How to Evade a No-Go Theorem in Flavor Symmetries [1]

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Abstract

A no-go theorem in flavor symmetries is reviewed. The theorem asserts that we cannot bring any flavor symmetry into mass matrix model in which number of Higgs scalars is, at most, one for each sector (e.g. H_u and H_d for up- and down-quark sectors, respectively). Such the strong constraint comes from the SU$(2)_L$ symmetry. Possible three options to evade the theorem are discussed.

1 Introduction

When we see a history of physics, we will find that "symmetries" always play a key role in the new physics. For investigating an origin of flavors, too, we may expect that an approach based on symmetries will be a powerful instrument. Especially, how to treat the flavor symmetry is a big concern in grand unification model-building.

However, when we want to introduce a flavor symmetry (e.g. discrete one, U(1), and so on) into our mass matrix model, we always encounter an obstacle, a no-go theorem in flavor symmetries [1]. The theorem asserts that we cannot bring any flavor symmetry into a mass matrix model in which number of Higgs scalars is, at most, one for each sector (e.g. H_u and H_d for up- and down-quark sectors, respectively). Such the strong constraint comes from the SU$(2)_L$ symmetry. We must take this theorem seriously. This theorem seems to require a new idea for the mass generation against a conventional idea "(masses)=(Yukawa coupling constants) $\times$(vacuum expectation value of Higgs scalar)". We should not consider this theorem to be negative, and we should utilize this theorem positively to investigate the origin of the flavor mass spectra.

Nevertheless, there are some optimists. Why? They know that, for example, the Yukawa interactions in the up- and down-quark sectors are independent of each other, and, besides, the Higgs scalars which contribute to each sector can be different (e.g. H_u and H_d, respectively). Therefore, they consider that we can apply the flavor symmetry to the up- and down-quark sectors separately. First, let check this.

In the standard model, the fermion masses are generated from the vacuum expectation values (VEVs) of the Higgs scalars:

$$H_Y = (Y_u)_{ij}\overline{Q}_{Li} H_u u_{Rj} + (Y_d)_{ij}\overline{Q}_{Li} H_d d_{Rj} + h.c. \tag{1.1}$$

where

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad H_u = \begin{pmatrix} H_u^0 \\ H_u^- \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix}. \tag{1.2}$$

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Therefore, the mass matrices M_u and M_d for the up- and down-quarks are given by

$$(M_u)_{ij} = (Y_u)_{ij}\langle H_u^0\rangle, \quad (M_d)_{ij} = (Y_d)_{ij}\langle H_d^0\rangle. \tag{1.3}$$

The requirement of a flavor symmetry means that the interactions (1.1) are invariant under the transformation of the flavor basis

$$\begin{array}{lcl} Q_L & \to & Q'_L = T_L Q_L, \\ u_R & \to & u'_R = T_R^u u_R, \\ d_R & \to & d'_R = T_R^d d_R. \end{array} \tag{1.4}$$

Then, the requirement, in general, imposes the following constraints on the Yukawa coupling constants Y_u and Y_d:

$$T_L^\dagger Y_u T_R^u = Y_u, \quad T_L^\dagger Y_d T_R^d = Y_d. \tag{1.5}$$

(Of course, the physical quark masses (1.3) are given below the energy scale $\mu = \Lambda_{EW}$, at which the SU(2)$_L$ symmetry is broken, so that the constraint (1.5) has a meaning above $\mu = \Lambda_{EW}$, i.e. for $Y_u(\mu)$ and $Y_d(\mu)$ at $\mu > \Lambda_{EW}$.) This constraint (1.5) does not always mean that the form of Y_u is the same as that of Y_d. A relation between the coupling constants Y_u and Y_d looks like free.

However, when we take notice of the Hermitian matrices $Y_u Y_u^\dagger$ and $Y_d Y_d^\dagger$, the situation will become clear:

$$\begin{array}{l} T_L^\dagger Y_u (Y_u)^\dagger T_L = Y_u (Y_u)^\dagger, \\ T_L^\dagger Y_d (Y_d)^\dagger T_L = Y_d (Y_d)^\dagger. \end{array} \tag{1.6}$$

Here, we should note that the flavor transformation operator T_L is identical both for up- and down-quark sectors. As we discuss in the next section, this will give a strong constraint for the Cabibbo-Kobayashi-Maskawa [2] (CKM) mixing matrix V, which is defined by

$$V = (U_L^u)^\dagger U_L^d, \tag{1.7}$$

where U_L^f are defined by

$$\begin{array}{l} (U_L^u)^\dagger M_u U_R^u = D_u \equiv \mathrm{diag}(m_u, m_c, m_t), \\ (U_L^d)^\dagger M_d U_R^d = D_d \equiv \mathrm{diag}(m_d, m_s, m_b), \end{array} \tag{1.8}$$

i.e.

$$\begin{array}{l} (U_L^u)^\dagger Y_u (Y_u)^\dagger U_L^u = \frac{1}{v_u^2} D_u (D_u)^\dagger, \\ (U_L^d)^\dagger Y_d (Y_d)^\dagger U_L^d = \frac{1}{v_d^2} D_d (D_d)^\dagger, \end{array} \tag{1.9}$$

and $v_u = \langle H_u^0\rangle$ and $v_d = \langle H_d^0\rangle$.

2 No-Go theorem in flavor symmetries

The no-go theorem in flavor symmetries is as follows [1]:

[Theorem] When a flavor symmetry is brought into a model within the framework of the standard model, the flavor mixing matrix (CKM matrix and/or neutrino mixing matrix) cannot describe a mixing among 3 families, and only a mixing between 2 families is allowed.

For example, the theorem asserts that we can obtain only a two-flavor mixing such as

$$V = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.1}$$

Such a strong constraint comes from the relations (1.6) and (1.9). When we define the following operators

$$T_u = (U_L^u)^\dagger T_L U_L^u, \quad T_d = (U_L^d)^\dagger T_L U_L^d, \tag{2.2}$$

from the flavor transformation operator T_L in the flavor symmetry, we can obtain a relation

$$T_u^\dagger D_u^2 T_u = D_u^2, \quad T_d^\dagger D_d^2 T_d = D_d^2, \tag{2.3}$$

because $Y_f Y_f^\dagger$ in (1.9) is express as

$$Y_f Y_f^\dagger = T_L^\dagger Y_f Y_f^\dagger T_L = T_L^\dagger \cdot U_L^f (1/v_f^2) D_f D_f^\dagger (U_L^f)^\dagger T_L = U_L^f T_f^\dagger (1/v_v^2) D_f D_f^\dagger T_f (U_L^f)^\dagger. \tag{2.4}$$

Therefore, if the eigenvalues of Y_f are non-zero and non-degenerate, the operator T_f must be expressed by a form of the phase matrix

$$T_f = P_f \equiv \mathrm{diag}(e^{i\delta_1^f}, e^{i\delta_2^f}, e^{i\delta_3^f}), \tag{2.5}$$

so that T_L is expressed as

$$T_L = U_L^u P_u (U_L^u)^\dagger = U_L^d P_d (U_L^d)^\dagger, \tag{2.6}$$

from the definition of T_f, Eq.(2.2). Therefore, the phase matrices P_u and P_d are related as

$$P_u = (U_L^u)^\dagger U_L^d P_d (U_L^d)^\dagger U_L^u = V P_d V^\dagger, \tag{2.7}$$

from Eq.(2.6) and the definition of the CKM matrix V, (1.7). Thus, we obtain a constraint on the CKM matrix V

$$P_u V - V P_d = 0, \tag{2.8}$$

i.e.

$$(e^{i\delta_i^u} - e^{i\delta_j^d}) V_{ij} = 0. \tag{2.9}$$

Therefore, if $\delta_i^u \neq \delta_j^d$, we obtain an unwelcome result $V_{ij} = 0$ [1].

We do not consider the case with $\delta_1^u = \delta_2^u = \delta_3^u$ and $\delta_1^d = \delta_2^d = \delta_3^d$, because the case corresponds to a trivial flavor transformation $T_L = \mathbf{1}$. For a non-trivial flavor transformation

T_L, we must choose, at least, one of δ_i^f differently from others. For example, for the case with $\delta_1^f = \delta_2^f \neq \delta_3^f$, we can obtain only a two-family mixing

$$V = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.10}$$

We can essentially obtain a similar result in the lepton sectors [1], although a stronger constraint will be added if the neutrino mass matrix is Majorana type.

Now, let us summarize the premises to derive the theorem:

(i) The $\mathrm{SU}(2)_L$ symmetry is unbroken;

(ii) There is only one Higgs scalar in each sector;

(iii) 3 eigenvalues of Y_f in each sector are non-zero and no-degenerate.

If one of them in a model is not satisfied, the model can evade the theorem.

For example, let us consider a model: (i) we consider an unbroken flavor symmetry at GUT scale; (ii) there is only one Higgs scalar in each sector, e.g. H_u and H_d; (iii) 3 eigenvalues of Y_f in each sector are completely different from each other and not zero at the GUT scale. Then, such the model is ruled out by the theorem. However, if the flavor symmetry is explicitly broken, i.e. the model has an explicit symmetry breaking term at the beginning, the present theorem does not affect such a model.

3 How to evade the no-go theorem

We will discuss three options to evade the no-go theorem:

(A) Model with multi-Higgs scalars;

(B) Model with an explicit broken flavor symmetry;

(C) Model in which Y's are fields.

Of course, a model in which all $\mathrm{SU}(2)_L$ doublets are singlets (i.e. $T_L = \mathbf{1}$) under the flavor symmetry can evade the no-go theorem.

In most phenomenological studies of flavor symmetries, models contain a phenomenological symmetry breaking term from the beginning, although we suppose that such a symmetry breaking term is spontaneously generated from the world of an unbroken flavor symmetry. Such a model belongs to the category (B). In the present studies, we do not discuss such a model in which the problem is postponed to future.

In most GUT models, more than two Higgs scalars which belong to different multiplets of the GUT group are assumed. Such models belong to the category (A). In such models, it is essential whether unwelcome components of the Higgs scalars can naturally be suppressed in the Higgs potential without any explicit symmetry breaking term.

3.1 Model with multi-Higgs scalars

A model with multi-Higgs scalars can evade the no-go theorem, where the Higgs scalars must have different transformation properties under the flavor symmetry. For example, we may consider a model:

$$M_{ij} = Y^a_{ij}\langle H_a\rangle + Y^b_{ij}\langle H_b\rangle + Y^c_{ij}\langle H_c\rangle. \tag{3.1}$$

However, generally, such a multi-Higgs model induces the so-called flavor-changing neutral currents (FCNC) problem. We must make those Higgs scalars heavy except for one of linear combinations of those scalars, e.g.

$$U_H \begin{pmatrix} H_a \\ H_b \\ H_c \end{pmatrix} = \begin{pmatrix} H_0 \\ H_1 \\ H_2 \end{pmatrix} \begin{array}{c} \sim 10^2\mathrm{GeV} \\ \sim 10^{16}\mathrm{GeV} \\ \sim 10^{16}\mathrm{GeV}, \end{array} \tag{3.2}$$

where U_H is a mixing matrix among H_a, H_b and H_c. Since the Higgs scalars H_a H_b and H_c have different quantum numbers of the flavor symmetry, such a mixing (3.2) breaks the flavor symmetry at a high energy scale, at which the mixing U_H is caused. Of course, such a mixing must be caused without any explicit symmetry breaking parameters.

However, at present, models which give a reasonable mixing mechanism are few. The mechanism must be proposed in the framework of the exact flavor symmetry. In most models, the suppression of unwelcome components are only assumptions by hand.

3.2 Model with an explicitly broken symmetry

We consider a model in which the symmetry is explicitly broken from the beginning. In other words, in such a model, there is no flavor symmetry from the beginning. Therefore, such a model can, of course, evade the no-go theorem.

In most phenomenological studies of flavor symmetries, models contain a phenomenological symmetry breaking term from the beginning, although we suppose that such a symmetry breaking term is spontaneously generated from the world of an unbroken flavor symmetry. Such a model belongs to the present category.

In any flavor symmetry, the symmetry finally has to be broken badly, because the observed flavor mass values are highly hierarchical. As an example of a model in which the flavor symmetry is badly broken at the beginning, let us review the following model [3]: We assume a U(3) flavor symmetry. For simplicity, we consider only a case of the charged lepton sector. The symmetry U(3) is broken by parameters $(Y_e)_{ij}$ explicitly:

$$W_Y = \sum_{i,j} (Y_e)_{ij} L_j E_i H_d. \tag{3.3}$$

(For convenience, hereafter, we will drop the index "e".) Also, we consider a U(3) nonet field Φ and we denote the superpotential for Φ as

$$W_\Phi = m_1 \mathrm{Tr}[\Phi\Phi] + m_2 \mathrm{Tr}^2[\Phi] + \lambda_1 \mathrm{Tr}[\Phi\Phi\Phi] + \lambda_2 \mathrm{Tr}[\Phi\Phi]\mathrm{Tr}[\Phi] + \lambda_3 \mathrm{Tr}^3[\Phi]. \tag{3.4}$$

We assume that the symmetry is also broken by a tadpole term with the same symmetry breaking parameter Y as follows:

$$W = W_\Phi - \mu^2 \mathrm{Tr}[Y\Phi] + W_Y. \tag{3.5}$$

Then, we obtain

$$\frac{\partial W}{\partial \Phi} = 0 = \frac{\partial W_\Phi}{\partial \Phi} - \mu^2 Y, \tag{3.6}$$

where

$$\frac{\partial W_\Phi}{\partial \Phi} = 3\lambda_1 \Phi\Phi + c_1(\Phi)\Phi + c_0(\Phi)\mathbf{1}, \tag{3.7}$$

$$c_1(\Phi) = 2(m_1 + \lambda_2 \mathrm{Tr}[\Phi]), \tag{3.8}$$

$$c_0(\Phi) = 2m_2 \mathrm{Tr}[\Phi] + \lambda_2 \mathrm{Tr}[\Phi\Phi] + 3\lambda_3 \mathrm{Tr}^2[\Phi]. \tag{3.9}$$

Now, we put an ansatz that our vacuum is given by the following specific solution of Eq.(3.6):

$$3\lambda_1 \Phi\Phi - \mu^2 Y = 0, \tag{3.10}$$

and

$$c_1(\Phi)\Phi + c_0(\Phi)\mathbf{1} = 0. \tag{3.11}$$

Eq.(3.10) leads to a bilinear mass formula

$$Y_{ij} = \frac{3\lambda_1}{\mu^2} \sum_k \langle \Phi_{ik} \rangle \langle \Phi_{kj} \rangle. \tag{3.12}$$

For non-zero and non-degenerate eigenvalues v_i of $\langle \Phi \rangle$, Eq.(3.11) requires $c_1 = 0$ and $c_0 = 0$. Thus, we can obtain a relation for the charged lepton masses by choosing a suitable form of W_Φ.

For example, when we assume [3]

$$W_\Phi = m \mathrm{Tr}[\Phi\Phi] + \lambda \mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}], \tag{3.13}$$

where $\Phi^{(8)}$ is an octet part of the nonet scalar Φ:

$$\Phi^{(8)} = \Phi - \frac{1}{3}\mathrm{Tr}[\Phi]\mathbf{1}, \tag{3.14}$$

we obtain

$$\mathrm{Tr}[\Phi\Phi] = \frac{2}{3}\mathrm{Tr}^2[\Phi], \tag{3.15}$$

from $c_0 = 0$, because

$$\mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] = \mathrm{Tr}[\Phi\Phi\Phi] - \mathrm{Tr}[\Phi]\left(\mathrm{Tr}[\Phi\Phi] - \frac{2}{9}\mathrm{Tr}^2[\Phi]\right). \tag{3.16}$$

Eq.(3.15) leads to the VEV relation

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2, \tag{3.17}$$

in the diagonal basis of $\langle \Phi_{ij} \rangle = \delta_{ij} v_i$. Therefore, from Eqs.(3.12) and (3.17), we obtain the charged lepton mass formula [4]

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2. \tag{3.18}$$

The formula (3.18) can give an excellent prediction $m_\tau = 1776.97$ MeV from the observed values of m_e and m_μ, which is in excellent agreement with the observed value [5] $m_\tau^{obs} = 1776.99^{+0.29}_{-0.26}$ MeV.

3.3 Model in which Y's are fields

We consider that Y_f in the Yukawa interaction (1.1) are fields, e.g.

$$H_Y = \frac{(Y_u)_{ij}}{M}\overline{Q}_{Li}H_u u_{Rj} + \frac{(Y_d)_{ij}}{M}\overline{Q}_{Li}H_d d_{Rj} + h.c. \tag{3.19}$$

Since the fields Y_f are transformed as

$$Y_f \to Y'_f = T_L Y_f (T_R^f)^\dagger, \tag{3.20}$$

under the transformation (1.4), the constraints (1.6) for $Y_f(Y_f)^\dagger$ disappears, so that we can again evade the no-go theorem.

For example, recently, Haba [6] has suggested that the effective Yukawa interaction originates in a higher dimensional term in Kähler potential K

$$K \sim \frac{1}{M^2} y_A A^\dagger_{ij} L_j E_i H_d, \tag{3.21}$$

which leads to an effective Yukawa interaction

$$(K)_D \sim \frac{1}{M^2} y_A (F_A^\dagger)_{ij} L_j E_i H_d. \tag{3.22}$$

A similar idea in the neutrino masses has been proposed by Arkani-Hamed, Hall, Murayama, Smith and Weiner [7].

For example, when we adopt an O'Raifeartaigh-type SUSY breaking mechanism [8]

$$W = W_\Phi(\Phi) + \lambda_A \mathrm{Tr}[A\Phi\Phi] + \lambda_B \mathrm{Tr}[B\Phi\Phi] - \mu^2 \mathrm{Tr}[\xi A], \tag{3.23}$$

where ξ (3×3 matrix) is a flavor breaking parameter, the scalar potential V is given by

$$V = \mathrm{Tr}\left[(\lambda_A\Phi\Phi - \mu^2\xi)(\lambda_A\Phi\Phi - \mu^2\xi)^\dagger\right] + |\lambda_B|^2 \mathrm{Tr}[\Phi\Phi\Phi^\dagger\Phi^\dagger]$$

$$+\mathrm{Tr}\left[\left(\frac{\partial W_\Phi}{\partial \Phi} + (\lambda_A A + \lambda_B B)\Phi + \Phi(\lambda_A A + \lambda_B B)\right)\left(\frac{\partial W_\Phi}{\partial \Phi} + (\lambda_A A + \lambda_B B)\Phi + \Phi(\lambda_A A + \lambda_B B)\right)^\dagger\right], \tag{3.24}$$

so that the conditions $\partial V/\partial A = 0$ and $\partial V/\partial B = 0$ give the constraint

$$\frac{\partial W_\Phi}{\partial \Phi} + (\lambda_A A + \lambda_B B)\Phi + \Phi(\lambda_A A + \lambda_B B) = 0, \tag{3.25}$$

and the condition $\partial V/\partial \Phi = 0$ gives

$$(|\lambda_A|^2 + |\lambda_B|^2)\Phi\Phi = \lambda_A^* \mu^2 \xi, \tag{3.26}$$

under the condition (3.25). Therefore, we can again obtain a bilinear form for the effective Yukawa coupling constant as follows:

$$-F_A^\dagger = \frac{\partial W}{\partial A} = \lambda_A \Phi\Phi - \mu^2 \xi = \lambda_B \frac{\lambda_B}{\lambda_A} \Phi\Phi \neq 0, \tag{3.27}$$

$$-F_B^\dagger = \frac{\partial W}{\partial B} = \lambda_B \Phi\Phi \neq 0. \tag{3.28}$$

However, since the present model leads to unwelcome situation that fermion parts $\psi_{C'}$ of the superfields $C' \equiv (\lambda_B A - \lambda_A B)/\sqrt{\lambda_A^2 + \lambda_B^2}$ become massless. In order to make those massless fermions harmless, we must change the Kähelar potential K into a non-canonical form with higher dimensional terms

$$-\frac{1}{M^2} \left(\mathrm{Tr}^2[A^\dagger A] + \mathrm{Tr}^2[B^\dagger B] \right). \tag{3.29}$$

Two conditions $\partial V/\partial A = 0$ and $\partial V/\partial B = 0$ have led to the same constraint (3.25) in the canonical Käheler potential, while, in the non-canonical Käheler potential with the higher dimensional terms (3.29), two conditions $\partial V/\partial A = 0$ and $\partial V/\partial B = 0$ lead to different constraints, and thereby, we can obtain

$$\langle A \rangle = \langle B \rangle = 0 \quad \text{and} \quad \langle \frac{\partial W_\Phi}{\partial \Phi} \rangle = 0. \tag{3.30}$$

Therefore, the massless fermions $\psi_{C'}$ can become harmless, because some dangerous amplitudes become zero due to $\langle A \rangle = \langle B \rangle = 0$. On the other hand, the VEV spectrum of Φ is practically determined by the constraint

$$\frac{\partial W_\Phi}{\partial \Phi} = 0, \tag{3.31}$$

which is derived from the conditions $\partial V/\partial A = 0$ and $\partial V/\partial B = 0$. By assuming a suitable form of W_Φ, we can again obtain the mass relation (3.18). For more details, see Ref.[9].

4 Summary

The no-go theorem in flavor symmetries asserts us that we cannot bring any flavor symmetry into a mass matrix model based on the standard model. We have demonstrated three options to evade the no-go theorem in the flavor symmetries:

(A) Model with multi-Higgs scalars;

(B) Model with an explicit broken symmetry;

(C) Model in which Y's are fields.

Models based on the scenario (A) have been proposed by many authors. In most GUT models, more than two Higgs scalars which belong to different multiplets of the GUT group are assumed. Therefore, if we can make those scalars heavy except one component, we will obtain a reasonable model which can evade the no-go theory. However, current most models have not demonstrated an explicit mechanism (Higgs potential) which makes unwelcome components of the Higgs scalars heavy except for one.

In most phenomenological studies of flavor symmetries, models contain a phenomenological symmetry breaking term from the beginning. Even if we suppose that such a symmetry breaking term is spontaneously generated from the world of an unbroken flavor symmetry, the model belongs to the category (B), unless we explicitly demonstrate it on the basis of a Higgs potential without any symmetry breaking term. In the scenario (B), there is no flavor symmetry from the beginning. The "flavor symmetry" is a faked one for convenience. However, if we once suppose a flavor symmetry, rather, we would like to consider that the symmetry is exact, and then it is broken spontaneously. Therefore, we are still unsatisfactory to the scenario (B).

Models based on the scenario (C) are interesting. However, in order to give an effective Yukawa interaction, we need a term with higher dimension

$$\frac{1}{M}(Y_e)_{ij} L_j E_i H_d$$

in the superpotential W, or

$$\frac{1}{M^2}(Y_e^\dagger)_{ij} L_j E_i H_d$$

in the Kähler potential K. However, we want a model without such higher dimensional terms as possible.

In conclusion, we have proposed three options to evade the no-go theorem. Those scenarios can evade the no-go theorem practically, but those still do not suit our feeling. We must seek for a more natural scenario which is free from the no-go theorem. Then, a hybrid model between (A) and (C), where there is a U(3)-flavor nonet Higgs doublet scalar and only one component becomes light, will be promising.

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