Empirical Neutrino Mass Matrix Related to Up-Quark Masses

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Abstract

Based on an approach to quark and lepton masses, where the mass spectra originate in vacuum expectation values of U(3) flavor nonet (gauge singlet) scalars, neutrino masses and mixing are investigated. As an offshoot of this approach, it is found that an empirical neutrino mass matrix which is described in terms of the up-quark and charged lepton masses can accommodate to a nearly tribimaximal mixing.

1 Introduction

The observed neutrino oscillation data are in favor of the so-called "tribimaximal mixing" [1]. Usually, the mixing matrix form has been understood based on a discrete symmetry. One of the motivation of the present paper is to investigate whether such a nearly tribimaximal mixing can be understood without assuming such a discrete symmetry. We will find an empirical neutrino mass matrix which is described in terms of up-quark and charged lepton mass matrices and which leads to a nearly tribimaximal mixing without assuming any discrete symmetry.

In this paper, we will discuss a non-standard approach to the masses and mixings of the quarks and leptons against the conventional mass matrix models. In this model, the mass spectra of the quarks and leptons originates in a vacuum expectation value (VEV) structure of a U(3)-flavor nonet scalar Φ [2, 3, 4]. In the present approach, we write a superpotential W for the U(3)-flavor nonet fields Y_f whose VEVs give effective Yukawa coupling constants $(Y_f^{eff})_{ij} = \langle (Y_f)_{ij}\rangle/\Lambda$ (Λ denotes an energy scale of the effective theory) and thereby we obtain relations among masses and mixings from SUSY vacuum conditions. Although, in Ref.[5], it has been tried to understand a charged lepton mass relation [6] on the basis of such the approach, the purpose of the present paper is not to understand such mass spectra, so that when we evaluate matrices $\langle Y_u\rangle$ and $\langle Y_e\rangle$, we will use the observed values of $\langle Y_u\rangle^D \propto \mathrm{diag}(m_u, m_c, m_t)$ and $\langle Y_e\rangle^D \propto \mathrm{diag}(m_e, m_\mu, m_\tau)$ (A^D denotes a diagonal form of a matrix A), respectively. Although our goal is a unified understanding of quark and lepton mass matrices, in this paper, our investigation will focus on the neutrino mass matrix. As a result, we will obtain a neutrino mass matrix

$$M_\nu = m_0^\nu \left(\langle Y_e^{-1}\rangle\langle Y_u^{1/2}\rangle + \langle Y_u^{1/2}\rangle\langle Y_e^{-1}\rangle + \xi_0 \mathbf{1}\right), \tag{1.1}$$

where the charged lepton and up-quark mass matrices M_e and M_u are given by $M_e = y_e\langle Y_e\rangle/\Lambda$ and $M_u = y_u\langle Y_u\rangle/\Lambda$. (For $Y_u^{1/2}$, see Sec.3.)

In order to estimate a neutrino mixing matrix, we must know an explicit form of (1.1) in a flavor basis in which a charged lepton mass matrix M_e is diagonal (we refer it as "e-basis"). Especially, we must know an explicit form $\langle Y_u^{1/2}\rangle$ in Eq.(1.1), although we know the form of $\langle Y_u\rangle$ in the "d-basis" in which a down-quark mass matrix M_d is diagonal, i.e. $\langle Y_u\rangle_d$ is given by $\langle Y_u\rangle_d = V^\dagger \langle Y_u\rangle_u V$, where $\langle A\rangle_f$ denotes the matrix form of the VEV matrix A on the f-basis, and V is the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. In the present paper, we do not assume a grand unification scenario, so that the e-basis cannot theoretically be related to the d-basis. Nevertheless, we will assume that the form $\langle Y_u^{1/2}\rangle_e$ is given by a relation

$$\langle Y_u^{1/2}\rangle_e = V^\dagger(\delta_{ue})\langle Y_u^{1/2}\rangle_u V(\delta_{ue}), \tag{1.2}$$

on analogy of $\langle Y_u^{1/2}\rangle_d$, where $V(\delta_{ue})$ denotes a mixing matrix in which the CP-violating phase δ in the CKM matrix $V(\delta)$ is replaced with a free parameter δ_{ue}. Then, we will find that the numerical results for the neutrino mass matrix (1.1) with the observed up-quark and charged lepton masses and CKM matrix parameter (except for δ) can give a nearly tribimaximal mixing when we take $\delta_{ue} \simeq \pi$. Of course, there is no theoretical ground in the assumption (1.2), and it is pure phenomenological one. Therefore, the neutrino mass matrix which gives a tribimaximal mixing is also completely empirical one. Nevertheless, we think that this will provide a promising clue to a unification mass matrix model of the quarks and leptons.

In the next section, for convenience of the present investigation, we will define an operators $U_{ff'}$ which transforms a matrix from a f-basis to another f'-basis. In Sec.3, we will assume a form of W_ν which is composed of cross terms not only between Y_ν and Y_e, but also between Y_e and $Y_u^{1/2}$, and thereby, we will discuss a neutrino mass matrix of a new type from the phenomenological point of view. The neutrino mass matrix is described in terms of the up-quark masses. Numerical study will be given in Sec.4. We will find that the mass matrix (1.1) with the phenomenological assumption (1.2) can accommodate to a nearly tribimaximal mixing [1] without assuming a discrete symmetry, but with assuming an empirical relation between the e- and d-bases. Finally, Sec.5 will be devoted to concluding remarks.

2 Flavor-basis transformation

Note that the matrix form $M_e = (y_e/\Lambda)\langle Y_e\rangle$ in the e-basis is diagonal from the definition of the e-basis, i.e. $M_e = (y_e/\Lambda)\langle Y_e\rangle_e = \mathrm{diag}(m_e, m_\mu, m_\tau)$, while the form in another basis is, in general, not diagonal. Let us begin the present investigation by defining useful notations on flavor bases. As we have already used, we define a name of a flavor basis as follows: when a VEV matrix $\langle Y_f\rangle$ takes a diagonal form in a basis, we refer this basis as "f-basis", and we denote a form of a matrix $\langle A\rangle$ on the f-basis as $\langle A\rangle_f$. And, we also define a flavor-basis transformation operator U_{ab} $(a, b = u, d, \nu, e)$ by

$$\langle A\rangle_b = U_{ab}^\dagger \langle A\rangle_a U_{ab}, \tag{2.1}$$

for an arbitrary Hermitian matrix[1] $\langle A\rangle$. The matrix U_{ab} satisfy the relations $U_{ab}^\dagger = U_{ba}$ and $U_{ab}U_{bc}U_{ca} = \mathbf{1}$. For example, when we assume that Y_f are Hermitian, we can express

$$\langle Y_u\rangle_d = V^\dagger \langle Y_u\rangle_u V \equiv V^\dagger \langle Y_u\rangle^D V, \tag{2.2}$$

$$\langle Y_\nu\rangle_e = U_\nu \langle Y_\nu\rangle_\nu U_\nu^\dagger \equiv U_\nu \langle Y_\nu\rangle^D U_\nu^\dagger, \tag{2.3}$$

where $\langle Y_f\rangle^D$ denote the diagonalized form of $\langle Y_f\rangle$, V is the CKM mixing matrix, and U_ν is a neutrino mixing matrix on the e-basis (note that since Y_ν corresponds to a Dirac mass matrix,

[1] If Y_f are not Hermitian, the definition (2.1) is replaced with $\langle A\rangle_b = U_{Lab}^\dagger \langle A\rangle_a U_{Rab}$. The assumption that Y_f are Hermitian is not essential in the present formulation, and the assumption is only one for convenience.

and not to Majorana mass matrix in the seesaw model, the mixing matrix U_ν does not always express the observed neutrino mixing matrix). Therefore, from the definition (2.1), we can regard U_{ud} and $U_{e\nu}$ as $U_{ud} = V$ and $U_{e\nu} = U_\nu$, respectively. If the d-basis is identical with the e-basis, the operator U_{ed} will be $U_{ed} = \mathbf{1}$, so that $U_{ue} = V$. In the present paper, we will be interested in whether U_{ed} is $\mathbf{1}$ or not. We illustrate our concern in Fig.1. As we see in Fig.1, if we can determine U_{ue} (or U_{ed}) in addition to the observed V and U_ν, whole relations among e-, ν-, u- and d-bases can completely be fixed. In the present paper, we will search for a possible form of U_{ue} from the phenomenological point of view.

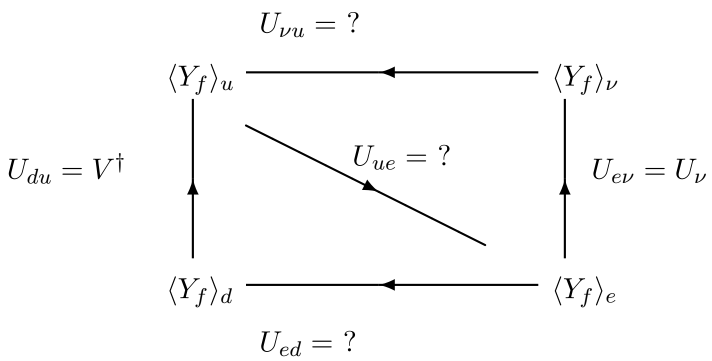

Figure 1: Illustration of our concern for flavor-basis transformation operators U_{ab}, which is defined by $\langle Y_f \rangle_b = U_{ab}^\dagger \langle Y_f \rangle_a U_{ab}$. We will search for a possible form of U_{ue} from the phenomenological point of view.

Thus, we try to build a mass matrix model not by investigating explicit structures of the Yukawa coupling constants (for example, with assuming some discrete symmetries), but by investigating a superpotential for U(3)-flavor nonet fields. Such a prescription seems to provide a new approach to quark and lepton masses and mixings. The purpose of the present paper is to investigate a possible structure of W_ν from the phenomenological point of view.

3 SUSY vacuum approach

In the present approach, the Yukawa coupling constants are understood as "effective" coupling constants $\langle Y_f \rangle / \Lambda$ from the following interactions:

$$W_Y = \sum_{i,j} \frac{y_u}{\Lambda} (Y_u)^i_j Q_i H_u U^j + \sum_{i,j} \frac{y_d}{\Lambda} (Y_d)^i_j Q_i H_d D^j$$

$$+ \sum_{i,j} \frac{y_\nu}{\Lambda} (Y_\nu)^i_j L_i H_u N^j + \sum_{i,j} \frac{y_e}{\Lambda} (Y_e)^i_j L_i H_d E^j + h.c. + y_R \sum_{i,j} N^i (M_R)_{ij} N^j, \qquad (3.1)$$

where Y_f $(f = u, d, \nu, e)$ are not coupling constants, but U(3)-flavor nonet fields [5], and Q and L are quark and lepton $\mathrm{SU}(2)_L$ doublet fields, respectively, and U, D, N, and E are $\mathrm{SU}(2)_L$

singlet matter fields. The mass parameter Λ denotes an energy scale of the effective theory. Therefore, the quark and lepton mass matrices M_f are given by

$$M_f = \frac{y_f}{\Lambda}\langle Y_f\rangle\langle H^0_{u/d}\rangle, \tag{3.2}$$

where $\langle Y_f\rangle/\Lambda \sim 1$. We consider that the VEVs of Y_f are completely determined by another $\mathrm{U}(3)_F$ nonet fields, Φ_e and Φ_u, as we show latter. For simplicity, we investigate a case that all VEV matrices Y_f $(f = u, d, \nu, e)$ are Hermitian.

In the present model, the superpotential W is given by

$$W = W_Y + W_e + W_u + W_\nu + W_d, \tag{3.3}$$

where W_f $(f = u, d, \nu, e)$ play a role in fixing the VEV structure $\langle Y_f\rangle$. Since we can easily show $\langle Q\rangle = \langle L\rangle = \langle U\rangle = \langle D\rangle = \langle N\rangle = \langle E\rangle = 0$, hereafter, we will drop the term W_Y from (3.3) when we investigate the VEV structures of Y_f. Since we focus on the neutrino mass matrix, we will discuss only W_u and W_ν.

We assume the following structure of W_u by introducing U(3)-nonet fields Y_u and Φ_u:

$$W_u = \lambda_u \mathrm{Tr}[\Phi_u\Phi_u\Phi_u] + m_u\mathrm{Tr}[\Phi_u\Phi_u] + \mu_u^2\mathrm{Tr}[\Phi_u] + \lambda_{Yu}\mathrm{Tr}[\Phi_u\Phi_u Y_u] + m_Y\mathrm{Tr}[Y_uY_u]. \tag{3.4}$$

(Although the similar form was assumed for W_e in Ref.[5], we will not mention the explicit form of W_e in the present model, because the purpose of the present paper is not to derive the charged lepton mass relation as in Ref.[5].) From the SUSY vacuum conditions, we obtain

$$\frac{\partial W}{\partial \Phi_u} = 3\lambda_u\Phi_u\Phi_u + 2m_u\Phi_u + \mu_u^2\mathbf{1} + \lambda_{Yu}(\Phi_uY_u + Y_u\Phi_u) = 0, \tag{3.5}$$

$$\frac{\partial W}{\partial Y_u} = \lambda_{Yu}\Phi_u\Phi_u + 2m_{Yu}Y_u = 0. \tag{3.6}$$

(For the moment, we take $W = W_u$.) Therefore, we can obtain a bilinear mass relation

$$Y_u = -\frac{\lambda_{Yu}}{2m_{Yu}}\Phi_u\Phi_u, \tag{3.7}$$

from Eq.(3.6). The operator $Y_u^{1/2}$ in Eq.(1.1) means the present operator Φ_u. Hereafter, we use Φ_u instead of $Y_u^{1/2}$. On the other hand, by substituting Eq.(3.7) into Eq.(3.5), we obtain

$$c_3\Phi_u\Phi_u\Phi_u + c_2\Phi_u\Phi_u + c_1\Phi_u + c_0\mathbf{1} = 0, \tag{3.8}$$

where $c_3 = \lambda_{Yu}^2/m_{Yu}$, $c_2 = -3\lambda_u$, $c_1 = -2m_u$ and $c_0 = -\mu_u^2$. Thus, if we give values of the coefficients c_n $(n = 3, 2, 1, 0)$, we can completely determine three eigenvalues of $\langle\Phi_u\rangle$, so that we can also completely determine three eigenvalues of $\langle Y_u\rangle$. We assume that the superpotential (3.3) does not include any explicit flavor symmetry breaking parameter. The most distinctive feature of the present model is that the U(3) flavor symmetry is spontaneously and completely broken by the non-zero and non-degenerate VEVs of $\langle\Phi_e\rangle$, without passing any subgroup of $U(3)_F$. (For example, differently from the present model, a $U(3)_F$-nonet scalar Φ in Ref.[4] is broken, not directly, but via a discrete symmetry S_4.)

Next, we investigate a possible form of W_ν which leads to phenomenologically successful neutrino mass matrix. For the moment, we neglect the term W_d from Eq.(3.3), i.e. we regard W as $W = W_e + W_u + W_\nu$. We suppose that the matrix Y_ν will be related to the up-type VEV matrix Φ_u by considering a correspondence $Y_e \leftrightarrow Y_d$ and $Y_\nu \leftrightarrow Y_u$. However, if Y_ν is described in terms of Φ_u and Y_u only, the matrix Y_ν is also diagonalized on the u-basis as well as Y_u and Φ_u. Since the observed neutrino mixing matrix is peculiarly different from the observed CKM matrix structure, we must consider that the ν-basis is different from the u-basis. Therefore, we consider that W_ν is a function not only of Y_u and Φ_u, but also of Y_e (and/or Y_d). By way of trial, we assume the following form of W_ν:

$$W_\nu = \frac{y_\nu}{\Lambda}\mathrm{Tr}[Y_e Y_\nu Y_e \Phi_0] + \lambda_{\nu 1}\mathrm{Tr}[(Y_e\Phi_u + \Phi_u Y_e)\Phi_0] + \lambda_{\nu 2}\mathrm{Tr}[Y_e Y_e \Phi_0], \tag{3.9}$$

where the new nonet field Φ_0 has been introduced in order that SUSY vacuum conditions for W_ν do not change relations (3.7) and so on, which are derived from SUSY vacuum conditions for W_e and W_u. (In the form (3.9), it is not a general form of possible terms which include Y_ν. Our concern is what specific form of W can lead to a successful phenomenology, and not what principle can lead to such a specific form of W.) From the SUSY vacuum condition

$$\frac{\partial W}{\partial Y_\nu} = 0 = \frac{y_\nu}{M} Y_e \Phi_0 Y_e, \tag{3.10}$$

we obtain

$$\langle\Phi_0\rangle = 0, \tag{3.11}$$

for $Y_e \neq 0$, so that we obtain

$$\frac{\partial W}{\partial Y_e} = 0 = \lambda_{Ye}\Phi_e\Phi_e + 2m_{Ye}Y_e + \frac{y_\nu}{\Lambda}(Y_\nu Y_e \Phi_0 + \Phi_0 Y_e Y_\nu)$$

$$+\lambda_{\nu 1}(\Phi_u\Phi_0 + \Phi_0\Phi_u) + \lambda_{\nu 2}(Y_e\Phi_0 + \Phi_0 Y_e) = \lambda_{Ye}\Phi_e\Phi_e + 2m_{Ye}Y_e. \tag{3.12}$$

On the other hand, from $\partial W/\partial\Phi_0 = 0$, we obtain

$$\frac{\partial W}{\partial \Phi_0} = 0 = \frac{y_\nu}{\Lambda} Y_e Y_\nu Y_e + \lambda_{\nu 1}(Y_e\Phi_u + \Phi_u Y_e) + \lambda_{\nu 2} Y_e Y_e, \tag{3.13}$$

i.e.

$$\frac{y_\nu}{\Lambda} Y_\nu = -\lambda_{\nu 1}(Y_e^{-1}\Phi_u + \Phi_u Y_e^{-1}) - \lambda_{\nu 2}\mathbf{1}. \tag{3.14}$$

The relation (3.14) means

$$(M_\nu^{Dirac})_{ij} = m_0^\nu \left[\left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}}\right)(\langle\Phi_u\rangle_e)_{ij} + \xi_0\delta_{ij}\right], \tag{3.15}$$

where m_{ei} are the charged lepton masses. From the definition (2.1) of the flavor-basis transformation, the form of $\langle\Phi_u\rangle_e$ is expressed by

$$\langle\Phi_u\rangle_e = U_{ue}^\dagger \langle\Phi_u\rangle_u U_{ue} = v_u U_{ue}^\dagger Z_u U_{ue}, \tag{3.16}$$

where $\langle\Phi_u\rangle = v_u Z_u \equiv v_u \mathrm{diag}(z_1^u, z_2^u, z_3^u)$ and

$$z_i^u = \frac{\sqrt{m_{ui}}}{\sqrt{m_{u1}+m_{u2}+m_{u3}}}, \tag{3.17}$$

(m_{ui} are up-quark masses) from Eq.(3.7).

The mass matrix (3.15) has a very peculiar form because the matrix includes up-quark masses $((\langle\Phi_u\rangle^D)_{ii} \propto \sqrt{m_{ui}})$. If we can know a form of U_{ue}, we can obtain an explicit form of the Dirac neutrino mass matrix (3.15) except for the common shift term (ξ_0-term), so that we can calculate the mixing matrix U_ν independently of the value of the parameter ξ_0. However, at this stage, the form (3.15) does not have any theoretical basis. Moreover, we have no principle to decide the form of U_{ue}. In the next section, we will investigate the mass matrix (3.15) from the phenomenological point of view, and we will demonstrate that the mass matrix (3.15) can give a nearly tribimaximal mixing when we assume a simple specific form of U_{ue}.

4 Phenomenological investigation of the neutrino mass matrix

In the present section, we assume a form of U_{ue}, and thereby, we investigate the mass matrix (3.15) from the phenomenological point of view.

4.1 Numerical study of the Dirac neutrino mass matrix

First, we investigate a case that the observed neutrinos are Dirac type and the mass matrix is given by (3.15). The simplest assumption for a form of U_{ue} is to consider that the d-basis is identical with the e-basis, so that we can regard U_{ue} as $U_{ue} = V$ because $U_{ud} = V$. Then, we can evaluate the form M_ν^{Dirac} except for the common shift term ξ_0, so that we can obtain the neutrino mixing angles $\sin^2 2\theta_{23}$ and $\tan^2\theta_{12}$. The numerical results are shown in Table 1 for the following input values: the up-quark masses [7] at $\mu = M_Z$, $m_{u1} = 0.00233$ GeV, $m_{u2} = 0.677$ GeV, $m_{u3} = 181$ GeV, and the CKM parameters [8], $|V_{us}| = 0.2257$, $|V_{cb}| = 0.0416$, $|V_{ub}| = 0.00431$. (Here, we have used the quark mass values at $\mu = M_Z$ because we have used the CKM parameter values at $\mu = M_Z$. For the energy scale dependency of the mass ratios and CKM parameters, for example, see Ref.[9].) The standard phase convention [8] has been adopted as a phase convention of V. The present experimental data [8] show $\delta \simeq \pi/3$. However, as seen in Table 1, the predicted values of $\sin^2 2\theta_{23}$ and $\tan^2\theta_{12}$ at $\delta \simeq \pi/3$ are in poor agreement with the observed values. Of course, the mixing matrix U_ν defined (2.3) is one for the Dirac neutrino matrix, it is not observed one if the observed neutrinos are Majorana type. However, when we take a seesaw neutrino mass matrix model, the predictions of $\sin^2 2\theta_{23}$ and $\tan^2\theta_{12}$ at $\delta \simeq \pi/3$ become all the more worse (even adjusting the parameter ξ_0) as we see later (in Table 2). Therefore, we cannot regard that the d-basis is identical with the e-basis, i.e. in other words, the matrix Y_d cannot simultaneously be diagonalized together with Y_e. We cannot regard U_{ue} as $U_{ue} = V$.

Table 1: δ_{ue} dependence of the neutrino Dirac mass matrix Y_ν. The numerical values of $(M_\nu^{Dirac})_{ij}$ are given in unit of $m_0^\nu v_u/m_0^e$ in Eq.(3.15) for the case of $\xi_0 = 0$. The values of $\sin^2 2\theta_{23}$ and $\tan^2\theta_{12}$ are estimated by $\sin^2 2\theta_{23} = 4|(U_\nu)_{23}|^2|(U_\nu)_{33}|^2/(1-|U_{13}|^2)$ and $\tan^2\theta_{12} = |(U_\nu)_{12}/(U_\nu)_{11}|^2$, respectively.

δ_{ue}	$(M_\nu)_{22}$	$(M_\nu)_{33}$	$(M_\nu)_{12}$	$(M_\nu)_{13}$	$\sin^2 2\theta_{23}$	$\tan^2\theta_{12}$	$\lvert U_{13}\rvert$
0	0.1579	0.1568	-3.526	1.264	0.3831	0.4170	0.0113
60°	0.1579	0.1568	$-3.547e^{i0.65°}$	$2.083e^{i28.3°}$	0.7545	0.4477	0.0085
90°	0.1577	0.1568	$-3.568e^{i0.074°}$	$2.660e^{i25.4°}$	0.9159	0.4730	0.0061
120°	0.1577	0.1568	$-3.589e^{-i0.64°}$	$3.134e^{-i18.4°}$	0.9813	0.4943	0.0039
180°	0.1576	0.1568	-3.609	3.544	0.9997	0.5125	0.0001

We still expect that $U_{ue} \simeq U_{ud}$, i.e. $U_{ed} \simeq \mathbf{1}$. Therefore, next, we investigate a possibility of $U_{ue} = V(\delta_{ue})$, where $V(\delta)$ is the standard expression of the CKM mixing matrix V with the CP violating phase δ. The observed data [8] on the CKM matrix parameters show $\delta \simeq \pi/3$. For simplicity, hereafter, we will regard the CKM matrix V as $V(\pi/3)$, and for U_{ue}, we will denote $U_{ue} = V(\delta_{ue})$, where we regard δ_{ue} as a free parameter. Then, we can show

$$U_{ed} = U_{ue}^\dagger U_{ud} = V^\dagger(\delta_{ue})V(\frac{\pi}{3}) = \mathbf{1} - \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ -\varepsilon_{13}^* & -\varepsilon_{23}^* & \varepsilon_{33} \end{pmatrix} = \mathbf{1} + \mathcal{O}(|V_{ub}|), \qquad (4.1)$$

where $\varepsilon_{11} = (1-e^{-i(\delta_{ue}-\pi/3)})c_{12}^2 s_{13}^2$, $\varepsilon_{22} = (1-e^{-i(\delta_{ue}-\pi/3)})s_{12}^2 s_{13}^2$, $\varepsilon_{33} = (1-e^{i(\delta_{ue}-\pi/3)})s_{13}^2$, $\varepsilon_{12} = (1-e^{-i(\delta_{ue}-\pi/3)})s_{12}c_{12}s_{13}^2$, $\varepsilon_{13} = (e^{-i\delta_{ue}} - e^{-i\pi/3})c_{12}s_{13}c_{13}$, and $\varepsilon_{23} = (e^{-i\delta_{ue}} - e^{-i\pi/3})s_{12}s_{13}c_{13}$ ($s_{ij} = \sin\theta_{ij}$ and $c_{ij}\cos\theta_{ij}$ are rotation parameters in the standard phase convention of the CKM matrix [8]).

As shown in Table 1, the cases $U_{ue} = V(\delta_{ue})$ with $(2/3)\pi \le |\delta_{ue}| \le \pi$ can give a reasonable set of $(\sin^2 2\theta_{23}, \tan^2\theta_{12})$ for the observed values $\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$ [10] and $\sin^2 2\theta_{23} = 1.00_{-0.13}$ [11]. Especially, we are interested in the cases, (i) $\delta_{ue} = \pi + \pi/3$ and (ii) $\delta_{ue} = \pi$. The case (i) gives $\sin^2 2\theta_{23} = 0.981$ and $\tan^2\theta_{12} = 0.494$, and it is likely that the form $U_{ue} = V(\pi + \delta_{CKM})$ can be understood in a future theoretical model. On the other hand, the case (ii) is also interesting, because the case can give a mixing highly close to the so-called tribimaximal mixing [1] (i.e. the case gives $\sin^2 2\theta_{23} = 1.000$ and $\tan^2\theta_{12} = 0.513$), and the mixing matrix U_{ue} is an orthogonal matrix (it does not include phase parameters), so that U_ν is also an orthogonal one.

More precisely speaking, the tribimaximal mixing takes place only when $(Y_\nu)_{22} = (Y_\nu)_{33}$ and $(Y_\nu)_{12} = \pm(Y_\nu)_{13}$. In the present model, the ratio $(Y_\nu)_{22}/(Y_\nu)_{33} \simeq 1$ is satisfied for any value of δ_{ue} in $U_{ue} = V(\delta_{ue})$. This is warranted by the observed fact

$$\frac{m_\mu}{m_\tau} \simeq \sqrt{\frac{m_c}{m_t}}. \tag{4.2}$$

On the other hand, the ratio $(Y_\nu)_{12}/(Y_\nu)_{13}$ is highly sensitive to the value of δ_{ue}, because

$$\frac{(Y_\nu)_{12}}{(Y_\nu)_{13}} \simeq \frac{V_{21}^* V_{22}\sqrt{m_c} + \cdots}{V_{31}^* V_{33}\sqrt{m_t} + \cdots} \simeq -\frac{|V_{us}|}{V_{31}^*}\sqrt{\frac{m_c}{m_t}}, \tag{4.3}$$

$$V_{31}^*(\delta_{ue}) \simeq |V_{us}||V_{cb}| - |V_{ub}|e^{-i\delta_{ue}}. \tag{4.4}$$

The relation $(Y_\nu)_{12}/(Y_\nu)_{13} \simeq -1$ with $\delta_{ue} = \pi$ is warranted by the fact that the relation

$$\sqrt{\frac{m_c}{m_t}} \simeq |V_{cb}| + \frac{|V_{ub}|}{|V_{us}|}, \tag{4.5}$$

is well satisfied with the observed values, $\sqrt{m_c/m_t} = 0.061$ [7] at $\mu = M_Z$, $|V_{cb}| = 0.0416$ and $|V_{ub}|/|V_{us}| = 0.0191$ [8].

4.2 Numerical study of the seesaw neutrino mass matrix

Next, we investigate a case that the observed neutrinos are Majorana neutrinos which are generated by a seesaw mechanism:

$$M_\nu = \left(\frac{y_\nu}{\Lambda} v_{Hu}\right)^2 Y_\nu M_R^{-1} Y_\nu^T. \tag{4.6}$$

In this case, the mixing matrix U_ν is not always identical with a mixing matrix $U_{e\nu}$ which is defined as

$$U_{e\nu}^\dagger \langle Y_\nu\rangle_e U_{e\nu} = \langle Y_\nu\rangle_\nu \equiv \langle Y_\nu\rangle^D. \tag{4.7}$$

In order to diagonalize the matrix (4.6), we must know a form of $\langle M_R\rangle_e$. For simplicity, we assume that the form of M_R is independent of the flavor basis, i.e. $M_R \propto \mathbf{1}$. Then, the mixing matrix U_ν is obtained by diagonalizing the matrix $\langle Y_\nu\rangle_e \langle Y_\nu\rangle_e^T$. When we denote $\langle Y_\nu\rangle_e$ as $\langle Y_\nu\rangle_e = Y_0 + \xi\mathbf{1}$, where $U_{e\nu}^\dagger Y_0 U_{e\nu} = D_0$ (D_0 is diagonal), we can show $U_{e\nu}^\dagger \langle Y_\nu\rangle_e \langle Y_\nu\rangle_e^T U_{e\nu}^* = (D_0 + \xi\mathbf{1})U_{e\nu}^\dagger U_{e\nu}^*(D_0 + \xi\mathbf{1})$. Since the transformation matrix $U_{e\nu}$ is orthogonal in the cases with $\delta_{ue} = 0$ and $\delta_{ue} = \pi$, the matrix $U_{e\nu}^\dagger U_{e\nu}^*$ becomes a unit matrix $\mathbf{1}$, so that the lepton mixing matrix U_ν is given by $U_\nu = U_{e\nu}$ as well as in the case of Dirac neutrinos. However, for the cases with $\delta_{ue} \neq 0$ and $\delta_{ue} \neq \pi$, the case of Majorana neutrinos cannot give the same results with the case of Dirac neutrinos. The numerical results are given in Table 2. Although, in the case of Dirac neutrinos, the case with $\delta_{ue} = \delta_{CKM} + \pi$ ($\simeq -120^\circ$) has been acceptable, in the present case, such a case with $\delta_{ue} \neq \pi$ is ruled out, because such case can give reasonable values for neither $\tan^2\theta_{12}$ nor $R \equiv \Delta m_{21}^2/\Delta m_{32}^2$ as seen in Table 2. (Also note that the value of $\tan^2\theta_{12}$ is sensitive to the value of ξ_0.)

Table 2: Dependence on δ_{ue} and ξ_0 for the predicted values $R \equiv \Delta m^2_{21}/\Delta m^2_{32}$, $\sin^2 2\theta_{23}$, $\tan^2\theta_{12}$ and $|U_{13}|$ in the seesaw mass matrix (4.6). The values of R in the case with $\delta_{ue} = 0$ and $\delta_{ue} = \pi$ have already been adjusted by fitting ξ_0 to the value $|R| \simeq 0.028$ and $\Delta m^2_{21} > 0$. For the cases $\delta_{ue} = \pi/3$ and $2\pi/3$, we cannot obtain such a small value as $|R| \simeq 0.028$, so that we denote only the cases with lower limits of R in Table.

δ_{ue}	$\xi_0/(v_u/m^e_0)$	R	$\sin^2 2\theta_{23}$	$\tan^2\theta_{12}$	$\lvert U_{13}\rvert$
0°	−25.57	−0.0282	0.3831	0.4170	0.0113
60°	−25.85	−0.1170	0.7035	0.0757	0.1349
60°	−25.99	−0.1156	0.7032	0.0381	0.1355
60°	−26.13	−0.1168	0.7029	0.0142	0.1360
120°	−25.99	−0.0705	0.9643	0.0889	0.0912
120°	−26.13	−0.0685	0.9641	0.0324	0.0912
120°	−26.27	−0.07000	0.9640	0.0054	0.0912
180°	−26.28	−0.0273	0.9997	0.5125	0.0001

Thus, if we consider that the observed neutrinos are Majorana types, only the case $U_{ue} = V(\pi)$ can give successful predictions:

$$U_\nu = \begin{pmatrix} 0.8131 & -0.5821 & -0.0001 \\ -0.4153 & -0.5803 & 0.7006 \\ 0.4079 & 0.5696 & 0.7136 \end{pmatrix}, \tag{4.8}$$

independently of the value of ξ_0. The result (4.8) is very close to the tribimaximal mixing

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{4.9}$$

(Here, we have taken a phase convention corresponding to (4.8).) We simply regard that $U_{e\nu} = U_{TB}$. The relations among four bases are illustrated in Fig. 2.

In the present model, the absolute values of the neutrino masses cannot be predicted because of the free parameter ξ_0. We can merely choose a suitable value of ξ_0 from the observed value

$$|R| = \frac{\Delta m^2_{21}}{|\Delta m^2_{32}|} = 0.028 \pm 0.004, \tag{4.10}$$

where we have used the observed values $\Delta m^2_{21} = (7.59 \pm 0.21) \times 10^{-5}\,\mathrm{eV}^2$ [10] and $|\Delta m^2_{32}| = (2.74^{+0.44}_{-0.26}) \times 10^{-3}\,\mathrm{eV}^2$ [11]. The numerical results are demonstrated in Table 3, where the values

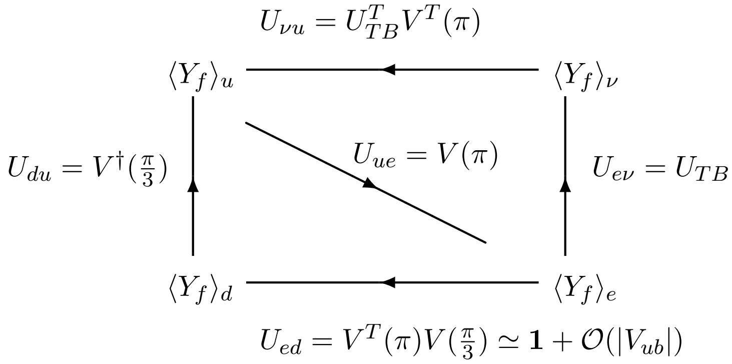

Figure 2: Flavor-basis transformation operators U_{ab}, which is defined by $\langle Y_f \rangle_b = U_{ab}^\dagger \langle Y_f \rangle_a U_{ab}$ $(a, b, c = u, d, \nu, e)$.

Table 3: Neutrino masses fitted from the observed values $R = \Delta m_{21}^2 / \Delta m_{32}^2$ and $m_{\nu 2} = \sqrt{|\Delta m_{32}^2|}$. For seesaw masses, the Majorana mass matrix M_R of the right-handed neutrinos is assumed as $M_R \propto \mathbf{1}$.

Type	$\xi_0/(v_u/m_0^e)$	R	$m_{\nu 1}$ [eV]	$m_{\nu 2}$ [eV]	$m_{\nu 3}$ [eV]	$\sum m_{\nu i}$ [eV]
Dirac mass	-26.48	-0.0276	0.0517	0.0523	0.0181	0.1221
Seesaw mass	-26.29	-0.0281	0.0516	0.0523	0.0062	0.1101

of ξ_0 are given in unit of v_u/m_0^e, $\langle (\Phi_u^D)_{ii} \rangle = v_u z_i^u \propto \sqrt{m_{ui}}$ and $m_{ei} = m_0^e (z_i^e)^2$, where z_i^e and z_i^u are normalized as $z_1^2 + z_2^2 + z_3^2 = 1$. Note that the present model gives an inverted mass hierarchy of neutrinos. The values of $m_{\nu i}$ are estimated by putting as $m_{\nu 2} = \sqrt{|\Delta m_{32}^2|} = 0.0523$ eV. As seen in Table 3, the numerical results $\sum m_{\nu i} = 0.12$ eV (Dirac neutrinos) and $\sum m_{\nu i} = 0.11$ eV (Majorana neutrinos) safely satisfies the cosmological lower bound $\sum m_{\nu i} < (0.2 - 0.4)$ eV (the recent cosmological neutrino mass bounds are listed, for example, in Ref.[12]). For a case of Majorana neutrinos, we can calculate the effective neutrino mass $\langle m_{ee} \rangle$ as

$$|\langle m_{ee} \rangle| = \left| \sum_i U_{ei}^2 m_{\nu i} \right| = 0.0164 \text{ eV}. \tag{4.11}$$

The value (4.11) will be observed in future neutrinoless double beta experiments.

5 Concluding remarks

In conclusion, based on a U(3)-flavor nonet scalar model, we have obtained a neutrino mass matrix (3.15) of a new type, where the matrix is described in terms of charged lepton and up-quark mass matrices. However, in order to evaluate the neutrino mixing matrix from the neutrino mass matrix (3.15), we must know the form of $\langle Y_u \rangle$ on the e-basis (not on the d-basis). Since we do not know it at present, we have assumed the form (1.2) from the phenomenological point of view. Then, we have found the neutrino mass matrix (3.15) with the phenomenological assumption (1.2) can give a nearly tribimaximal mixing. (Therefore, as shown in the present title, the neutrino mass matrix is not one which is derived from a model, but it is an empirical one.) Nevertheless, it is worthwhile noticing because the form is one of a new type which is related to the up-quark masses and which successfully leads to the nearly tribimaximal mixing without assuming any discrete symmetry. Inversely speaking, this phenomenological success suggests a possibility that we can understand the CKM matrix and quark mass spectrum by starting from a discrete symmetry which gives the observed tribimaximal mixing for the lepton sectors.

If we accept the empirical neutrino mass matrix (3.15), in order that the neutrino mass matrix Y_ν gives successful results, we cannot regard that the e-basis is identical with the d-basis, and we must take $U_{ue} = V(\delta_{ue})$ with $2\pi/3 \leq \delta_{ue} \leq \pi$ for Dirac neutrinos and with $\delta_{ue} = V(\pi)$ for the seesaw (Majorana) neutrinos, although the e-basis is still very near to the d-basis, i.e. $U_{ed} = \mathbf{1} + \mathcal{O}(|V_{ub}|)$, Eq.(4.1). The present model gives an inverse hierarchy of the neutrino masses as seen in Table 3. The reason why U_{ue} takes the form $V(\delta_{ue})$ is, at present, an open question, and it is only a phenomenological conclusion.

In this paper, we have not investigate a possible form of W_d which will give relations of the field Y_d to other fields[2]. Since we have given U_{ue}, the relative relations among four flavor-bases are fixed each other. Therefore, if we give a form of W_d, we can give not only an explanation of the down-quark masses, but also "predictions" for other masses and mixings. However, in order to give an explicit form of W_d, we must put further assumptions, so that we have not discussed the explicit form of W_d because the purpose of the present paper is to report an empirical neutrino mass matrix of a new type. A possible model for Y_d will be given elsewhere.

By the way, we have not discussed a possibility that the present model is extended to a grand unification (GUT) scenario. In the present model, since all Y_f are assumed as U(3)-flavor nonets, the model cannot be extended to GUT scenario, because Y_u, for example, will be a 6-plet of $U(3)_F$ in a GUT model, because $\mathbf{3} \times \mathbf{3} = \mathbf{6}_S + \mathbf{3}^*_A$ (not $\mathbf{3} \times \mathbf{3}^* = \mathbf{1} + \mathbf{8}$). When Y_u is a 6-plet of $U(3)_F$, it is hard to lead such a bilinear relation as Eq.(3.7). If we want a formulation similar to the present prescription, we may consider, for example, $O(3)_F$ instead of $U(3)_F$. Then, the nonet fields in the present model will be replaced with $(\mathbf{1} + \mathbf{5})_S + \mathbf{3}_A$ of $O(3)_F$.[3] How to extend the present model to a GUT model is also our future task.

[2]For example, in Ref.[5], a model for W_d has been proposed. However, in the model, since the d-basis is identical with the e-basis, we cannot apply the model to the present model straightforwardly.

[3]Note added: Based on an O(3) flavor symmetry, an extended version [13] of the present model has recently proposed. The essential structure of the O(3) model is similar to that in the present U(3) model, and the substantial formulations have already been given in the present paper. Since the VEV matrices $\langle Y_f \rangle$ are diagonalized as $U_f^\dagger \langle Y_f \rangle U_f = \langle Y_f \rangle^D$ and $U_f^T \langle Y_f \rangle U_f = \langle Y_f \rangle^D$ in the U(3) and O(3) models, respectively, we can use $\mathrm{Tr}[\langle \Phi_u \rangle] = \mathrm{Tr}[\langle \Phi_u \rangle^D] \propto \mathrm{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$ and $\mathrm{Tr}[\langle \Phi_u \rangle \langle \Phi_u \rangle] = \mathrm{Tr}[\langle \Phi_u \rangle^D \langle \Phi_u \rangle^D] \propto \mathrm{diag}(m_u, m_c, m_t)$ in the U(3) model, while we cannot use such relations in the O(3) model because $U_f U_f^T \neq \mathbf{1}$ in general. Therefore, the U(3) model still has a considerable advantage compared with the O(3) model.

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