we do not discuss whether the flavor symmetry is gauged or global. We discuss the charged lepton mass relations not with the order of 10^{-5} but with the order of 10^{-3}. Since our interest is only in the VEV values (v_1, v_2, v_3) of the eigenvalues of $\langle\Phi\rangle$, we denote the scalar potential $V(\Phi)$ in terms of v_i as follows:

$$V_S = \lambda_1(v_1^2 v_3^2 + v_2^2 v_2^2 + v_3^2 v_1^2) + \lambda_2(v_1 v_3 + v_2 v_2 + v_3 v_1)^2$$

$$+\lambda_3(v_1^4 + v_2^4 + v_3^4) + \lambda_4(v_1^2 + v_2^2 + v_3^2)^2, \tag{1.3}$$

where we have denoted only typical terms which are presumed from the original Sumino potential [6]. In the original Sumino model [6], the potential $V(\Phi)$ has been given only by λ_1 and λ_3 terms. In contrast to the Sumino model, in order to derive the Sumino relation (1.2), the author [7] has recently proposed a model based on a so-called "supersymmetric yukawaon" model, where a superpotential $W(\Phi)$ is practically given by the λ_1, λ_2 and λ_4 terms, and the λ_3 term is forbidden by R-charge conservation. In any cases, the Sumino relation (1.1) is derived from the first term in the potential (1.3), so that the first term must be a dominant term in the model, and other terms play a role in giving small corrections to the Sumino relation (1.2). Hereafter, we refer to the potential (1.3) as an extended Sumino potential.

The Sumino model is based on a flavor symmetry U(9) which is reduced to U(3)×O(3). On the other hand, the potential form (1.3) in the supersymmetric yukawaon model [7] is based on a U(3) flavor symmetry (however, by assuming many subsidiary fields). In this paper, we will also not refer to an explicit model for the potential form (1.3) from a practical point of view. Since the purpose of the present paper is to discuss only a substantial framework of the extended Sumino potential (1.3), we deal with Φ as if it is a plain 3×3 matrix and we do not discuss what flavor symmetry is supposed in a model concerned and to what multiplet the field Φ belongs. For example, when we consider a diagonal form of $\langle\Phi\rangle$ and a form $\langle\bar{\Phi}\rangle \equiv T\langle\Phi\rangle T$ as follows

$$\Phi = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_3 \end{pmatrix}, \quad \bar{\Phi} = \begin{pmatrix} v_3 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_1 \end{pmatrix}, \tag{1.4}$$

by assuming a transformation matrix

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{1.5}$$

(here and hereafter, for simplicity, we denote the VEV matrix $\langle \Phi \rangle$ as Φ simply), the $\lambda_1, \cdots$ and λ_4 terms in the potential (1.3) can be expressed as $\text{Tr}[\Phi\bar{\Phi}\Phi\bar{\Phi}]$, $\text{Tr}[\Phi\bar{\Phi}]\text{Tr}[\Phi\bar{\Phi}]$, $\text{Tr}[\Phi\Phi\Phi\Phi]$, and $\text{Tr}[\Phi\Phi]\text{Tr}[\bar{\Phi}\bar{\Phi}]$, respectively. On the other hand, if we choose a flavor basis in which a form of $\langle \Phi \rangle$ is given by

$$\Phi = \begin{pmatrix} 0 & 0 & v_1 \\ 0 & v_2 & 0 \\ v_3 & 0 & 0 \end{pmatrix}, \quad \Phi^T = \begin{pmatrix} 0 & 0 & v_3 \\ 0 & v_2 & 0 \\ v_1 & 0 & 0 \end{pmatrix}, \tag{1.6}$$

we can express those terms in (1.3) as $\text{Tr}[\Phi\Phi\Phi\Phi]$, $\text{Tr}[\Phi\Phi]\text{Tr}[\Phi\Phi]$, $\text{Tr}[\Phi\Phi^T\Phi\Phi^T]$, and $\text{Tr}[\Phi\Phi^T]\text{Tr}[\Phi\Phi^T]$, respectively. In any cases, in order to build a realistic model, we will be obliged to give an explicit model of the flavor symmetry and to introduce some additional fields. However, the purpose of the present paper is to investigate a hint for a further extension of the Sumino model, so that we will not refer to such a realistic model.

In the Sumino model [6], the relation (1.2) has been derived on the condition that the relation (1.1) is exactly satisfied. On the other hand, in the supersymmetric yukawaon model [7], the relation (1.1) is satisfied only as an approximate relation for the running masses. Another interest in the present paper is to see what happens in the charged lepton mass spectrum when we put a potential term V_K which leads to the relation (1.1) in addition to the Sumino potential (1.3). In the present paper, we adopt the following form [3] as V_K:

$$V_K = \mu^2 \text{Tr}[\Phi\Phi] + \lambda_K \text{Tr}[\hat{\Phi}\hat{\Phi}](\text{Tr}[\Phi])^2$$

$$= \mu^2(v_1^2 + v_2^2 + v_3^2) + \lambda_K \left[v_1^2 + v_2^2 + v_3^2 - \frac{1}{3}(v_1 + v_2 + v_3)^2 \right](v_1 + v_2 + v_3)^2, \tag{1.7}$$

where $\hat{\Phi}$ is a traceless matrix which is defined as $\hat{\Phi} = \Phi - \frac{1}{3}\text{Tr}[\Phi]$ in the expression (1.4). [In Ref.[3], the form (1.7) has been given by assuming that the mass term (the μ^2 term) is given in terms of a nonet ($\mathbf{8} + \mathbf{1}$) form of U(3), while the dimension 4 term (the λ_K term) is given only in terms of a form "octet-octet".] In the next section, we will investigate a possible VEV spectrum of Φ under the potential $V = V_K + V_S$. Since we have 4 parameters in the potential (1.3), we cannot predict the charged lepton mass spectrum under the general form (1.3) of V_K. However, when we build an explicit model, we can drop some of the terms in the potential (1.3) by assuming some additional symmetries. The purpose is to see whether we can obtain a more plausible result by dropping some of the terms in the potential (1.3) or not. Then, the result will provide us a promising clue to a realistic lepton mass matrix model.

2 Minimizing conditions of the potential

In this section, we investigate minimizing conditions of $V = V_K + V_S$. From the explicit form of V_K, (1.7), but without assuming an explicit form of V_S, we obtain the following conditions

$$\frac{\partial V}{\partial v_i} = 2v_i \left[A + \frac{1}{v_i}(B + B_i) \right] = 0, \tag{2.1}$$

where $i = 1, 2, 3$ and

$$A = \mu^2 + \lambda_K (v_1 + v_2 + v_3)^2, \tag{2.2}$$

$$B = \lambda_K (v_1 + v_2 + v_3) \left[v_1^2 + v_2^2 + v_3^2 - \frac{2}{3}(v_1 + v_2 + v_3)^2 \right], \tag{2.3}$$

$$B_i = \frac{1}{2} \frac{\partial V_S}{\partial v_i}. \tag{2.4}$$

Hereafter, we investigate only a case with $v_i \neq 0$ for all $i = 1, 2, 3$.

If the extended Sumino potential (1.3) is absent, i.e. $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$, the condition $A + B/v_1 = A + B/v_2 = A + B/v_3 = 0$ leads to $A = B = 0$, so that we obtain

$$K \equiv \frac{v_1^2 + v_2^2 + v_3^2}{(v_1 + v_2 + v_3)^2} = \frac{2}{3}, \tag{2.5}$$

together with

$$(v_1 + v_2 + v_3)^2 = -\frac{\mu^2}{\lambda_K}. \tag{2.6}$$

The result (2.5) means the charged lepton mass relation (1.1). However, when the Sumino potential (1.3) exists, the relation (2.5) is not satisfied as we see below.

When V_S exists, independently of the explicit form of V_S, we obtain three conditions

$$-A = \frac{1}{v_1}(B + B_1) = \frac{1}{v_2}(B + B_2) = \frac{1}{v_3}(B + B_3). \tag{2.7}$$

From Eq.(2.7), we obtain $(B + B_1)/v_1 = (B + B_3)/v_3$, i.e.

$$B = \frac{1}{v_1 - v_3}(v_3 B_1 - v_1 B_3). \tag{2.8}$$

By using a general formula $k = (c_1 a + c_2 b)/(c_1 + c_2)$ for a given relation $k = a = b$, we can write A as follows:

$$-A = \frac{1}{v_1 + v_3} \left(v_1 \frac{B + B_1}{v_1} + v_3 \frac{B + B_3}{v_3} \right) = \frac{1}{v_1 + v_3}(2B + B_1 + B_3). \tag{2.9}$$

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When we eliminate B from $(B + B_2)/v_2 = (2B + B_1 + B_3)/(v_1 + v_3)$ by using Eq.(2.8), we can obtain a general formula for v_i

$$(v_3 - v_2)B_1 + (v_2 - v_1)B_3 + (v_1 - v_3)B_2 = 0. \tag{2.10}$$

Also, from Eq.(2.9), we obtain

$$A = -\frac{B_1 - B_3}{v_1 - v_3}. \tag{2.11}$$

Hereafter, we will use the conditions (2.8), (2.10) and (2.11) instead of the three conditions $\partial V/\partial v_i = 0$ as minimizing conditions of $V = V_K + V_S$. We will refer to the conditions (2.11), (2.8) and (2.10) as the conditions A, B and C, respectively. Note that the condition C is satisfied independently of the values of A and B which depend on the parameters μ^2 and λ_K in V_K.

For the explicit form of V_S, (1.3), we obtain

$$\lambda_1 S(v_1, v_2, v_3) + \lambda_2(v_2^2 + 2v_1v_3)(2v_2 - v_1 - v_3)$$

$$+\lambda_3[(2v_2^2 - v_1^2 - v_3^2)v_2 - S(v_1, v_2, v_3)] = 0, \tag{2.12}$$

where the factor $S(v_1, v_2, v_3)$ is defined by

$$S(v_1, v_2, v_3) = v_2^3 + v_1v_3v_2 - v_1v_3(v_1 + v_3), \tag{2.13}$$

and $S(v_1, v_2, v_3) = 0$ gives the Sumino mass relation (1.2).

Note that the λ_4 term (and also a term with a form $(v_1+v_2+v_3)^n$) does not affect the general formula (2.10). However, this does not mean that the λ_4 term does not affect the minimizing conditions. The λ_4 term contributes to the factors B_i as follows:

$$
\begin{aligned}
B_1 &= 2\lambda_1 v_1 v_3^2 + 2\lambda_2(v_2^2 + 2v_1v_3)v_3 + 2\lambda_3 v_1^3 + 2\lambda_4(v_1^2 + v_2^2 + v_3^2)v_1, \\
B_3 &= 2\lambda_1 v_1^2 v_3 + 2\lambda_2(v_2^2 + 2v_1v_3)v_1 + 2\lambda_3 v_3^3 + 2\lambda_4(v_1^2 + v_2^2 + v_3^2)v_3, \\
B_2 &= 2\lambda_1 v_2^3 + 2\lambda_2(v_2^2 + 2v_1v_3)v_2 + 2\lambda_3 v_2^3 + 2\lambda_4(v_1^2 + v_2^2 + v_3^2)v_2.
\end{aligned} \tag{2.14}
$$

Therefore, the λ_4 term contributes to the condition B, Eq.(2.11), as follows:

$$A = 2[\lambda_1 v_1 v_3 + \lambda_2(v_2^2 + 2v_1v_3) - \lambda_3(v_1^2 + v_1v_3 + v_3^2) - \lambda_4(v_1^2 + v_2^2 + v_3^2)]. \tag{2.15}$$

In contrast to the condition A, (2.15), the λ_4 term does not contribute to the condition B, (2.8), i.e.

$$B = -2(v_1 + v_3)[\lambda_1 v_1 v_3 + \lambda_2(v_2^2 + 2v_1v_3) - \lambda_3 v_1 v_3]. \tag{2.16}$$

Since we have no information on the values A and B which are defined by Eqs.(2.2) and (2.3), respectively, what we can use to investigate the structure of V_K is only the condition C, (2.12). For reference, we denote numerical values of Eq.(2.12):

$$0.00127398\,\lambda_1 - 0.0453018\,\lambda_2 - 0.198278\,\lambda_3 = 0. \tag{2.17}$$

Here, we have used the observed pole mass values [4] and the numerical values have been given in the unit of v_0^3 [$v_0^2 \equiv v_1^2 + v_2^2 + v_3^2$, and $(v_1, v_2, v_3)/v_0 = (0.0164734, 0.236879, 0.971400)$]. If we assume an exact Sumino mechanism (cancellation between photon and family gauge boson diagrams), the relation (2.17) must be satisfied exactly. Even if we does not require such a cancellation, the numerical relation (2.17) must approximately be satisfied because we know that the deviation from the mass formula (1.1) is only 10^{-3} for running masses at a typical energy scale ($\mu \sim 10^3 - 10^{16}$ GeV) .

We readily find that there is no solution of $(\lambda_1, \lambda_2, \lambda_3)$ except for a hierarchical case. For example, if we suppose a case $\lambda_2 = 0$, which is a case in the Sumino model [6], we obtain $\lambda_3/\lambda_1 = 0.006422$, and if we suppose a case $\lambda_3 = 0$, which is a case in the yukawaon model [7], we obtain $\lambda_2/\lambda_1 = 0.028122$. Thus, we cannot have a solution with the same order of λ_i.

In this paper, we will investigate a case with a more plausible value of λ_i/λ_j. For such the purpose, we will discuss the remaining conditions A and B in the next section.

3 Possible form of V_S

In this section, we discuss the conditions A, (2.15), and B, (2.16), in addition to the condition C, (2.12). First, we investigate a possibility of $B = 0$, so that the condition B is given by

$$\lambda_1 v_1 v_3 + \lambda_2(v_2^2 + 2v_1 v_3) - \lambda_3 v_1 v_3 = 0. \tag{3.1}$$

In the present paper, the relation (2.5) is not always exactly satisfied. For convenience, let us define the following parameter ξ:

$$K \equiv \frac{v_1^2 + v_2^2 + v_3^2}{(v_1 + v_2 + v_3)^2} = \frac{2}{3}(1 + \xi), \tag{3.2}$$

so that the parameter B is expressed as

$$B = \frac{2}{3}\lambda_K(v_1 + v_2 + v_3)^3 \xi. \tag{3.3}$$

For example, the value of ξ is $\xi = (0.011 \pm 0.014) \times 10^{-3}$ for pole masses, while $\xi = (1.95 \pm 0.02) \times 10^{-3}$ and $\xi = (2.30 \pm 0.02) \times 10^{-3}$ at $\mu = 10^3$ GeV and $\mu = 10^{12}$ GeV, respectively, for running masses in a SUSY model with $\tan\beta = 10$. (See Ref.[7].) If we accept the Sumino mechanism, the value of ξ must exactly be zero, so that we can regard the value of B as $B = 0$. (Of course, the case $\xi = 0$ cannot be derived from the present formulation.) Then, we can use the second condition (3.1) in addition to the first condition (2.12). We have two conditions for three parameters λ_1, λ_2 and λ_3, so that we can completely determine the relative rations among λ_i. When we use the pole mass values for m_{ei}, we obtain numerical results

$$\frac{\lambda_2}{\lambda_1} = -0.173249, \quad \frac{\lambda_3}{\lambda_1} = 0.046009. \tag{3.4}$$

Again, we obtain a hierarchical result for λ_i. However, it is not likely that these values (3.4) can be understood by considering a specific model.

Next, we investigate a case with $A = 0$. By way of trial, we assume that the μ^2 term in V_K is always canceled by the term $\lambda_K(v_1 + v_2 + v_3)^2$, i.e. Eq.(2.6). Then, the condition A, (2.15), with the value $A = 0$ leads to a condition

$$\lambda_1 v_1 v_3 + \lambda_2(v_2^2 + 2v_1 v_3) - \lambda_3(v_1^2 + v_1 v_3 + v_3^2) - \lambda_4(v_1^2 + v_2^2 + v_3^2) = 0. \tag{3.5}$$

The condition (3.5) has a special solution $\lambda_2/\lambda_1 = \lambda_3/\lambda_1 = -\lambda_4/\lambda_1 = -1/3$ independently of values of v_i. However, the solution does not satisfy the condition (2.12), so that the solution is ruled out. Since the condition (3.5) contains four parameters $\lambda_1, \cdots, \lambda_4$, we cannot determine the relative ratios among λ_i unless we put an ansatz in addition to the conditions (2.12) and (3.5).

Since the λ_1 term is a main term in the Sumino relation, we consider $\lambda_1 \neq 0$. Then, we eliminate λ_1 from (2.12) and (3.5), we obtain

$$\lambda_2(v_2^2 + 2v_1 v_3)(v_1 v_3 - v_2^2)v_2 + \lambda_3[(v_1^2 + v_3^2)S + v_1 v_3 v_2(2v_2^2 - v_1^2 - v_3^2)]$$

$$+\lambda_4(v_1^2 + v_2^2 + v_3^2)S = 0, \tag{3.6}$$

where $S = S(v_1, v_2, v_3)$ defined by Eq.(2.13). A numerical relation of (3.6) for the pole mass values is expressed as $-0.83719\lambda_2 - 1.95002\lambda_3 + 1.27398\lambda_4 = 0$, i.e.

$$\lambda_4 = \frac{1}{1.521735}\lambda_2 + 1.530652\,\lambda_3. \tag{3.7}$$

In the yukawaon model [7], the λ_3 term is forbidden because of R charge conservation, so that the condition (3.7) suggests $\lambda_2 = (2/3)\lambda_4$, which is a likely case. [In the yukawaon model, the λ_1 term includes a contribution from another mechanism, so that the value of $|\lambda_1|$ can be much larger than the values $|\lambda_i|$ ($i = 2, 3$). For numerical fitting on the case with $\lambda_2 = (2/3)\lambda_4$, see Ref.[7], where the numerical fitting for running masses at several energy scales has been demonstrated.] Also, we may speculate a solution $\lambda_3 = (2/3)\lambda_4$ for a case $\lambda_2 = 0$. Anyhow, the relation (3.7) suggests an interesting constraint on the coefficients λ_i:

$$\lambda_3 - \frac{2}{3}\lambda_4 + \left(\frac{2}{3}\right)^2 \lambda_2 = 0. \tag{3.8}$$

[Note that the numerical relation (3.7) does not need to be exactly satisfied because we deal with running masses $m_{ei}(\mu)$ whose values are dependent on model parameters such as $\mu = \Lambda$, $\tan\beta$, and so on.]

For a case with $A = B = 0$, we cannot find a suggestive relation among λ_i because the conditions are too tight.

4 Concluding remarks

In conclusion, by stimulating Sumino's charged lepton mass relation, (1.2), we have investigated a possible scalar potential form of the field Φ, which plays a role in giving the charged lepton mass spectrum via $m_{ei} \propto v_i^2$ (v_i are eigenvalues of $\langle\Phi\rangle$). We have obtained a general

formula for v_i, Eq.(2.10), independently of a form of the potential $V(\Phi)$. For an explicit form of V_S, (1.3), we obtain the relation (2.12). For a model with $\lambda_2 = \lambda_3 = \lambda_4 = 0$, we obtain $S(v_1, v_2, v_3) = 0$, so that we get the Sumino relation (1.2) which is fairly satisfied with realistic charged lepton masses (for both cases of pole masses and running masses). For the pole masses, the condition (2.12) is numerically expressed as Eq.(2.17). As seen in Eq.(2.17), we must consider that the λ_2 and λ_3 terms must be considerably suppressed compared with the first term (λ_1 term) in order to protect the Sumino relation $S \simeq 0$ from destroying badly.

Also, we have obtained relations (2.11) (Condition A) and (2.8) (Condition B) which are dependent on V_K, and which are given by Eqs.(2.15) and (2.16) for the explicit form of V_K, (1.3). The condition (2.16) is rewritten as

$$\lambda_1 - \lambda_3 + \lambda_2 \frac{v_2^2 + 2v_1 v_3}{v_1 v_3} = -\frac{1}{3}\lambda_K \xi \frac{(v_1 + v_2 + v_3)^3}{(v_1 + v_3)v_1 v_3}, \tag{4.1}$$

which is numerically expressed as

$$\lambda_1 - \lambda_3 + 5.5065\,\lambda_2 = 38.738\,\lambda_K \xi, \tag{4.2}$$

where, for convenience, we have used the pole mass values for v_i although the value of ξ for pole mass values is zero. Since the observed value of ξ is $\xi \sim 10^{-3}$ for the running masses, the left hand side of Eq.(4.2) must sufficiently be canceled among λ_1, λ_2 and λ_3, or all of λ_i must be of the order of $\lambda_K \times 10^{-2}$. For a special case in which the Sumino mechanism holds exactly, i.e. $\xi = 0$, the condition (3.1) is required in addition to the relation(2.12). The simultaneous equations (2.12) and (3.1) require the numerical ratios among λ_i as (3.4). It does not seems that the numerical results (3.4) are understood from a simple model.

In contrast to the case $B = 0$, the case $A = 0$ leads to an interesting constraint for λ_i, (3.8), i.e. $\lambda_2/\lambda_4 = 2/3$ for a case with $\lambda_3 = 0$, and $\lambda_3/\lambda_4 = 2/3$ for a case with $\lambda_2 = 0$. Such cases are likely in building an explicit model. [However, we will need an additional ansatz which gives $A = 0$, Eq.(2.6).] It seems to be worthwhile searching a model with $A = 0$ even if we pay a cost of such an additional ansatz.

The conditions A and B are dependent on a structure of V_K. For example, in the original Sumino model [6], the relation (1.1) is exactly derived at an energy scale at which U(9) flavor symmetry is broken into U(3)×O(3). The potential V_S effectively appears at a level of U(3)×O(3). In his model, only the relation C is satisfied, but the conditions A and B are free. Even in such a case, the general formula (2.7) is exactly required, so that the use of the formula (2.7) will be useful for building a model of leptons.

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