Phenomenological Meaning of a Neutrino Mass Matrix Related to Up-Quark Masses

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Abstract

Recently, a curious neutrino mass matrix has been proposed: it is related to up-quark masses, and it can excellently give a nearly tribimaxial mixing. It is pointed out that, in order to obtain such successful results, three phenomenological relations among masses and CKM parameters must be simultaneously satisfied. This suggests that there must be a specific flavor-basis in which down-quark and charged lepton mass matrices are simultaneously diagonalized.

1 Introduction

Recently, a curious neutrino mass matrix has been proposed by the author [1]: the mass matrix is related to up-quark masses as follows:

$$M_\nu = M_D M_R^{-1} M_D^T, \tag{1.1}$$

where the neutrino Dirac mass matrix M_D is given by $M_D \propto M_e$ (M_e is a charged lepton mass matrix), and the right-handed neutrino Majorana mass matrix M_R is given by

$$M_R \propto M_e M_u^{1/2} + M_u^{1/2} M_e. \tag{1.2}$$

The mass matrix (1.1) with (1.2) has been derived from an idea that the origin of the mass spectra (i.e. effective Yukawa coupling constants) is due to vacuum expectation values (VEV) structures of gauge singlet scalars Φ_{ij}. (The details are reviewed in the next section.) In order to obtain the lepton mixing matrix U, one must know forms of M_D and $M_u^{1/2}$ in the "e-basis" (we refer to a diagonal basis of the mass matrix M_f as "f-basis"). The form $M_D = M_e$ is given by $M_e = \mathrm{diag}(m_e, m_\mu, m_\tau)$ in the e-basis. For the form $M_u^{1/2}$, by analogy with the relation $M_u = V^T D_u V$ in the d-basis, where $D_u = \mathrm{diag}(m_u, m_c, m_t)$ and V is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix (and note that a mass matrix M_f is diagonalized as a form $U_f^T M_f U_f = D_f$ in the present model because we assume an O(3) flavor symmetry as we mention it in Sec.2), we assume that $M_u^{1/2}$ in the e-basis is given by a form

$$M_u^{1/2} = V^T(\delta) D_u^{1/2} V(\delta), \tag{1.3}$$

where we have adopted the standard expression VSD [2] as a phase convention of the CKM matrix $V(\delta)$. In order to estimate the form M_ν, we use the following observed up-quark masse values at an energy scale of the weak interactions $\mu = m_Z$ [3], $m_u = 0.00127$ GeV, $m_c = 0.619$ GeV, $m_t = 171.7$ GeV, and the observed CKM mixing parameters (best-fit values) [4] $|V_{us}| =$

Table 1: δ dependence of predicted values in the standard phase convention of $V(\delta)$. Here, $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ have been used as three input values of the four independent parameters of $V(\delta)$. The best-fit value of δ in the quark sector is $\delta_q = 69.8°$ from the observed CKM matrix data.

δ	$\sin^2 2\theta_{23}$	$\tan^2\theta_{12}$	$\|U_{13}\|$	$\Delta m^2_{21}/\Delta m^2_{32}$
0	0.4803	0.4745	0.01042	0.00196
60°	0.7631	0.4801	0.00844	0.00139
69.8°	0.8127	0.4851	0.00781	0.00127
90°	0.9028	0.5017	0.00615	0.00102
120°	0.9688	0.5277	0.00386	0.00081
180°	0.9952	0.5525	0.00094	0.00068

0.2257, $|V_{cb}| = 0.0415$ and $|V_{ub}| = 0.00359$ together with the observed charged lepton masses. (Here, since we use the values at $\mu = m_Z$ for the CKM matrix parameters, we also use the running mass values at $\mu = m_Z$.) Then, one can successfully obtain a nearly tribimaximal mixing [5],

$$U = \begin{pmatrix} +0.8026 & -0.5966 & -0.0009 \\ -0.4356 & -0.5871 & +0.6823 \\ +0.4076 & +0.5472 & +0.7311 \end{pmatrix}, \tag{1.4}$$

for $\delta = \pi$, i.e.

$$\sin^2 2\theta_{23} = 0.9952, \quad \tan^2\theta = 0.5525, \quad |U_{13}| = 0.00094. \tag{1.5}$$

For reference, we give phase-dependence of the numerical results in Table 1. The best-fit values [4] of the CKM mixing parameters show $\delta = 69.8°$. However, as seen in Table 1, the predicted value of $\sin^2 2\theta_{23}$ at $\delta \simeq 69.8°$ is in poor agreement with the observed value $\sin^2 2\theta_{23} = 1.00_{-0.13}$ [6], although the predicted value of $\tan^2\theta_{12}$ is roughly in agreement with the observed value $\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$ [7]. As stated in the next section, since the flavor-basis transformation matrix is confined to an orthogonal matrix because the present model is based on an O(3) flavor symmetry, the phase parameter δ must be 0 or π.

We also list numerical results for the original Kobayashi-Maskawa phase convention [8] in Table 2. As seen in Table 2, not only the both cases, $\delta = 0$ and $\delta = \pi$, but also any values of δ cannot give a reasonable value of $\sin^2 2\theta_{23}$. Thus, we find that the phenomenological success is only for the case of $V(\delta) = V_{SD}(\delta)$ (not for the original KM phase convention of CKM matrix).

In order to obtain the phenomenological success, it is essential to assume not only the neutrino mass matrix form (1.1) with (1.2), but also forms of flavor-basis transformation matrices U_{ud} and U_{ue}

$$\begin{aligned} U_{ud} &= V_{SD}(\delta_q) \quad (\delta_q \simeq 70°), \\ U_{ue} &= V_{SD}(\delta_\ell) \quad (\delta_\ell = 180°), \end{aligned} \tag{1.6}$$

where $U_{ff'}$ transforms a matrix in an f-basis into that in an f'-basis, and $V_{SD}(\delta)$ is the standard

Table 2: δ dependence of predicted values in the original Kobayashi-Maskawa phase convention of $V(\delta)$. Here, $|V_{us}|$, $|V_{td}|$ and $|V_{ub}|$ have been used as three input values of the four independent parameters of $V(\delta)$. The best-fit value of δ in the quark sector is $\delta_q = 90.8°$ from the observed CKM matrix data.

δ	$\sin^2 2\theta_{23}$	$\tan^2 \theta_{12}$	$\|U_{13}\|$	$\Delta m^2_{21}/\Delta m^2_{32}$
0	0.7821	0.5074	0.00769	0.00093
60°	0.8088	0.3587	0.0303	0.0052
90°	0.8781	0.1862	0.0614	0.04269
120°	0.8482	0.3523	0.03303	0.00752
180°	0.8369	0.5028	0.00329	0.00169

phase convention of the CKM matrix with the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ as three input values of the four independent parameters of V_{SD}.

In Sec.2, we give a short review of the model which leads to the mass matrix (1.1) with (1.2). In Sec.3, we investigate relations between conditions for tribimaximal mixing and the empirical neutrino mass matrix (1.1) with (1.2) and (1.3) from the phenomenological point of view. One will find that three phenomenological relations among the masses and CKM matrix parameters must simultaneously be satisfied in order to get a nearly tribimaximal mixing. As we state in Sec.3, it is hard to consider that such the simultaneous coincidence are accidental, so that it should be considered that such phenomenological relations originate in a common law. In Sec.4, we speculate possible forms of the mass matrices M_d and M_e. It is concluded that there is a flavor basis in which the down-quark and charged lepton mass matrices, M_d and M_e, are simultaneously diagonalized. Finally, Sec.5 is devoted to summary and remarks.

2 Model

The neutrino mass matrix related to up-quark masses has first been derived on the basis of a U(3) flavor symmetry model [9], and then, the form (1.1) with (1.2) has been derived on the basis of an O(3) flavor symmetry model [1]. In this section, we give a short review of the O(3) model.

It is assumed that effective Yukawa coupling constants Y_f^{eff} are given by VEVs $\langle Y_f \rangle$ of gauge singlet scalars Y_f (for convenience, we refer to those fields as "Yukawaons") which belong to $(\mathbf{3}\times\mathbf{3})_S = \mathbf{1}+\mathbf{5}$ of an O(3) flavor symmetry:

$$W_Y = \sum_{i,j} \frac{y_u}{\Lambda} U_i (Y_u)_{ij} Q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} D_i (Y_d)_{ij} Q_j H_d$$

$$+ \sum_{i,j} \frac{y_\nu}{\Lambda} L_i (Y_\nu)_{ij} N_j H_u + \sum_{i,j} \frac{y_e}{\Lambda} L_i (Y_e)_{ij} E_j H_d + h.c. + \sum_{i,j} y_R N_i (Y_R)_{ij} N_j, \tag{2.1}$$

where Q and L are quark and lepton $SU(2)_L$ doublet fields of $O(3)_F$ triplets, and U, D, N, and E are $SU(2)_L$ singlet matter fields of $O(3)_F$ triplets, and Λ is an energy scale of an effective theory.

Since we assume the O(3) flavor symmetry, the Yukawaons Y_f $(f = u, d, e, \nu, R)$ are symmetric. Under this definition of $(Y_f)_{ij}$ given by Eq.(1.1), the VEV matrix $\langle Y_f \rangle$ are diagonalized as $U_f^T \langle Y_f \rangle U_f = \langle Y_f \rangle^D$, where the index D means that the matrix is diagonal, and the quark and lepton mixing matrices V and U are given by $V = U_u^\dagger U_d$ and $U = U_e^\dagger U_\nu$, respectively. In order to distinguish the Yukawaons Y_f from each other, the following $\mathrm{U}(1)_X$ charges are assigned: $Q_X(Y_f) = x_f$ $(f = u, d, \nu, e)$, $Q_X(U) = -x_u$, $Q_X(E) = -x_e$, and so on. The field Y_R has a charge $Q_X(Y_R) = 2x_\nu$.

One writes a superpotential W under the following conditions: (i) Terms consist of, at most, holomorphic cubic terms of the fields, and do not contain higher dimensional terms, except for the Yukawa interaction terms W_Y; (ii) Those are invariant under the $\mathrm{O}(3)_F$ and $\mathrm{U}(1)_X$ symmetries. (iii) Yukawaons Y_f always behave as a combination of **1**+**5**, so that, for example, **5** alone never appears in the interaction terms.

The VEV spectra $\langle Y_f \rangle$ are evaluated from supersymmetric (SUSY) vacuum conditions for a superpotential $W = W_Y + W_u + W_d + W_e + W_\nu + W_R$, where W_f $(f = u, d, \nu, e, R)$ play a role in fixing the VEV structures $\langle Y_f \rangle$. (Since one can easily show $\langle Q \rangle = \langle L \rangle = \langle U \rangle = \langle D \rangle = \langle N \rangle = \langle E \rangle = 0$, hereafter, the term W_Y is dropped from the superpotential W as far as the VEV structures of Y_f are investigated.) For example, a spectrum of $\langle Y_u \rangle$ is obtained from the following superpotential terms W_u:

$$W_u = \lambda_u \mathrm{Tr}[\Phi_u \Phi_u A_u] + \mu_u \mathrm{Tr}[Y_u A_u] + W_{\Phi u}, \tag{2.2}$$

where a new filed A_u has $\mathrm{U}(1)_X$ charge $Q_X = -x_u$. Here, the term $W_{\Phi u}$ has been introduced in order to fix eigenvalues of $\langle \Phi_u \rangle$. Since the purpose of the present paper is not to discuss quark and lepton mass spectra, an explicit form of $W_{\Phi u}$ is given in Appendix A. Since $W_{\Phi u}$ contains Y_u and Φ_u as shown in Appendix A, SUSY vacuum conditions $\partial W/\partial Y_u = 0$ and $\partial W/\partial \Phi_u = 0$ will be discussed in Appendix A. From a SUSY vacuum condition $\partial W/\partial A_u = 0$ (for the moment, one regards W_u as W), one obtains

$$\frac{\partial W}{\partial A_u} = 0 = \lambda_u \Phi_u \Phi_u + \mu_u Y_u, \tag{2.3}$$

so that one obtains a bilinear relation

$$\langle Y_u \rangle = -\frac{\lambda_u}{\mu_u} \langle \Phi_u \rangle \langle \Phi_u \rangle, \tag{2.4}$$

i.e. the field Φ_u plays a role of $M_u^{1/2}$ in Eq.(1.2). For convenience, we refer to Φ_f as "ur-Yukawaons". The ur-Yukawaons Φ_f play a role in fixing VEV spectra of Yukawaons. Although we consider 5 Yukawaons Y_f $(f = u, d, e, \nu, R)$, we will consider only 2 ur-Yukawaons Φ_e and Φ_u in the present model. Note that, since the matrix $\langle \Phi_u \rangle$ is not Hermitian, the relation

$$U_u^T \langle Y_u \rangle U_u = \langle Y_u \rangle^D \propto \mathrm{diag}(m_u, m_c, m_t), \tag{2.5}$$

does not always mean

$$U_u^T \langle \Phi_u \rangle U_u = \langle \Phi_u \rangle^D \propto \mathrm{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t}), \tag{2.6}$$

where D denotes that the matrix is diagonal. As one sees later, one needs the relation (2.6). Therefore, we assume the field Φ_u (and also Y_u) is real, so that the matrix U_u is orthogonal matrix.

For the charged lepton sector, we also assume superpotential term W_e similar to the up-quark sector:

$$W_e = \lambda_e \mathrm{Tr}[\Phi_e \Phi_e A_e] + \mu_e \mathrm{Tr}[Y_e A_e] + W_{\Phi e}, \tag{2.7}$$

where Φ_e, Y_e and A_e have U(1)$_X$ charges $\frac{1}{2}x_e$, x_e and $-x_e$, respectively, so that one obtains relations

$$Y_e = -\frac{\lambda_e}{\mu_e}\Phi_e \Phi_e, \tag{2.8}$$

with $\Phi_e^D \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, where one has again assumed that the field Φ_e is real. (Here and hereafter, for simplicity, we will sometimes express VEV matrices $\langle M \rangle$ as simply M.)

Next, let us investigate a possible form of W_R. We introduce a new field A_R with U(1)$_X$ charge $Q_X = -2x_\nu$. In order to obtain the relation (1.2), we assume the following form of W_R:

$$W_R = \lambda_R \mathrm{Tr}[(Y_e \Phi_u + \Phi_u Y_e) A_R] + \mu_R \mathrm{Tr}[Y_R A_R] + \lambda_R \xi \mathrm{Tr}[Y_\nu Y_\nu A_R], \tag{2.9}$$

where we have assumed a relation among the U(1)$_X$ charges,

$$2x_\nu = \frac{1}{2}x_u + x_e. \tag{2.10}$$

From SUSY vacuum conditions $\partial W/\partial Y_R = 0$, one obtains $A_R = 0$. Then, the requirement $\partial W/\partial Y_e = 0$ leads to the condition $\partial W_e/\partial Y_e = 0$, so that one obtains the relation (2.8). From $\partial W/\partial A_R = 0$, one obtains

$$Y_R = -\frac{\lambda_R}{\mu_R}(Y_e \Phi_u + \Phi_u Y_e + \xi Y_\nu Y_\nu). \tag{2.11}$$

The third term (ξ-term) does not affect a form of the lepton mixing matrix U because the term gives a constant term proportional to a unit matrix **1** as shown later. Thus, one can obtain the desirable form (1.2) of Y_R.

Next, we discuss how to obtain $\langle Y_\nu \rangle = \langle Y_e \rangle$. The simplest assumption to obtain a relation $M_D \propto M_e$ (i.e. $Y_\nu \propto Y_e$) is to assume that the fields N and E have the same U(1)$_X$ charges (i.e. $x_\nu = x_e$), and to consider a model without Y_ν. However, then, one obtains $x_\nu = x_e = x_u/2$ from the relation (2.10), so that Y_e and Φ_u (and also Y_u and Y_R) have the same U(1)$_X$ charges. This brings some additional terms into W_u, W_e and W_R due to the mixings between Y_e and Φ_u and between Y_u and Y_R, so that one cannot obtain desirable relations without ad hoc selections of those terms. Therefore, in order to obtain the relation $Y_\nu \propto Y_e$ with $x_\nu \neq x_e$, we assume the following structure of W_ν:

$$W_\nu = \lambda_{\nu\nu}\phi_\nu \mathrm{Tr}[Y_\nu A_\nu] + \lambda_{\nu e}\phi_e \mathrm{Tr}[Y_e A_\nu], \tag{2.12}$$

where ϕ_ν and ϕ_e are gauge- and flavor-singlet fields, and we assign $\mathrm{U}(1)_X$ charges as $Q_X(A_\nu) = x_\nu$, $Q_X(\phi_\nu) = -(x_\nu + x_\nu)$ and $Q_X(\phi_e) = -(x_e + x_\nu)$. From $\partial W/\partial\phi_\nu = 0$ and $\partial W/\partial\phi_e = 0$, one obtains $A_\nu = 0$. From $\partial W/\partial A_\nu = 0$, one obtains

$$Y_\nu = -\frac{\lambda_{\nu e}\phi_e}{\lambda_{\nu\nu}\phi_\nu} Y_e. \tag{2.13}$$

In order to obtain a neutrino mixing matrix form in the e-basis, one must know a matrix form of $\langle\Phi_u\rangle$ in the e-basis, although the form $\langle\Phi_u\rangle^D$ on the u-basis is given by Eq.(2.6). (Now, "f-basis" is defined as a flavor basis in which the VEV matrix $\langle Y_f\rangle$ is diagonal.) Let us define a transformation of a VEV matrix $\langle Y_f\rangle$ from a b-basis to an a-basis as

$$\langle Y_f\rangle_a = U_{ba}^T \langle Y_f\rangle_b U_{ba}, \tag{2.14}$$

where U_{ab} are unitary matrices, and $\langle Y_f\rangle_a$ denotes a VEV matrix form on the a-basis. The unitary matrices U_{ab} satisfy $U_{ab}^\dagger = U_{ba}$ and $U_{ab}U_{bc} = U_{ac}$. (These operators U_{ab} are not always members of O(3) flavor-basis transformations.) Since $Y_f^T = Y_f$ in the present model, the VEV matrix $\langle Y_f\rangle$ are diagonalized as $U_f^T\langle Y_f\rangle U_f = \langle Y_f\rangle^D \equiv \langle Y_f\rangle_f$. Therefore, $\langle Y_u\rangle_d$ is given by

$$\langle Y_u\rangle_d = V^T(\delta)\langle Y_u\rangle_u V(\delta), \tag{2.15}$$

where $V(\delta)$ is the standard expression of the CKM matrix. The simplest assumption is to consider that the d-basis is identical with the e-basis, and then, one can regard U_{ue} as $U_{ue} = V$ because $U_{ud} = V$. However, since one has assumed that Y_u and Y_e are real, the flavor-basis transformation matrix U_{ue} must be orthogonal, i.e. the phase parameter δ is 0 or π even if one assumes the form $U_{ue} = V(\delta)$. As one has already seen in Table 1, the case with $\delta = \pi$ can give reasonable numerical results.

Anyhow, one assumes the form

$$\langle\Phi_u\rangle_e = U_{ue}^T\langle\Phi_u\rangle_u U_{ue} = V^T(\delta)\langle\Phi_u\rangle^D V(\delta), \tag{2.16}$$

one can obtain the following phenomenological neutrino mass matrix

$$\begin{aligned}\langle M_\nu\rangle_e &= k_\nu Y_e^D \left[V^T(\delta)\Phi_u^D V(\delta) Y_e^D + Y_e^D V^T(\delta)\Phi_u^D V(\delta) + \xi Y_\nu Y_\nu\right]^{-1} Y_e^D \\ &= k_\nu \left[(Y_e^D)^{-1} V^T(\delta)\Phi_u^D V(\delta) + V^T(\delta)\Phi_u^D V(\delta)(Y_e^D)^{-1} + \xi_0 \mathbf{1}\right]^{-1},\end{aligned} \tag{2.17}$$

where $Y_e^D \propto \mathrm{diag}(m_e, m_\mu, m_\tau)$ and $\Phi_u^D \propto \mathrm{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$. The third term ($\xi_0$ term) does not affect the lepton mixing matrix U. Rather, the existence of the ξ_0 term is useful to adjust the value of $\Delta m_{21}^2/\Delta m_{32}^2$ because the predicted values in Table 1 were considerably small compared to the observed value $|R| = 0.028 \pm 0.004$, where one has used the observed values $\Delta m_{21}^2 = (7.59 \pm 0.21) \times 10^{-5}\,\mathrm{eV}^2$ [7] and $|\Delta m_{32}^2| = (2.74^{+0.44}_{-0.26}) \times 10^{-3}\,\mathrm{eV}^2$ [6].

3 Conditions for a tribimaximal mixing

In this section, we investigate what phenomenological relations are required for the mass matrix (2.17) in order to give a nearly tribimaximal mixing. Since one know [10] that a mixing matrix for $(M_\nu)^{-1}$ is given by U^* when a mixing matrix for M_ν is given by U, for the purpose to obtain conditions for a tribimaximal mixing, one can investigate the following matrix

$$M = (Y_e^D)^{-1} V^T(\delta) \Phi_u^D V(\delta) + V^T(\delta) \Phi_u^D V(\delta) (Y_e^D)^{-1} + \xi_0 \mathbf{1}, \tag{3.1}$$

i.e.

$$M_{ij} = \left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}} \right) \sum_k \sqrt{m_{uk}} V_{ki} V_{kj}, \tag{3.2}$$

instead of the mass matrix (2.17). Since the ξ_0-term is not essential for evaluating the mixing matrix U, hereafter, we put $\xi_0 = 0$. (Although a similar study has been done in Ref.[9] based on a U(3) flavor symmetry, where the VEV matrix $\langle \Phi_u \rangle_e$ has been given by $\langle \Phi_u \rangle_e = V^\dagger(\delta) \langle \Phi_u \rangle_u V(\delta)$, in the present O(3) model, the VEV matrix $\langle \Phi_u \rangle_e$ is given by $\langle \Phi_u \rangle_e = V^T(\delta) \langle \Phi_u \rangle_u V(\delta)$.)

As shown in Appendix, the conditions to obtain the maximal $2 \leftrightarrow 3$ mixing, i.e.

$$\sin^2 2\theta_{23} \equiv 4|U_{23}|^2 |U_{33}|^2 = 1, \quad |U_{13}|^2 = 0, \tag{3.3}$$

are

$$|M_{12}| = |M_{13}|, \tag{3.4}$$

and

$$|M_{22}| = |M_{33}|, \tag{3.5}$$

From Eq.(3.2), one obtains

$$M_{12} \simeq \frac{\sqrt{m_c}}{m_e} V_{21} V_{22}, \tag{3.6}$$

$$M_{13} \simeq \frac{\sqrt{m_t}}{m_e} V_{31} V_{33}, \tag{3.7}$$

$$M_{22} \simeq 2 \frac{\sqrt{m_t}}{m_\mu} V_{33}^2, \tag{3.8}$$

$$M_{33} \simeq 2 \frac{\sqrt{m_t}}{m_\tau} V_{33}^2, \tag{3.8}$$

where one has assumed a hierarchical structure of $|V_{ij}|$ similar to the observed CKM matrix. The condition (3.5) requires

$$\sqrt{\frac{m_c}{m_t}} \simeq \frac{m_\mu}{m_\tau}, \tag{3.9}$$

The left- and right-hand sides of Eq.(3.9) give values [3] 0.060 and 0.059, respectively. Therefore, the condition (3.5) is phenomenologically well satisfied. On the other hand, the condition (3.4) requires

$$\sqrt{\frac{m_c}{m_t}} \simeq \frac{|V_{31}|}{|V_{21}|}. \tag{3.10}$$

In order to evaluate the relation (3.10), one uses a relation

$$\frac{V_{31}}{V_{21}} = -\left(\frac{V_{23}^*}{V_{33}^*} + \frac{V_{11}}{V_{21}}\frac{V_{13}^*}{V_{33}^*}\right), \tag{3.11}$$

from the unitary relation $V_{11}V_{13}^* + V_{21}V_{23}^* + V_{31}V_{33}^* = 0$. For a standard expression of the CKM matrix, Eq.(3.11) leads to

$$\frac{V_{31}}{V_{21}} \simeq -\left(|V_{cb}| - \frac{|V_{ub}|}{|V_{us}|}e^{i\delta}\right). \tag{3.12}$$

The left-hand side of Eq.(3.10) is 0.060, and the right-hand side is $0.0412 + 0.0174 = 0.0586$ for $\delta = \pi$. Thus, the condition is also well satisfied. Note that if the observed value of $|V_{td}|$, $|V_{td}| = 0.00874$ [4], as the value $|V_{31}|$ is used, the condition (3.10) cannot be satisfied. This is a reason for that when one used the original Kobayashi-Maskawa phase convention instead of the standard CKM matrix expression, one could not give a nearly tribimaximal mixing as seen in Table 2.

Next, we check the condition to give $\tan^2\theta_{12} = 1/2$,

$$\eta^2\left((M_{22}M_{33})^{1/2} + M_{23}\right) - M_{11} = \eta(M_{12}M_{13})^{1/2}, \tag{3.13}$$

(see (B.16) in Appendix B). From Eqs.(3.6) - (3.8) and

$$M_{11} \simeq 2\frac{\sqrt{m_t}}{m_e}\left(\sqrt{\frac{m_c}{m_t}}V_{21}^2 + \sqrt{\frac{m_u}{m_t}}V_{11}^2\right), \tag{3.14}$$

$$M_{23} \simeq \frac{\sqrt{m_t}}{m_\mu}V_{32}V_{33}, \tag{3.15}$$

one finds $|M_{22} + M_{23}| \ll |M_{11}$, so that the condition (3.13) requires $M_{23} \simeq M_{11}$ ($\eta = -1$ in the present case). The condition $M_{23} \simeq M_{11}$ requires

$$|V_{us}| + \frac{1}{|V_{us}|}\sqrt{\frac{m_u}{m_t}} \simeq \frac{1}{2}. \tag{3.16}$$

The left-had side of Eq.(3.16) gives $0.2257 + 02007 = 0.4264$. Considering the present rough approximation, one may consider that the condition (3.13) is roughly satisfied.

In conclusion, in order to obtain a tribimaximal mixing, the three phenomenological relations (3.9), (3.10) and (3.16) must simultaneously be satisfied. It is hard to consider that such the simultaneous coincidences are accidental. Rather, it should be considered that such the phenomenological relations originate in a common law. Also, one must note that, in order to satisfy the condition (3.10), one must take the standard expression of the CKM matrix and use the observed values $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ in order to fix the three rotation angles in the CKM matrix. This suggests that the down-quark mass matrix M_d has a similar structure with the charged lepton mass matrix M_e. In the next section, we will investigate a possible relation between M_d and M_e.

4 Possible structures of M_d and M_e

In this section, we speculate possible mass matrix forms of the down-quark and charged lepton mass matrices M_d and M_e which lead to the assumption (1.6).

Generally, there are 9 phase conventions of the CKM matrix V [11]:

$$V(m,n) = R_m P_\ell R_\ell R_n \qquad (m \neq \ell \neq n), \tag{4.1}$$

where $m, n, \ell = 1, 2, 3$, and

$$R_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \quad R_2(\theta) = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix}, \quad R_3(\theta) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.2}$$

$$\begin{aligned} P_1 &= \mathrm{diag}(e^{i\delta},\ 1,\ 1), \\ P_2 &= \mathrm{diag}(1,\ e^{i\delta},\ 1), \\ P_3 &= \mathrm{diag}(1,\ 1,\ e^{i\delta}). \end{aligned} \tag{4.3}$$

($c = \cos\theta$ and $s = \sin\theta$). For example, the standard expression V_{SD} of the CKM matrix

$$V_{SD}(\delta) = R_1(\theta_{23}) P_3(\delta) R_2(\theta_{13}) P_3(-\delta) R_3(\theta_{12}), \tag{4.4}$$

is rewritten as

$$V_{SD}(\delta) = e^{i\delta} P_1(-\delta) R_1(\theta_{23}) P_2(-\delta) R_2(\theta_{13}) R_3(\theta_{12}) P_3(-\delta), \tag{4.5}$$

because $P_3(\delta) = e^{i\delta} P_1(-\delta) P_2(-\delta)$. Since the factors $e^{i\delta} P_1(-\delta)$ and $P_3(-\delta)$ in the left- and right-hand sides can be absorbed into the unobservable phases of up- and down-quarks, respectively, the standard expression V_{SD} corresponds to the expression $V(1,3)$ defined in (4.1).

In the O(3) model, where the mass matrices are symmetric, the mass matrices M_u and M_d are diagonalized as

$$U_u^T M_u U_u = D_u, \quad U_d^T M_d U_d = D_d, \tag{4.6}$$

and the CKM matrix V is given by

$$V = U_u^\dagger U_d. \tag{4.7}$$

As seen in the general expressions of V given in (4.1), one can always find a flavor basis (we refer to it as a "x-basis") in which the CP-violating phases are factorized as

$$\langle Y_u \rangle_x = P_n(\delta_u) \langle \widetilde{Y}_u \rangle_x P_n(\delta_u), \quad \langle Y_d \rangle_x = P_n(\delta_d) \langle \widetilde{Y}_d \rangle_x P_n(\delta_d), \tag{4.8}$$

where $\langle \widetilde{Y}_u \rangle_x$ and $\langle \widetilde{Y}_d \rangle_x$ are real matrices, and they are diagonalized by rotation matrices R_u and R_d as

$$R_u^T \langle \widetilde{Y}_u \rangle_x R_u = D_u, \quad R_d^T \langle \widetilde{Y}_d \rangle_x R_d = D_d, \tag{4.9}$$

respectively. Then, since $U_u = P_n(-\delta_u)R_u$ and $U_d = P_n(-\delta_d)R_d$, one obtains the expression of the flavor-basis transformation operator U_{ud}

$$U_{ud} = V = R_u^T P_n(\delta_u - \delta_d) R_d. \tag{4.10}$$

Similarly, one can obtain an expression of U_{ue} as follows:

$$U_{ue} = R_u^T P_n(\delta_u - \delta_e) R_e. \tag{4.11}$$

Now, let us return to our model. As seen in Sec.3, the requirement (1.6) for a nearly tribimaximal mixing is rewritten as

$$\begin{aligned} U_{ud} &= R_1(\theta_{23})P_3(\delta_q)R_2(\theta_{13})P_3(-\delta_1)R_3(\theta_{12}), \\ U_{ue} &= R_1(\theta_{23})P_3(\delta_\ell)R_2(\theta_{13})P_3(-\delta_\ell)R_3(\theta_{12}), \end{aligned} \tag{4.12}$$

where the rotation angles are fixed by the observed CKM mixing data as

$$\begin{aligned} \theta_{13} &= \sin^{-1}|V_{ub}|, \\ \theta_{23} &= \sin^{-1}(|V_{cb}|/\sqrt{1-|V_{ub}|^2}), \\ \theta_{12} &= \sin^{-1}(|V_{us}|/\sqrt{1-|V_{ub}|^2}), \end{aligned} \tag{4.13}$$

and the phase parameters are taken as $\delta_q \simeq 70^\circ$ and $\delta_\ell = 180^\circ$. This suggests that the mass matrices M_d and M_e in the x-basis are diagonalized by the same rotation matrix

$$R_d = R_2(\theta_{13}^d)R_3(\theta_{12}), \tag{4.14}$$

while the up-quark mass matrix M_u in the x-basis is diagonalized by

$$R_u = R_2^T(\theta_{13}^u)R_1^T(\theta_{23}), \tag{4.15}$$

where $\theta_{13} = \theta_{13}^d - \theta_{13}^u$, θ_{23} and θ_{12} are given by (4.13), and the phase parameters are given by

$$\delta_q = \delta_d - \delta_u \simeq 70^\circ, \quad \delta_\ell = \delta_e - \delta_u = 180^\circ. \tag{4.16}$$

(Since we have assumed that Y_u and Y_e are real in the present model, the phase factors δ_u and δ_e must be 0 or π.) Therefore, forms of the mass matrices M_u, M_d and M_e in the x-basis are given by

$$\begin{aligned} \langle Y_u\rangle_x &= P_2(\delta_u)R_2^T(\theta_{13}^u)R_1^T(\theta_{23})D_u R_1^T(\theta_{23})R_2(\theta_{13}^u)P_2(\delta_u), \\ \langle Y_d\rangle_x &= P_2(\delta_d)R_2(\theta_{13}^d)R_3(\theta_{12})D_u R_3^T(\theta_{12})R_2^T(\theta_{13}^d)P_2(\delta_d), \\ \langle Y_e\rangle_x &= P_2(\delta_e)R_2(\theta_{13}^d)R_3(\theta_{12})D_e R_3^T(\theta_{12})R_2^T(\theta_{13}^d)P_2(\delta_e). \end{aligned} \tag{4.17}$$

In other words, one can choose such a x-basis in which the mass matrices M_d and M_e are diagonalized simultaneously, and CP-violating phase factors are factorized as shown in (4.17).

5 Concluding remarks

When one consider a neutrino mass matrix form

$$M_\nu \propto (\langle\Phi_e\rangle^m \langle\Phi_u\rangle^n + \langle\Phi_u\rangle^n \langle\Phi_e\rangle^m)^{-1} , \tag{5.1}$$

one can find that a case which can give a reasonable lepton mixing is only a case with $m = -2$ and $n = 1$, even if one consider any form of U_{ue}. (This is related to the observed fact $\sqrt{m_c/m_t} \simeq m_\mu/m_\tau$.) One also find that the case with $m = -2$ and $n = 1$ can lead to a nearly tribimaximal mixing only when one assume $U_{ue} = V_{SD}(\pi)$, where $V_{SD}(\delta)$ is the standard expression of the CKM matrix with the inputs $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$. Therefore, in the present paper, it has been investigated what structure of the neutrino mass matrix form (1.2) play an essential role in giving a nearly tribimaximal mixing. We have found that, in order to obtain such a nearly tribimaximal mixing, we need to accept the three phenomenological relations (3.9), (3.10) and (3.16). It is hard to consider that such the relations accidentally hold, so that we consider that the ad hoc assumption $U_{ue} = V(\delta_\ell)$ has an underlying meaning. In Sec.4, we have investigated possible structures of the down-quark and charged lepton mass matrices. We have concluded that there must be a specific flavor basis in which the down-quark and charged mass matrices are simultaneously diagonalized.

In the present model, an O(3) flavor symmetry has been assumed. Relations which are obtained from the O$(3)_F$ invariant superpotential by using SUSY vacuum conditions hold only in flavor bases which are connected by an orthogonal transformation. Therefore, in order to use those relations in the e-basis and/or u-basis, it has been assumed that $\langle\Phi_e\rangle$ and $\langle\Phi_u\rangle$ are real and the e-basis and u-basis can be connected by an orthogonal transformation U_{ue}. On the other hand, one knows that $\langle Y_d\rangle$ cannot be real because of the observation of CP violating phenomena in the quark sector. Therefore, one cannot use the relations from the SUSY vacuum conditions in the d-basis. (However, this does not mean that one cannot build a down-quark mass matrix model. Relations including Yukawaon Y_d still hold in the u-basis.)

In spite of such disadvantage of the O$(3)_F$ model, the reason that one consider O(3) flavor symmetry is as follows: If we consider a U(3) flavor symmetry, the Yukawaon Y_R (and also Y_u in a grand unification scenario) must be **6** of U$(3)_F$. It is difficult to build a U$(3)_F$ invariant superpotential for Y_R without considering higher dimensional terms. (For example, a Yukawaon model based on a U$(3)_F$ symmetry is found in Ref.[9]. However, the superpotential term for Y_R in the U$(3)_F$ model is somewhat intricate.) In order to build a simpler model for Y_R, one will be obliged to adopt an O$(3)_F$ model.

In the present scenario, it is assumed that there are no higher dimensional terms with $(1/\Lambda)^n$ ($n \geq 1$) in the superpotential except for the effective Yukawa interaction terms W_Y, Eq.(2.1). Although we want to build a model of W_Y without any higher dimensional terms, at present, we have no idea for such a model. It is a future task to us.

So far, we have not discussed a structure of W_d which gives a down-quark mass matrix $\langle Y_d\rangle$, although an attempt to give such W_d has been proposed in Ref.[1]. Since this is not the question of the moment in the present paper, we did not discuss. We will discuss a possible structure of W_d elsewhere.

Although the present approach to the masses and mixings of quarks and leptons is not conventional and not yet established, this approach will become one of the promising approaches because one can treat the masses and mixings without discussing explicit forms of the Yukawa coupling constants.

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Appendix A: An example of $W_{\Phi f}$

The superpotential term $W_{\Phi u}$ in Eq.(2.2) has been introduced to fix the VEV spectrum of the ur-Yukawaon Φ_u. In this appendix, we demonstrate an example of $W_{\Phi u}$.

When one introduces a further new field B_u with a $U(1)_X$ charge $Q_X = -(3/2)x_u$, one can have a term $\mathrm{Tr}[\Phi_u Y_u B_u]$. However, of course, if one has only this term, one cannot fix the eigenvalues of $\langle\Phi_u\rangle$, because one needs a cubic equation in $\langle\Phi_u\rangle$. Therefore, one assume existence of $\mathrm{Tr}[A]\mathrm{Tr}[BC]$, $\mathrm{Tr}[B]\mathrm{Tr}[CA]$ and $\mathrm{Tr}[C]\mathrm{Tr}[AB]$ in addition to the term $\mathrm{Tr}[ABC]$ only for the term $W_{\Phi u}$. Then, the superpotential W_u for the up-quark sector is given by

$$W_u = \lambda_u \mathrm{Tr}[\Phi_u\Phi_u A_u] + \mu_u \mathrm{Tr}[Y_u A_u] + W_{\Phi u}, \tag{A.1}$$

$$\begin{aligned} W_{\Phi u} = y_u \mathrm{Tr}[(\Phi_u Y_u + Y_u\Phi_u)B_u] + 2y_{1u}\mathrm{Tr}[\Phi_u]\mathrm{Tr}[Y_u B_u] \\ +2y_{2u}\mathrm{Tr}[Y_u]\mathrm{Tr}[\Phi_u B_u] + 2y_{3u}\mathrm{Tr}[B_u]\mathrm{Tr}[\Phi_u Y_u]. \end{aligned} \tag{A.2}$$

The SUSY vacuum condition $\partial W/\partial A_u = 0$ has already been investigated in Sec.2. In this appendix, we will investigate $\partial W/\partial Y_u = 0$, $\partial W/\partial \Phi_u = 0$ and $\partial W/\partial B_u = 0$.

From the conditions $\partial W/\partial Y_u = 0$ and $\partial W/\partial \Phi_u = 0$, one obtains

$$\frac{\partial W}{\partial Y_u} = 0 = \mu_u A_u + y_u(\Phi_u B_u + B_u\Phi_u) + 2y_{1u}\mathrm{Tr}[\Phi]\, B_u + 2y_{2u}\mathrm{Tr}[\Phi B_u]\,\mathbf{1} + 2y_{3u}\mathrm{Tr}[B_u]\,\Phi_u, \tag{A.3}$$

$$\frac{\partial W}{\partial \Phi_u} = 0 = \lambda_u(\Phi_u A_u + A_u\Phi_u) + y_u(Y_u B_u + B_u Y_u) + y_{1u}\mathrm{Tr}[Y_u B_u]\,\mathbf{1} + y_{2u}\mathrm{Tr}[Y_u]\, B_u + y_{3u}\mathrm{Tr}[B_u]\, Y_u. \tag{A.4}$$

Since one searches a vacuum with $\Phi_u \neq 0$ and $Y_u \neq 0$, one can obtain

$$A_u = B_u = 0, \tag{A.5}$$

by requiring Eqs.(A.3) and (A.4) simultaneously. On the other hand, from $\partial W/\partial B_u = 0$, one obtains

$$\frac{\partial W}{\partial B_u} = 0 = y_u(\Phi_u Y_u + Y_u\Phi_u) + 2y_{1u}\mathrm{Tr}[\Phi_u]\, Y_u + 2y_{2u}\mathrm{Tr}[Y_u]\,\Phi_u + 2y_{3u}\mathrm{Tr}[\Phi_u Y_u]\,\mathbf{1}. \tag{A.6}$$

By substituting $Y_u \propto \Phi_u\Phi_u$, Eq.(2.4), one obtains a cubic equation in Φ_u:

$$y_u\Phi_u^3 + y_{1u}\mathrm{Tr}[\Phi_u]\,\Phi_u^2 + y_{2u}\mathrm{Tr}[\Phi_u^2]\,\Phi_u + y_{3u}\mathrm{Tr}[\Phi_u^3]\,\mathbf{1} = 0. \tag{A.7}$$

Since the coefficient of Φ_u, $y_{1u}\mathrm{Tr}[\Phi_u]/2y_u$, in a cubic equation (A.7) must be equal to $-\mathrm{Tr}[\Phi_u]$, one obtains a restriction

$$y_{1u} = -y_u. \tag{A.8}$$

Also, from constraints for the coefficients of Φ and $\mathbf{1}$ in the cubic equation, one obtains

$$\frac{y_{2u}}{y_u}\mathrm{Tr}[\Phi_u^2] = \frac{1}{2}\left(\mathrm{Tr}[\Phi_u]^2 - \mathrm{Tr}[\Phi_u^2]\right), \tag{A.9}$$

and

$$\frac{y_{3u}}{y_u}\mathrm{Tr}[\Phi_u^3] = -\mathrm{det}\Phi_u, \tag{A.10}$$

respectively. The constraints (A.9) and (A.10) lead to formulas

$$\frac{\mathrm{Tr}[\Phi_u^2]}{\mathrm{Tr}[\Phi_u]^2} = \frac{1}{1+2y_{2u}/y_u}, \tag{A.11}$$

and

$$\mathrm{det}\Phi_u = \frac{y_{3u}/y_u}{2(1+3y_{3u}/y_u)}\mathrm{Tr}[\Phi_u]\left(\mathrm{Tr}[\Phi_u]^2 - 3\mathrm{Tr}[\Phi_u^2]\right), \tag{A.12}$$

respectively. Thus, the VEV spectrum can completely be determined by the coefficients y_{1u}/y_u, y_{2u}/y_u and y_{3u}/y_u.

We also assume the same structure W_e as W_u for the charged lepton sector. Then, if one takes $y_{2e}/y_e = 1/4$, one obtains $\mathrm{Tr}[\Phi_e^2]/\mathrm{Tr}[\Phi_e]^2 = 2/3$, so that one can obtain an interesting charged lepton mass relation [12]. However, since the purpose of the present paper is not to discuss the mass spectra of quarks and leptons, we do not touch this problem.

Appendix B: Mass matrix form for a tribimaximal mixing

A general mass matrix form which gives a tribimaximal mixing [5] has been given by He and Zee [13]. We summarize the general form for a case of the tribimaximal mixing matrix with phases, and we discuss conditions for $\sin^2 2\theta_{23} = 1$ and $\tan^2\theta_{12} = 1/2$ separately.

An orthogonal mixing matrix U which gives a maximal $2 \leftrightarrow 3$ mixing

$$\sin^2 2\theta_{23} = 1 \ \text{ and } \ U_{13} = 0, \tag{B.1}$$

is given by a form

$$\widetilde{U} = \begin{pmatrix} c & s & 0 \\ -\frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{B.2}$$

where $c = \cos\theta$ and $s = \sin\theta$. Since a mixing matrix U with $U_{13} = 0$ cannot contain a CP violating phase, an extended form U from the orthogonal mixing matrix $\widetilde{U}$ to a unitary mixing matrix is given by

$$U = P(\alpha)\widetilde{U}P(\beta), \tag{B.3}$$

where

$$P(\delta) = \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}). \qquad (B.4)$$

When one defines a mass matrix M with $M^T = M$ which is diagonalized by U as follows:

$$U^T M U = D \equiv \mathrm{diag}(m_1, m_2, m_3), \qquad (B.5)$$

one can obtain

$$\widetilde{U}^T \widetilde{M} \widetilde{U} = P^2(-\beta)\, D \equiv \mathrm{diag}(\widetilde{m}_1, \widetilde{m}_2, \widetilde{m}_3) \equiv \widetilde{D}, \qquad (B.6)$$

where

$$\widetilde{M} = P(\alpha) M P(\alpha). \qquad (B.7)$$

The matrix $\widetilde{M}$ which is diagonalized by an orthogonal matrix is real except for a common phase factor, so that the eigenvalues $\widetilde{m}_i$ are also real. As seen in Eq.(B.6), the phases β_i in $\widetilde{m}_i = m_i e^{-2i\beta_i}$ are the so-called Majorana phases, they are unobservable in neutrino oscillation experiments. Hereafter, for convenience, we denote $\widetilde{m}_i$ as m_i simply. Then, one can obtain the explicit form of $\widetilde{M}$ from $\widetilde{M} = \widetilde{U}\widetilde{D}\widetilde{U}^T$ as

$$\begin{aligned}
\widetilde{M}_{11} &= \tfrac{1}{2}(m_2 + m_1) - \tfrac{1}{2}(m_2 - m_1)\cos 2\theta, \\
\widetilde{M}_{22} = \widetilde{M}_{33} &= \tfrac{1}{2} m_3 + \tfrac{1}{4}(m_2 + m_1) + \tfrac{1}{4}(m_2 - m_1)\cos 2\theta, \\
\widetilde{M}_{12} = \widetilde{M}_{13} &= \tfrac{1}{2\sqrt{2}}(m_2 - m_1)\sin 2\theta, \\
\widetilde{M}_{23} &= -\tfrac{1}{2} m_3 + \tfrac{1}{4}(m_2 + m_1) + \tfrac{1}{4}(m_2 - m_1)\cos 2\theta.
\end{aligned} \qquad (B.8)$$

Therefore, the conditions that the mass matrix $\widetilde{M}$ gives the maximal $2 \leftrightarrow 3$ mixing (B.1) are

$$\widetilde{M}_{12} = \widetilde{M}_{13} \quad \text{and} \quad \widetilde{M}_{22} = \widetilde{M}_{33}, \qquad (B.9)$$

i.e.

$$M_{12} e^{i\alpha_2} = M_{13} e^{i\alpha_3} \quad \text{and} \quad M_{22} e^{2i\alpha_2} = M_{33} e^{2i\alpha_3}. \qquad (B.10)$$

The conditions (B.10) are rewritten as

$$\left(\frac{M_{12}}{M_{13}}\right)^2 = \frac{M_{22}}{M_{33}} = e^{2i(\alpha_3 - \alpha_2)}. \qquad (B.11)$$

On the other hand, the mixing angle $\theta \equiv \theta_{12}$ is obtained from

$$\tan 2\theta = \frac{2\sqrt{2}\widetilde{M}_{12}}{\widetilde{M}_{33} + \widetilde{M}_{23} - \widetilde{M}_{11}}, \qquad (B.12)$$

i.e.

$$\tan 2\theta = \frac{2\sqrt{2}\eta (M_{12} M_{13})^{1/2}}{\eta^2 \left((M_{22} M_{33})^{1/2} + M_{23}\right) - M_{11}}, \qquad (B.13)$$

where

$$\eta = \exp i \left(-\alpha_1 + \frac{\alpha_2 + \alpha_3}{2} \right) . \tag{B.14}$$

Therefore, the conditions for a tribimaximal mixing, i.e. constraints (B.1) and

$$\tan^2 \theta = \frac{1}{2}, \tag{B.15}$$

require the conditions (B.11) and

$$\eta^2 \left((M_{22} M_{33})^{1/2} + M_{23} \right) - M_{11} = \eta (M_{12} M_{13})^{1/2}, \tag{B.16}$$

respectively.

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