Nearly Tribimaximal Neutrino Mixing without Discrete Symmetry

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Abstract

Based on a new approach to quark and lepton masses, where the mass spectra originate in vacuum expectation values of U(3)-flavor nonet (gauge singlet) scalars, a neutrino mass matrix of a new type is speculated. The mass matrix is described in terms of the up-quark and charged lepton masses, and it can lead to a nearly tribimaximal mixing without assuming any discrete symmetry. Quark mass relations are also discussed based on the new approach.

One of the most challenging problems in contemporary particle physics is to clarify the origin of flavors. For this purpose, searching for a unified description of the observed quark and lepton mass spectra will provide a promising clue to us. In conventional mass matrix model, the quark and lepton mass matrices M_f are given by the forms $(M_f)_{ij} = (Y_f)_{ij} v_H$, where $(Y_f)_{ij}$ are coupling constants of the Yukawa interactions $\bar{f}_{Li} f_{Rj} H^0$ and v_H is a vacuum expectation value (VEV) of the neutral component of the Higgs scalar H, $v_H = \langle H^0 \rangle$. Usually, each matrix Y_f has many independent parameters. Against this conventional approach, there is another idea: the origin of the mass spectra is due to VEV structures of Higgs scalars H_{ij} [1], i.e. $(M_f)_{ij} = y_f \langle (H^0)_{ij} \rangle$. In the present paper, we will investigate an extended model by separating the role of H_{ij} into two roles: one of the roles is to cause $\mathrm{SU}(2)_L$ symmetry breaking at the energy scale $\mu \sim 10^2$ GeV, and the conventional $\mathrm{SU}(2)_L$ doublet Higgs scalars H_u and H_d still play the role in this scenario; another one is to give an origin of the mass spectra, and we consider gauge-singlet scalars $(Y_f)_{ij}$ whose VEVs give effective Yukawa coupling constants $\langle (Y_f)_{ij} \rangle / \Lambda$ (Λ is an energy scale of the effective theory).

In this model, we assume U(3)-flavor nonet scalars [2], and we consider the following superpotential terms:

$$W_Y = \sum_{i,j} \frac{y_u}{\Lambda} U^i (Y_u)_i{}^j Q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} D^i (Y_d)_i{}^j Q_j H_d$$

$$+ \sum_{i,j} \frac{y_\nu}{\Lambda} L^i (Y_\nu)_i{}^j N_j H_u + \sum_{i,j} \frac{y_e}{\Lambda} L^i (Y_e)_i{}^j E_j H_d + h.c. + \sum_{i,j} y_R N_i (\bar{Y}_R)^{ij} N_j, \tag{1}$$

where Y_f ($f = u, d, \nu, e$) are U(3)-flavor nonet (gauge singlet) fields [3], and Q and L are quark and lepton $\mathrm{SU}(2)_L$ doublet fields, respectively, and U, D, N, and E are $\mathrm{SU}(2)_L$ singlet matter fields. In Eq.(1), we have assigned Q, E and N to $\mathbf{3}$ of $\mathrm{U}(3)_F$ and L, U and D to $\bar{\mathbf{3}}$ of $\mathrm{U}(3)_F$, respectively, so that all "would-be Yukawa-coupling-constant fields" Y_f ($f = u, d, \nu, e$) are assigned to nonet of $\mathrm{U}(3)_F$, while $\bar{Y}_R$ is assigned to $\bar{\mathbf{6}}$. Therefore, hereafter, we will regard Y_f as Hermitian and $\bar{Y}_R$ as symmetric. (Note that the present scenario with this assignment cannot apply to a grand unification theory (GUT) scenario, because, for example, in SU(5)-GUT, the fields Q and U should be assigned to the same multiplet $\mathbf{3}$, so that Y_u must be $\bar{\mathbf{6}}$, not nonet.) In order to distinguish the nonet fields Y_f ($f = u, d, \nu, e$) from each other, we assign additional U(1) charges q_f to Y_f ($f = u, d, \nu, e$), and $-q_u$ to U, $-q_e$ to E, and so on. The field $\bar{Y}_R$ has the charge $-2q_\nu$.

In the present approach, we will investigate relations among Y_f and Y_R by using supersymmetric (SUSY) vacuum conditions for the superpotential $W = W_u + W_d + W_\nu + W_e + W_R + W_Y$, where W_f ($f = u, d, \nu, e$) and W_R determine the VEV structures of Y_f and Y_R, respectively. (Since we can easily show $\langle Q \rangle = \langle L \rangle = \langle U \rangle = \langle D \rangle = \langle N \rangle = \langle E \rangle = 0$, hereafter, we will drop the term W_Y from W when we investigate the VEV structures of Y_f.) Such an approach to quark and lepton mass matrices has first been adopted by Ma [4] and has been developed by the author within a context of U(3)-flavor nonet model [3]. In the conventional mass matrix approach, the investigation has now been on a level with theoretically reliable ground via a long period of phenomenological investigations. However, the present approach is still in its beginning stage, so that we need more phenomenological investigations. Therefore, we adopt the following strategy in this approach: (i) First, we search for a possible form of the superpotential W which can successfully provide relations among the observed masses and mixings from the phenomenological point of view; (ii) Next, we investigate what symmetries or quantum number assignments can explain such a specific form of W. In this paper, we will investigate a possible form of W by putting weight on the step (i).

For convenience later, let us define a name of a flavor basis as follows: when a VEV matrix $\langle Y_f \rangle$ takes a diagonal form on a basis, we call the basis "f-basis", and we denote a form of a matrix A on the f-basis as $(A)_f$. (Since the ν-basis is defined for the Dirac neutrino mass matrix Y_ν, the basis is practically meaningless in a Majorana neutrino mass matrix model.)

Recently, as a byproduct in such approach, an interesting neutrino mass matrix form [5] has been reported: the form is given by $M_\nu \propto Y_e^{-1} Y_u^{1/2} + Y_u^{1/2} Y_e^{-1} + \xi_0 \mathbf{1}$; current models which give a tribimaximal neutrino mixing [6] have been proposed based on discrete symmetries, while the above mass matrix M_ν can provides a nearly tribimaximal neutrino mixing without assuming any discrete symmetry. On the other hand, in general, if a neutrino mass matrix M_ν can give reasonable masses and mixing, a neutrino mass matrix $\tilde{M}_\nu$ with an inverse form of M_ν, $\tilde{M}_\nu = m_0^2 M_\nu^{-1}$, can also give reasonable predictions, because, by taking the inverse of $U^\dagger M_\nu U^* = M_\nu^D \equiv \mathrm{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$, we can obtain $U^T \tilde{M}_\nu U = m_0^2 (M_\nu^D)^{-1} = \mathrm{diag}(m_0^2/m_{\nu 1}, m_0^2/m_{\nu 2}, m_0^2/m_{\nu 3})$, i.e. we obtain the mixing matrix U^* instead of U and neutrino masses $(m_0^2/m_{\nu 1}, m_0^2/m_{\nu 2}, m_0^2/m_{\nu 3})$ with a normal (inverse) hierarchy instead of neutrino masses $(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$ with an inverse (normal) hierarchy. Therefore, in this paper, instead of the model $M_\nu \propto Y_e^{-1} Y_u^{1/2} + Y_u^{1/2} Y_e^{-1} + \xi_0 \mathbf{1} = Y_e^{-1}(Y_u^{1/2} Y_e + Y_e Y_u^{1/2} + \xi_0 Y_e Y_e) Y_e^{-1}$, we will investigate a neutrino mass matrix with a seesaw-type

$$M_\nu = \frac{y_\nu^2 v_{Hu}^2}{y_R \Lambda^2} Y_\nu \bar{Y}_R^{-1} Y_\nu^T, \tag{2}$$

where $\bar{Y}_R$ and Y_ν are given by

$$\bar{Y}_R \propto Y_u^{1/2} Y_e + Y_e Y_u^{1/2} + \xi_0 Y_e Y_e, \tag{3}$$

and $Y_\nu \propto Y_e$, respectively. In the previous model [5], the matrix M_ν was for Dirac neutrinos, while the present M_ν is for Majorana neutrinos. The previous model could not provide a reasonable mass spectrum without adjusting the parameter ξ_0, while, in this paper, we will give a reasonable mass spectrum without the ξ_0-term. Note that, in the present scenario, since the Dirac neutrino mass matrix Y_ν is identical with the charged lepton mass matrix Y_e, the nearly tribimaximal mixing originates in the structure of $\bar{Y}_R$, (3).

In order to obtain the relation $Y_\nu \propto Y_e$, in this paper, we assume that the U(1) charge of Y_ν is the same with that of Y_e, i.e. $q_\nu = q_e$, so that the field Y_e can couple to the Dirac neutrino sector. For simplicity and from the economical point of view, in this paper, we identify Y_ν as Y_e. On the other hand, in order to give the operator $Y_u^{1/2}$ in the expression (3), we introduce additional nonet fields Φ_u and Φ_{u0} with the U(1) charges $\frac{1}{2}q_u$ and $-q_u$, respectively. Then, we can write down the superpotential for the u-sector

$$W_u = \lambda_u \mathrm{Tr}[\Phi_u \Phi_u \Phi_{u0}] + m_u \mathrm{Tr}[Y_u \Phi_{u0}] + W_{\Phi u}(\Phi_u). \tag{4}$$

From SUSY vacuum conditions(for the moment, we regard W_u as W), we obtain

$$\frac{\partial W}{\partial \Phi_{u0}} = 0 = \lambda_u \Phi_u \Phi_u + m_u Y_u, \tag{5}$$

$$\frac{\partial W}{\partial Y_u} = 0 = m_u \Phi_{u0}, \tag{6}$$

$$\frac{\partial W}{\partial \Phi_u} = 0 = \lambda_u (\Phi_u \Phi_{u0} + \Phi_{u0} \Phi_u) + \frac{\partial W_{\Phi u}}{\partial \Phi_u}. \tag{7}$$

From the condition (5), we obtain a bilinear relation

$$\langle Y_u \rangle = -\frac{\lambda_u}{m_u} \langle \Phi_u \rangle \langle \Phi_u \rangle, \tag{8}$$

so that the VEV values of Φ_u are given by

$$\langle \Phi_u \rangle^D \propto \mathrm{diag}(\sqrt{m_{u1}}, \sqrt{m_{u2}}, \sqrt{m_{u3}}), \tag{9}$$

where D denotes that the matrix is on its diagonal basis. (Hereafter, for simplicity, we will express VEV matrices $\langle A \rangle$ as simply A.) From the condition (6), we obtain

$$\Phi_{u0} = 0. \tag{10}$$

Therefore, from the condition (7), we obtain $\partial W_{\Phi u}/\partial \Phi_u = 0$. We assume that three eigenvalues of $\langle \Phi_u \rangle$ can completely be determined by this condition $\partial W_{\Phi u}/\partial \Phi_u = 0$. However, for this purpose, the superpotential term $W_{\Phi u}$ will include U(1) symmetry breaking terms. In this paper, we do not discuss the explicit form of $W_{\Phi u}$. We assume that the VEV values are suitably given by Eq.(9) with the observed up-quark masses m_{ui}. For convenience, for the e-sector, we also assume superpotential terms W_e similar to the u-sector:

$$W_e = \lambda_e \mathrm{Tr}[\Phi_e \Phi_e \Phi_{e0}] + m_e \mathrm{Tr}[Y_e \Phi_{e0}] + W_{\Phi e}(\Phi_e), \tag{11}$$

where Φ_e, Φ_{e0} and Y_e have U(1) charges $\frac{1}{2}q_e$, $-q_e$ and q_e, respectively, so that we obtain relations

$$Y_e = -\frac{\lambda_e}{m_e}\Phi_e\Phi_e, \tag{12}$$

with $\Phi_e^D \propto \mathrm{diag}(\sqrt{m_{e1}}, \sqrt{m_{e2}}, \sqrt{m_{e3}})$ and so on.

Next, let us investigate a possible form of W_R. Since $\bar{Y}_R$ is $\bar{\mathbf{6}}$ of $\mathrm{U}(3)_F$, in order to compose $\mathrm{U}(3)_F$ singlet together with $\bar{Y}_R$ in the superpotential, we need a field X_0 with $\mathbf{6}$ of $\mathrm{U}(3)_F$. We also assume fields X_1 and X_2 with $\mathbf{6}$ and $\bar{X}$ with $\bar{\mathbf{6}}$. Then, we find that the following form of W_R can lead to the relation (3):

$$W_R = m_R \mathrm{Tr}[X_0 \bar{Y}_R] + \frac{y_{eu}}{\Lambda}\mathrm{Tr}[Y_e X_0 \Phi_u^T \bar{X} + \Phi_u X_0 Y_e^T \bar{X}]$$

$$+\lambda_1 \mathrm{Tr}[\Phi_u X_1 \bar{X} - X_1 \Phi_u^T \bar{X}] + \lambda_2 \mathrm{Tr}[\Phi_e X_2 \bar{X} - X_2 \Phi_e^T \bar{X}], \tag{13}$$

where we have assumed the U(1) charges $Q(X_0) = -Q(\bar{Y}_R) = 2q_e$, $Q(\bar{X}) = -3q_e - \frac{1}{2}q_u$, $Q(X_1) = 3q_e$ and $Q(X_2) = \frac{1}{2}(5q_e + q_u)$. Note that, under this U(1) charge assignment, a term $\mathrm{Tr}[X_0\bar{Y}_R X_0 \bar{Y}_R]$ is also allowed. However, as shown later, we will choose the vacuum with $X_0 = 0$, so that the term $\mathrm{Tr}[X_0\bar{Y}_R X_0 \bar{Y}_R]$ is harmless. Therefore, we have dropped such harmless terms from the superpotential (13). Also, note that the field $\mathbf{9}_k^{\ l}$ between $\bar{\mathbf{6}}^{ik}$ and $\mathbf{6}_{lj}$ is expressed as $(\bar{\mathbf{6}}\cdot\mathbf{9}\cdot\mathbf{6})_j^{\ i}$, while $\mathbf{9}_l^{\ k}$ between $\mathbf{6}_{ik}$ and $\bar{\mathbf{6}}^{lj}$ is expressed as $(\mathbf{6}\cdot\mathbf{9}^T\cdot\bar{\mathbf{6}})_i^{\ j}$. From SUSY vacuum conditions $\partial W/\partial \bar{Y}_R = 0$ and $\partial W/\partial \bar{X} = 0$, where $W = W_u + W_e + W_R$, we obtain $X_0 = 0$. Then, the requirement $\partial W/\partial Y_e = 0$ leads to the condition $\partial W_e/\partial Y_e = 0$, so that we obtain the relation (12). From $\partial W/\partial X_0 = 0$, we obtain

$$\bar{Y}_R = -\frac{y_{eu}}{m_R\Lambda}(\Phi_u^T \bar{X} Y_e + Y_e^T \bar{X}\Phi_u). \tag{14}$$

The condition $\partial W/\partial X_1 = \lambda_1(\bar{X}\Phi_u - \Phi_u^T \bar{X}) = 0$ demands that the matrix $\bar{X}$ is diagonal on the u-basis, while $\partial W/\partial X_2 = \lambda_2(\bar{X}\Phi_e - \Phi_e^T\bar{X}) = 0$ demands that $\bar{X}$ is also diagonal on the e-basis. Since we consider that the e-basis and u-basis are different bases each other, $\bar{X}$ must be a unit matrix: $\langle\bar{X}\rangle = v_X\mathbf{1}$. Thus, we can obtain the desirable form (3) of $\bar{Y}_R$ (without the ξ_0-term).

Next, in order to obtain the neutrino mixing matrix form on the e-basis, we must know a matrix form of Φ_u on the e-basis although the form $(\Phi_u)_u$ on the u-basis is given by Eq.(9). Since the fields defined in Eq.(1) transform under a flavor-basis transformation as $\mathbf{3}\to T\cdot\mathbf{3}$, $\bar{\mathbf{3}}\to\bar{\mathbf{3}}\cdot T^\dagger$, $\mathbf{9}\to T\cdot\mathbf{9}\cdot T^\dagger$, $\bar{\mathbf{6}}\to T^*\cdot\bar{\mathbf{6}}\cdot T^\dagger$ and $\mathbf{6}\to T\cdot\mathbf{6}\cdot T^T$, a transformation of a VEV matrix Y_f from a b-basis to an a-basis is expressed as

$$(Y_f)_a = T_{ab}(Y_f)_b T_{ab}^\dagger, \tag{15}$$

Table 1: The δ dependency of predicted values in the case $T_{ue} = V(\delta)$. The values of $\sin^2 2\theta_{23}$ and $\tan^2\theta_{12}$ are estimated by $\sin^2 2\theta_{23} = 4|(U_\nu)_{23}|^2|(U_\nu)_{33}|^2$ and $\tan^2\theta_{12} = |(U_\nu)_{12}/(U_\nu)_{11}|^2$, respectively. The numerical results in the case $T_{ue} = V(-\delta)$ are identical with the case $T_{ue} = V(\delta)$.

δ	$\sin^2 2\theta_{23}$	$\tan^2\theta_{12}$	$\lvert U_{13}\rvert$	$\Delta m^2_{21}/\Delta m^2_{32}$
0	0.3831	0.4170	0.01132	0.00262
60°	0.7565	0.4178	0.00917	0.00172
90°	0.9174	0.4459	0.00645	0.00119
120°	0.9817	0.4806	0.00384	0.00091
180°	0.9997	0.5125	0.00010	0.00074

where $T^\dagger_{ab} = T_{ba}$ and $T_{ab}T_{bc} = T_{ac}$. On the other hand, since the VEV matrices Y_f $(f = u, d)$ are diagonalized as $U_f^\dagger Y_f U_f = Y_f^D \equiv (\Lambda/y_f v_H)\mathrm{diag}(m_{f1}, m_{f2}, m_{f3})$ $(f = u, d)$, we obtain $T_{du} = V^\dagger$, where V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix defined by $V = U^\dagger_{uL}U_{dL}$. The VEV matrix Y_e defined in Eq.(1) is diagonalized as $U_e^T Y_e U_e^* = Y_e^D$, the transformation T_{eu} is given by $T_{eu} = U_e^T U_u$. (In this definition, the neutrino seesaw matrix M_ν is diagonalized as $U_\nu^T M_\nu U_\nu = M_\nu^D$, and the neutrino mixing matrix U is given by $U = U_e^\dagger U_\nu$.)

The simplest assumption is to consider that the d-basis is identical with the e-basis, so that we can regard T_{ue} as $T_{ue} = V$ because $T_{ud} = V$. Then, we can evaluate the neutrino mass matrix (2) with $(\bar{Y}_R)_e \propto (\Phi_u^T)_e(Y_e)_e + (Y_e^T)_e(\Phi_u)_e$ by using the form $(\Phi_u)_e = T_{eu}(\Phi_u)_u T^\dagger_{eu} = V^\dagger(\delta)\Phi_u^D V(\delta)$. In the numerical calculation of M_ν, we adopt the standard phase convention $V(\delta)$ [8] of the CKM matrix V, and use the following input values: the up-quark masses [7] at the energy scale $\mu = M_Z$, $m_{u1} = 0.00233$ GeV, $m_{u2} = 0.677$ GeV, $m_{u3} = 181$ GeV, and the CKM parameters [8], $|V_{us}| = 0.2257$, $|V_{cb}| = 0.0416$, $|V_{ub}| = 0.00431$. (Here, we have used the quark mass values at $\mu = M_Z$ because we have used the CKM parameter values at $\mu = M_Z$. For the energy scale dependency of the mass ratios and CKM parameters, for example, see Ref.[9].) As seen in Table 1, the results are dependent on the CP violating phase parameter δ. The present experimental data [8] on the CKM matrix favor $\delta \simeq \pi/3$. However, as seen in Table 1, the predicted value of $\sin^2 2\theta_{23}$ at $\delta \simeq \pi/3$ is in poor agreement with the observed value $\sin^2 2\theta_{23} = 1.00_{-0.13}$ [10], although the predicted value of $\tan^2\theta_{12}$ is roughly in agreement with the observed value $\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$ [11]. Therefore, we cannot regard that the e-basis is identical with the d-basis.

However, as seen in Table 1, note that the case with $\delta \geq 2\pi/3$ can give a nearly tribimaximal mixing. If we assume a form $T_{ue} = V(\delta_{ue})$ suggested from $T_{ud} = V(\delta)$, we can show [5] $T_{ed} = T^\dagger_{ue}T_{ud} = V^\dagger(\delta_{ue})V(\delta) = \mathbf{1} + \mathcal{O}(|V_{ub}|)$, so that we can still consider $T_{ue} \simeq T_{ud}$. Especially, we are interested in the case $\delta_{ue} = \pi/3 + \pi$. Since $T_{ue} = U_u^\dagger U_e^*$ and $T_{ud} = U_u^\dagger U_d$, it is likely that the difference between U_e^* and U_d yields a phase shift π. However, in this paper, we a priori assume the form $T_{ue} = V(\pi/3 + \pi)$ as a phenomenological requirement suggested in Table 1. Again, we summarize our phenomenological neutrino mass matrix which can lead to a nearly tribimaximal mixing for $|\delta_{ue}| \geq 2\pi/3$ as follows:

$$(M_\nu)_e = k_\nu Y_e^D \left[\left(V^\dagger(\delta_{ue}) \Phi_u^D V(\delta_{ue}) \right)^T Y_e^D + (Y_e^D)^T \left(V^\dagger(\delta_{ue}) \Phi_u^D V(\delta_{ue}) \right) \right]^{-1} Y_e^D, \qquad (16)$$

where $Y_e^D \propto \mathrm{diag}(m_e, m_\mu, m_\tau)$ and $\Phi_u^D \propto \mathrm{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$. (For the phenomenological reason why the mass matrix (16) can give a nearly tribimaximal mixing, see Ref.[5].)

As seen in Table 1, the predicted value of $R = \Delta m^2_{21}/\Delta m^2_{32}$ is considerably small compared to the observed value $|R| = 0.028 \pm 0.004$, where we have used the observed values $\Delta m^2_{21} = (7.59 \pm 0.21) \times 10^{-5}\,\mathrm{eV}^2$ [11] and $|\Delta m^2_{32}| = (2.74^{+0.44}_{-0.26}) \times 10^{-3}\,\mathrm{eV}^2$ [10]. The value R can be adjusted by taking the ξ_0-term in Eq.(3) into consideration. (It is easy to bring the ξ_0-term into the present model.) However, the smallness of Δm^2_{21} can also become mild by considering the renormalization group equation (RGE) effects. Since, so far, we have not fixed the energy scale Λ, the values without RGE effects have been listed in Table 1. We consider that the RGE effects can give a reasonable value of R without the ξ_0 term. By the way, the present neutrino masses are normal hierarchical, so that, if regard $m_{\nu 3}$ as $m_{\nu 3} = \sqrt{\Delta m^2_{32}} = 0.0523$ eV, we can obtain neutrino masses $m_{\nu 1} = 0.78$ meV, $m_{\nu 2} = 1.76$ meV and $m_{\nu 1} = 52.3$ meV for the case $\delta_{ue} = \pi/3 + \pi$.

So far, we did not discuss a structure of W_d. Although the purpose of the present paper is not to give a structure of W_d, here, let us discuss a possible structure of W_d lightly. Although we have assumed that the ν-basis is identical with the e-basis, and we have regarded that Y_ν is identical with Y_e, we cannot put such an assumption in the d-sector. We assume the following superpotential W_d:

$$W_d = \lambda_{du} \left(\mathrm{Tr}[\Phi_d]\mathrm{Tr}[\Phi_u \Phi_{d0}] + \mathrm{Tr}[\Phi_u]\mathrm{Tr}[\Phi_d \Phi_{d0}] \right) + m_d \mathrm{Tr}[Y_d \Phi_{d0}] + \lambda_d \det\Phi_d, \qquad (17)$$

where we have assumed U(1) charges $Q(\Phi_{d0}) = -Q(Y_d) \equiv -q_d$ and $Q(\Phi_d) = q_d - \frac{1}{2}q_u$. Under this charge assignment, the term $\mathrm{Tr}[\Phi_d \Phi_u \Phi_{d0}]$ is also allowed. So far, in W_u, W_e and W_R, we have not considered cubic terms of a type $\mathrm{Tr}[A]\mathrm{Tr}[BC]$, while, in W_d, we have assumed such a cubic term $\mathrm{Tr}[A]\mathrm{Tr}[BC]$ instead of a cubic term $\mathrm{Tr}[ABC]$. At present, the form (17) is merely a phenomenological assumption. Also note that the cubic term $\det\Phi_d$ breaks the U(1) symmetry. From the condition $\partial W/\partial \Phi_d = 0$, we obtain

$$Y_d = -\frac{\lambda_{du}}{m_d} (\mathrm{Tr}[\Phi_d]\Phi_u + \mathrm{Tr}[\Phi_u]\Phi_d). \qquad (18)$$

Since we have already taken $\partial W_u/\partial \Phi_u = 0$ in Eq.(7), we obtain $\Phi_{d0} = 0$ for $\Phi_d \neq 0$ from the condition $\partial W/\partial \Phi_u = \lambda_{du}(\mathrm{Tr}[\Phi_d \Phi_{d0}]\mathbf{1} + \mathrm{Tr}[\Phi_d]\Phi_{d0}) + \partial W_u/\partial \Phi_u = 0$. Then, from the condition $\partial W/\partial \Phi_d = 0$, we obtain $0 = \partial \det\Phi_d/\partial \Phi_d = \Phi_d \Phi_d - \mathrm{Tr}[\Phi_d]\Phi_d + (1/2)\left(\mathrm{Tr}^2[\Phi_d] - \mathrm{Tr}[\Phi_d \Phi_d]\right)\mathbf{1}$, where we have used the formula for a 3×3 Hermitian matrix A, $\det A = (1/3)\mathrm{Tr}[AAA] - (1/2)\mathrm{Tr}[AA]\mathrm{Tr}[A] + (1/6)\mathrm{Tr}^3[A]$. Therefore, the requirement $\partial \det\Phi_d/\partial \Phi_d = 0$ demands that the matrix Φ_d is a rank-1 matrix. Such a rank-1 matrix is generally expressed as $((\Phi_d)_u)_{ij} = v_d x_i x_j^*$, where $|x_1|^2 + |x_2|^2 + |x_3|^2 = 1$. Therefore, Y_d is expressed as

$$((Y_d)_u)_{ij}(v_{Hd}/\Lambda) = k_d \left[\delta_{ij}\sqrt{m_{ui}} + x_i x_j^* (\sqrt{m_{u3}} + \sqrt{m_{u2}} + \sqrt{m_{u1}}) \right]. \qquad (19)$$

From the trace of (19), we obtain $m_{d3} \simeq 2k_d\sqrt{m_{u3}}$. From $(\mathrm{Tr}^2[Y_d] - \mathrm{Tr}[Y_dY_d])/2$, we also obtain $m_{d3}m_{d2} \simeq 2k_d^2\sqrt{m_{u3}m_{u2}}$, where we have regarded $|x_2|^2 \ll \sqrt{m_c/m_t}$, so that we obtain a relation

$$\frac{m_s}{m_b} \simeq \frac{1}{2}\sqrt{\frac{m_c}{m_t}}. \tag{20}$$

The observed values are $m_s/m_b \simeq 0.031$ and $\sqrt{m_c/m_t} \simeq 0.061$ at $\mu = M_Z$ [7], so that the relation (20) is in good agreement with the observed value. Also we can obtain $m_d/m_s \simeq \sqrt{m_u/m_c}$ from $\det Y_d$, but the result is sensitive to the values of $|x_i/x_j|$, so that we do not discuss no more details of m_{di}/m_{dj} in this paper.

In conclusion, we have proposed a new approach to the masses and mixings of quarks and leptons, and we have found a neutrino mass matrix of a new type as a byproduct of this approach. In the new approach, we write a superpotential W for U(3)-flavor nonet fields Y_f whose VEVs give effective Yukawa coupling constants $\langle (Y_f)_{ij}\rangle/\Lambda$ and we obtain relations among masses and mixings from the SUSY vacuum conditions. In this approach, we cannot predict the absolute values of the masses and mixings, but we can obtain relations among the VEV matrices Y_f and $\bar{Y}_R$. However, in the present investigation, we have not derived our relations among Y_f and $\bar{Y}_R$ from a general form of the superpotential W under some principles (symmetries, and so on), and we have assumed a specific form of W from the phenomenological point of view. Especially, since we have assumed the ad hoc form $T_{ue} = V(\delta_{ue})$, we cannot assert that the neutrino mass matrix M_ν given in Eq.(24) has been derived theoretically. Nevertheless, it is worthwhile noticing because the form is of a new type which is related to the up-quark masses and which successfully leads to the nearly tribimaximal mixing without assuming any discrete symmetry.

Since the present approach is still in its beginning stage, we have many tasks to investigate: for example, (i) investigation of the explicit structures of $W_{\Phi u}(\Phi_u)$ and $W_{\Phi e}(\Phi_e)$, which completely determine the eigenvalues of $\langle\Phi_u\rangle$ and $\langle\Phi_e\rangle$, i.e. $(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$ and $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, respectively; (ii) investigation of the explicit structure of W_d in order to give more definite quark mass relations and CKM matrix parameters (Y_d in this paper has included free parameters x_i, so that we cannot derive definite conclusions because we can adjust the parameters x_i to the observed values freely); (iii) investigation of symmetries and quantum number assignments which can uniquely derive the present specific (phenomenological) form of W; (iv) re-formulation for a case with Y_f of $\mathrm{U}(6)_F$ $\mathbf{6}$ and/or $\bar{\mathbf{6}}$ (not nonet) in order to accommodate the present scenario to a GUT scenario.

In the present scenario, most of the fields Φ_u, Φ_e, Y_f ($f = u, d, e$), $\bar{Y}_R$, and so on, take VEV of the order of Λ, and their masses are also of the order Λ. However, some components of those fields are massless in the SUSY limit, and, under the SUSY breaking at $\mu \sim 1$ TeV, they have masses of the order $\mu \sim 1$ TeV. Since those particles are gauge singlets, in principle, they are harmless in the low energy phenomenology. However, in TeV region physics, we may expect fruitful phenomenology about flavor-mediated (but gauge-singlet) processes. This approach will shed a new light on the quark and lepton masses and mixings and on a TeV scale flavor physics.

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References

[1] Y. Koide, Mod. Phys. Lett. **A5** (1990) 2319; Y. Koide and M. Tanimoto, Z. Phys. C **72** (1996) 333; Y. Koide, Phys. Rev. **D73** (2006) 057901.

[2] Y. Koide, Eur. Phys. J. **C52** (2007) 617; JHEP **08**, 086 (2007).

[3] Y. Koide, Phys. Lett. **B662** (2008) 43.

[4] E. Ma, Phys. Lett. **B649** (2007) 287.

[5] Y. Koide, arXiv:0803.3101 [hep-ph].

[6] S. Pakvasa and H. Sugawara, Phys. Lett. **B82** (1979) 105; Y. Yamanaka, H. Sugawara and S. Pakvasa, Phys. Rev. **D25** (1982) 1895; P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. **B458** (1999) 79; Phys. Lett. **B530** (2002) 167; Z. Z. Xing, Phys. Lett. **B533** (2002) 85; P. F. Harrison and W. G. Scott, Phys. Lett. **B535** (2003) 163; Phys. Lett. **B557** (2003) 76; E. Ma, Phys. Rev. Lett. **90** (2003) 221802; C. I. Low and R. R. Volkas, Phys. Rev. **D68** (2003) 033007.

[7] H. Fusaoka and Y. Koide, Phys. Rev. D **57** (1998) 3986.

[8] Particle Data Group, J. Phys. G **33** (2006) 1.

[9] M. Olechowski and S. Pokorski, Phys. Lett. **B257** (1991) 388.

[10] D. G. Michael *et al.*, MINOS collaboration, Phys. Rev. Lett. **97** (2006) 191801; J. Hosaka, *et al.*, Super-Kamiokande collaboration, Phys. Rev. **D74** (2006) 032002.

[11] S. Abe, *et al.*, KamLAND collaboration, arXiv:0801.4589.