Dark Energy and Neutrino Model in SUSY

International Workshop on Neutrino Masses and Mixings
—Toward Unified Understanding of Quark and Lepton Mass Matrices—
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RT, M. Tanimoto, PLB 633 (2006) 675
RT, M. Tanimoto, JHEP 0605 (2006) 021
RT, M. Tanimoto, PRD 74 (2006) 055002

Collaborator

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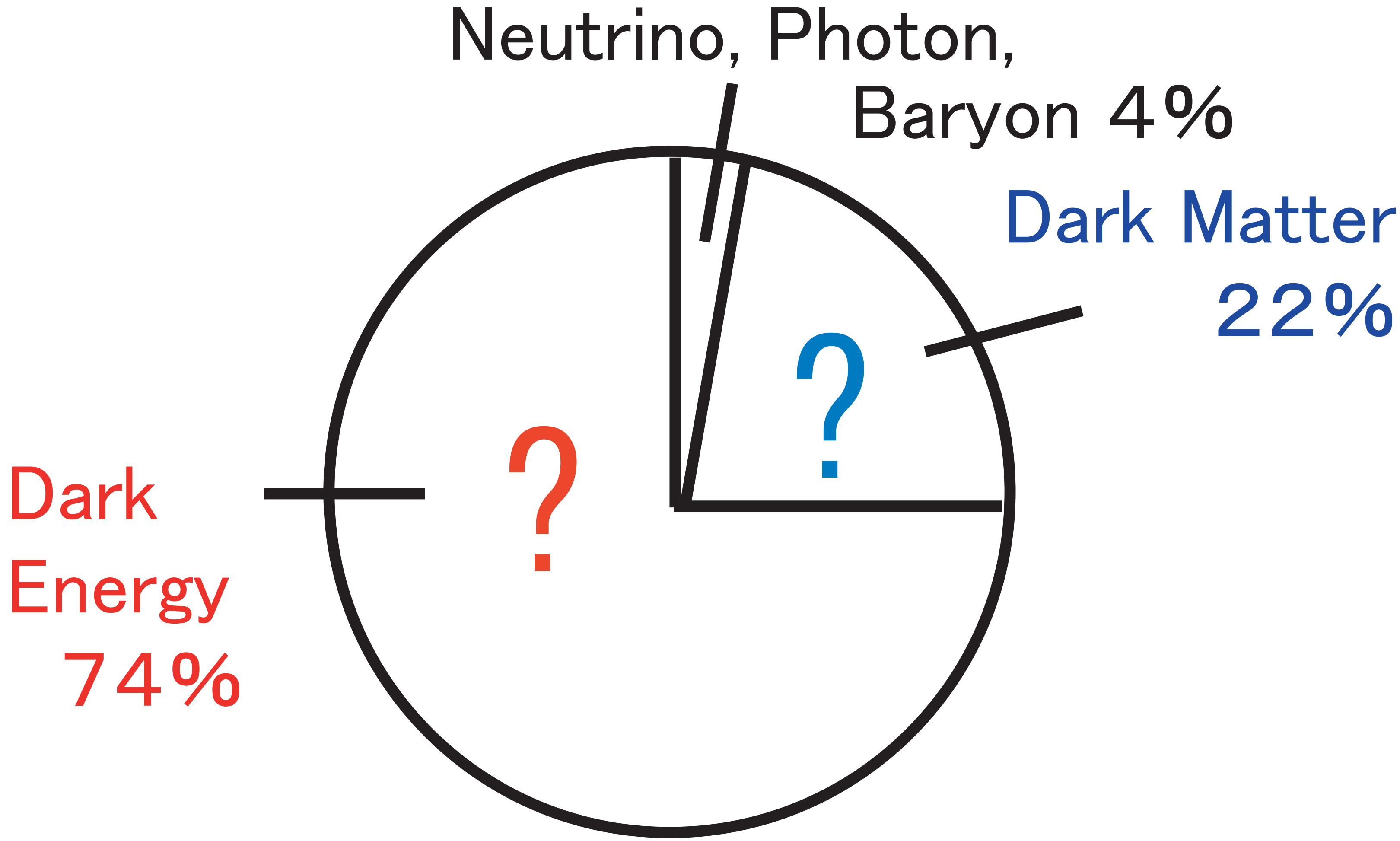
1. Introduction

Cosmic Energy Budget



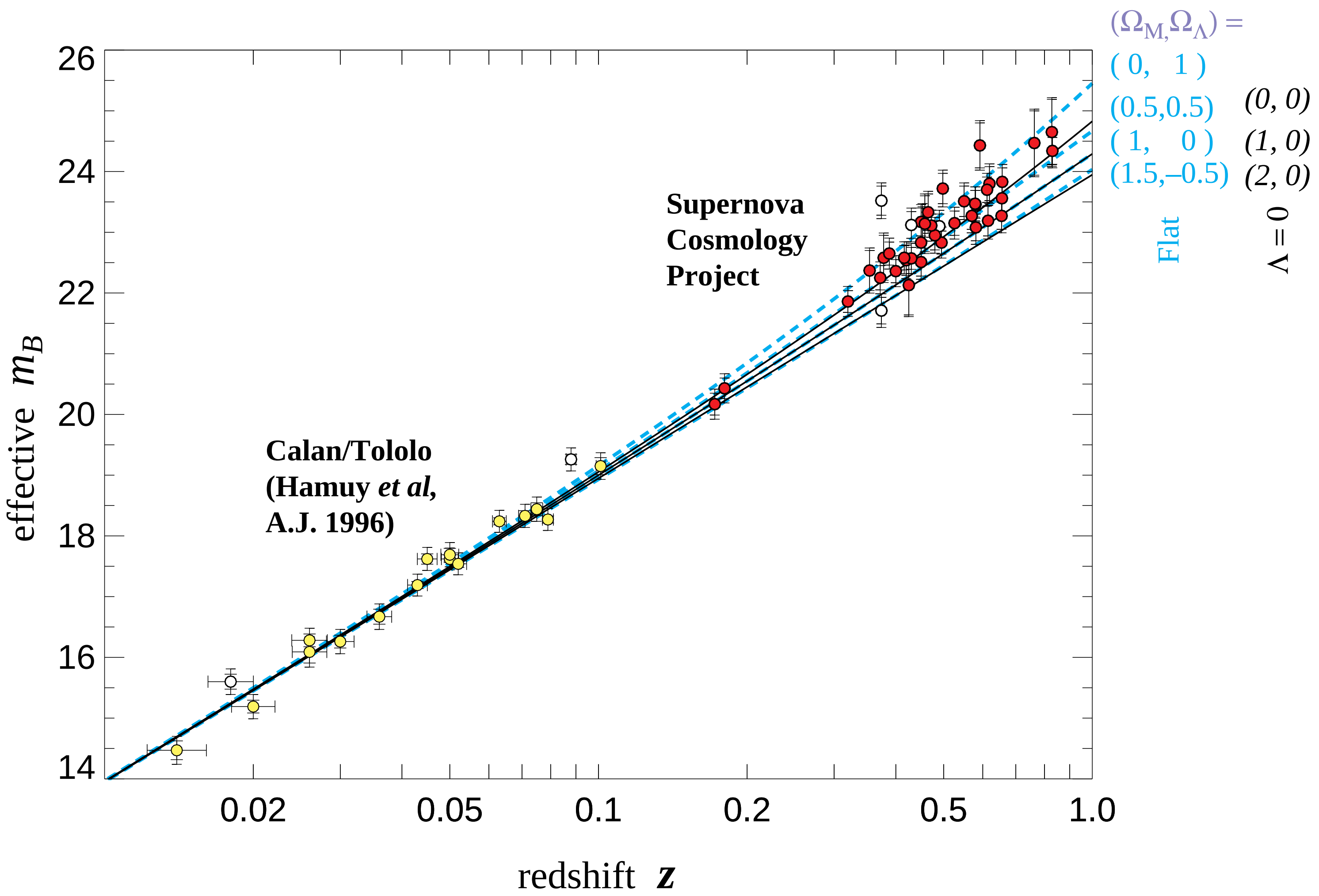
Properties of the Dark Energy

Positive energy density, Negative pressure

Fluid having negative pressure ⇒ Cosmic acceleration

Observations of type IA Supernovae

$$(\Omega_i \equiv \frac{\rho_i}{\rho_c},\ \rho_c \equiv \frac{3H_0^2}{8\pi G})$$



(S. Perlmutter *et. al.*, Astrophys. J. 517 (1999) 565)

Important Parameter

Equation of state parameter :

$$w = \frac{p_{\rm DE}}{\rho_{\rm DE}} = \begin{cases} -0.97^{+0.07}_{-0.09} & \text{(Flat, WMAP-3+SNLS)} \\ -1.06^{+0.13}_{-0.08} & \text{(WMAP-3+LSS \& SN)} \end{cases}$$

$$\rho \propto a^{-3(w+1)}$$

Cosmological Constant :

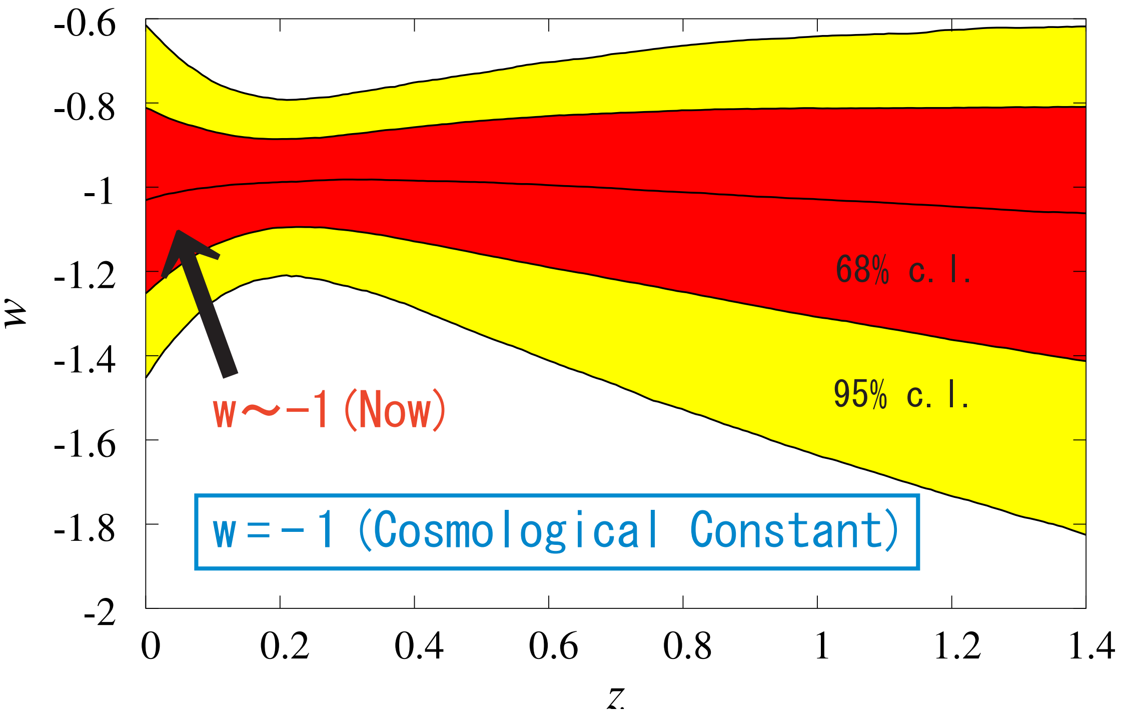
$$\rho_\Lambda : \text{Constant} \Rightarrow w = -1$$

Matter (e.g. Baryon, CDM) :

$$\rho_{\rm m} \propto a^{-3} \Rightarrow w = 0$$

Radiation (e.g. Photon) :

$$\rho_\gamma \propto a^{-4} \Rightarrow w = \frac{1}{3}$$



(U. Seljak, *et al.*, Phys. Rev. D71, 103515 (2005))

Candidates of the Dark Energy

$w > -1$

- Quintessence (scalar field : $m_\phi \sim 10^{-33}$eV)
- Mass Varying Neutrinos (MaVaNs)

⋮

$w < -1$

- Phantom Energy

⋮

Variable Neutrino Mass

- Neutrino dark matter

Kawasaki, Murayama, Yanagida (1992)

- Relation between the neutrino and the dark energy

Gu, Wang, Zhang (2003)

Fardon, Nelson, Weiner (2003)

Implication of MaVaNs

- Neutrino oscillation
- Solar neutrinos
- Baryogenesis
- Leptogenesis

⋮

2. Mass Varying Neutrinos Scenario

Gu, Wang, Zhang, PRD 68(2003)087301

Fardon, Nelson, Weiner, JCAP 10(2004)005

Why do we relate the dark energy with neutrinos?

- The energy density and masses of neutrinos are uncertain compared with other components of the Universe (e.g. baryons...).

- The mass scale of the neutrino is close to the dark energy scale.

Neutrino mass scale

$$\Delta m^2_{\rm sol} \sim 8.0 \times 10^{-5}\ {\rm eV}^2 \quad , \quad \Delta m^2_{\rm atm} \sim 2.5 \times 10^{-3}\ {\rm eV}^2$$

[KamLAND, SNO] [K2K, SK]

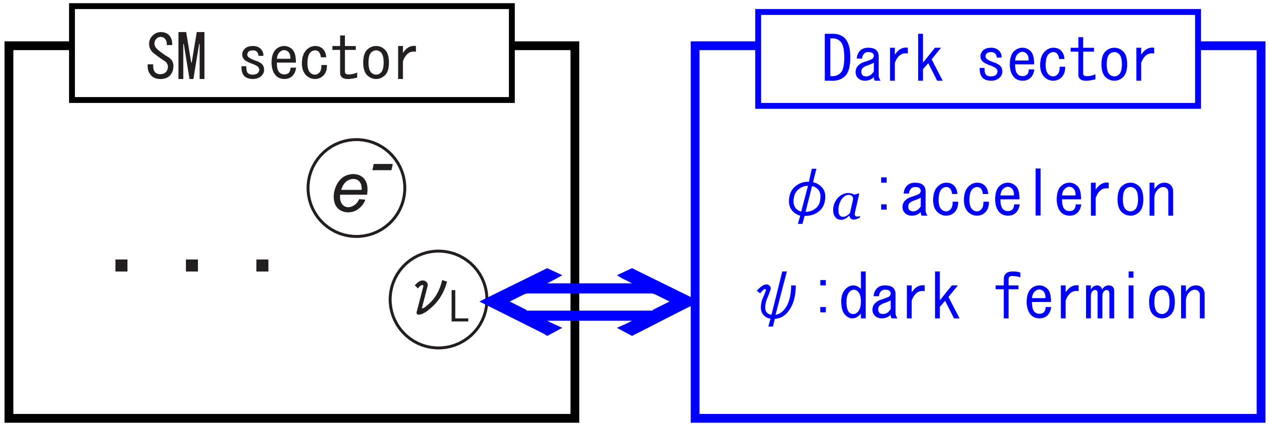
Dark energy scale

$$\Lambda_{\rm DE} \sim 2 \times 10^{-3}\ {\rm eV}$$

—Mass Varying Neutrinos Scenario—

Assumptions

(i) A dark energy sector interacts with the standard model sector only through neutrinos.



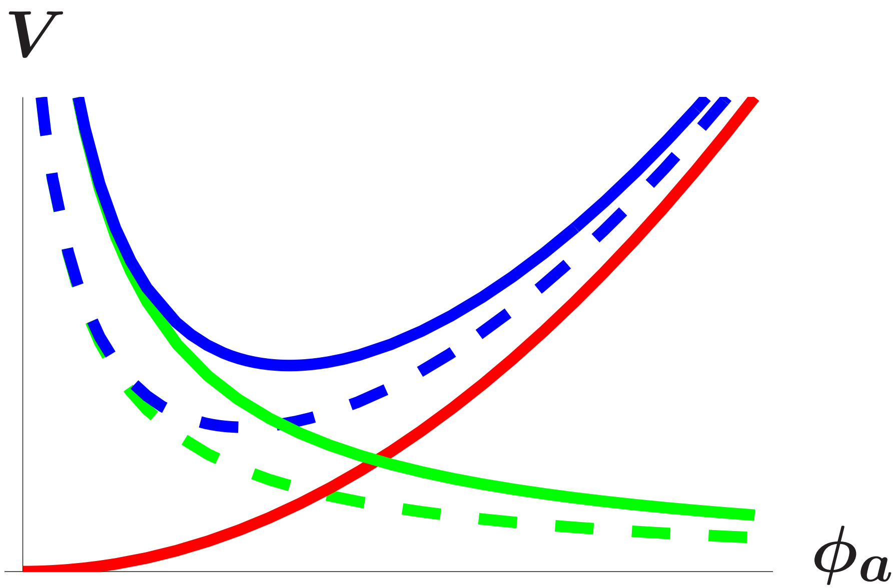
Example)

$$\mathcal{L}_{mass} = \lambda\phi_a\psi\psi + m_D\bar{\nu}_L\psi + h.c. \Rightarrow m_\nu = m_\nu(\phi_a) = \frac{m_D^2}{\lambda\phi_a}$$

(ii) The dark energy is the sum of the scalar potential and the energy density of neutrinos.

$$\rho_{\mathrm{DE}} = V(\phi_a) + \rho_\nu(m_\nu(\phi_a))$$

Consequences



For non-relativistic neutrinos

$$\begin{aligned}\rho_{\rm DE} &= V(\phi_a) + \rho_\nu(m_\nu(\phi_a)) \\ &= V(\phi_a) + m_\nu(\phi_a) n_\nu \\ &= V(\phi_a) + \frac{m_D^2}{\lambda \phi_a} n_\nu\end{aligned}$$

As the universe expands, n_ν decreases:

— $\rho_{\rm DE}$ $\Rightarrow$ - - $\rho'_{\rm DE}$

— ρ_ν - - ρ'_ν

—$V(\phi_a)$

$\Rightarrow$ $\rho_{\rm DE}$ & $< \phi_a >$ vary on cosmological time scale.

$\Rightarrow$ $m_\nu(\phi_a)$ also varies.

Statiaonary Condition

$$\frac{\partial \rho_{\rm DE}}{\partial \phi_a} = 0 \implies \frac{\partial \rho_{\rm DE}}{\partial m_\nu} = 0 \quad \left(\frac{\partial m_\nu}{\partial \phi_a} \neq 0\right)$$

1 generation model

R. D. Peccei, Phys. Rev. D71 (2005) 023527

Energy density of neutrinos

$$\rho_\nu = T^4 F(\xi) \qquad \text{(For non-relativistic neutrinos : } \rho_\nu = m_\nu n_\nu)$$

$$\xi = m_\nu / T$$

$$F(\xi) = \frac{1}{\pi^2}\int_0^\infty \frac{dy y^2 \sqrt{y^2+\xi^2}}{e^y+1}$$

Equation of state for the dark energy

- Energy conservation law : $\dot{\rho}_{\rm DE} = -3H(\rho_{\rm DE} + p_{\rm DE})$
- Stationary condition : $\frac{\partial \rho_{\rm DE}}{\partial m_\nu} = \frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial V(\phi_a(m_\nu))}{\partial m_\nu} = 0$

$$w + 1 = \frac{4 - h(\xi)}{3\left[1 + \frac{V(\phi_a(m_\nu))}{T^4 F(\xi)}\right]} \Longrightarrow \frac{m_\nu n_\nu}{\rho_{\rm DE}} \quad \text{(Non-rel. limit)}$$

$$h(\xi) \equiv \frac{\xi(\partial F(\xi)/\partial \xi)}{F(\xi)}$$

Stationary condition

$$\frac{\partial \rho_{\mathrm{DE}}}{\partial m_\nu} = \frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial V(\phi_a(m_\nu))}{\partial m_\nu} = 0$$
$$= T^3 \frac{\partial F}{\partial \xi} + \frac{\partial V(\phi_a(m_\nu))}{\partial m_\nu} = 0$$

$$\Downarrow$$

♠ **Once a scalar potential is given, one can find the temperature (time) dependence of m_ν & w.**

However...
there are two severe constraints on the scalar potential.

[Smallness] & [Flatness]

Constraints on a MaVaNs model (Scalar potential)

(i) Observations : $\Omega^0_{\rm DE} = \rho^0_{\rm DE}/\rho_c \simeq 0.74$

$$\Rightarrow V(\phi^0_a(m^0_\nu)) = 0.74\rho_c - \rho^0_\nu \simeq 2.96 \times 10^{-11}\,\mathrm{eV}^4$$

[Smallness]

(ii) Stationary condition (@ the present) :

$$\Rightarrow \left.\frac{\partial V(\phi_a(m_\nu))}{\partial m_\nu}\right|_{m_\nu = m^0_\nu} = -T^3 \left.\frac{\partial F}{\partial \xi}\right|_{m_\nu = m^0_\nu,\ T = T_0}$$

$$\simeq -n^0_\nu$$

$$\simeq -8.82 \times 10^{-13}\,\mathrm{eV}^3$$

[Flatness]

♠ The present value of a scalar potential must be small, and its gradient must be flat.

Constraints on a MaVaNs model ($m^0_{\phi_a}$ & m^0_ν)

♠ In order that the acceleron does not vary significantly on distance of inter-neutrino spacing, the present acceleron mass must be less than $\mathcal{O}(10^{-4}\text{eV})$ ($\sim (n^0_\nu)^{1/3}$).

In this talk...

- Present Acceleron mass :

$$m^0_{\phi_a} = 10^{-4}\,\text{eV}$$

- Present neutrino mass :

$$m^0_\nu \sim \mathcal{O}(10^{-2}\,\text{eV})$$

Constraints on a MaVaNs model (Speed of sound in $\nu - \phi_a$ fluid)

♠ MaVaNs contains a catastrophic instability ($c_s^2 < 0$) which occurs when ν become non-relativistic ($\rho_\nu = m_\nu n_\nu$).

Afshordi, Zaldarriage, Kohri, PRD 72(2005)065024

$$c_s^2 = \frac{\dot{p}_{\rm DE}}{\dot{\rho}_{\rm DE}} = \frac{\dot{w}\rho_{\rm DE} + w\dot{\rho}_{\rm DE}}{\dot{\rho}_{\rm DE}} = \frac{\dot{m}_\nu n_\nu}{m_\nu \dot{n}_\nu} < 0 \quad \text{(at the non-rel. limit)}$$

♠ Since a fluid having $c_s^2 < 0$ is gravitationally unstable, it condensates into clusters and thus cannot act as the dark energy.

♠ There are some models which avoid this instability ($c_s^2 < 0$), when $m_\nu^0 \sim \mathcal{O}(10^{-2}\text{eV})$, $T_0 \simeq 1.69 \times 10^{-4}\text{eV}$ ($\Rightarrow \xi \sim \mathcal{O}(10^2)$).

R.T., Tanimoto, JHEP 0605 (2006) 021

$$\rho_\nu = \frac{T^4}{\pi^2}\int_0^\infty \frac{dy y^2\sqrt{y^2+\xi^2}}{e^y+1} \simeq m_\nu n_\nu + a\frac{n_\nu T}{\xi} + \mathcal{O}\left(\frac{1}{\xi^2}\right) + \cdots$$

$$\Downarrow$$

$$\Downarrow \xi \equiv m_\nu/T, \quad a \simeq 6.47$$

$$\Downarrow$$

$$c_s^2 \simeq \frac{\frac{\partial m_\nu}{\partial z} n_\nu}{m_\nu \frac{\partial n_\nu}{\partial z}} + \frac{\frac{5}{3} a n_\nu \left(\frac{5T_0}{\xi} - \frac{1}{\xi^2}\frac{\partial m_\nu}{\partial z}\right)}{m_\nu \frac{\partial n_\nu}{\partial z}}$$

$\Uparrow$ Negative $\qquad\qquad$ $\Uparrow$ Positive

Positive c_s^2 condition $[\mathcal{O}(1/\xi)]$

$$\underbrace{\frac{\partial m_\nu}{\partial z}\left(1-\frac{5aT^2}{3m_\nu^2}\right)}_{\Uparrow\ \text{Negative}}+\underbrace{\frac{25aT_0^2(z+1)}{3m_\nu}}_{\Uparrow\ \text{Positive}}>0$$

♠ A model which leads to small time variation of the neutrino mass $(\partial m_\nu/\partial z)$ is favored.

Realization of small $\partial m_\nu/\partial z$

- Small power-law potential

$$V(\phi_a) = A\left(\frac{\phi_a}{\phi_a^0}\right)^{k}$$

$$\frac{\partial m_\nu}{\partial z} \sim -ck\ , \qquad c \sim \mathcal{O}(1),\ k \ll 1$$

- Constant dominant $m_\nu(\phi_a)$

$$m_\nu(\phi_a) = c' + f(\phi_a), \qquad f(\phi_a) \ll c' : \text{Constant}$$

$$\frac{\partial m_\nu}{\partial z} \sim 0$$

Another way to avoid the instability

- The lightest (relativistic) ν_i are the source of the dark energy.

Fardon, Nelson, Weiner, JCAP 10(2004) 005

Summary of MaVaNs scenario

Constraints on a MaVaNs model

- $V(\phi_a^0) \sim \mathcal{O}(10^{-11})$ eV^4 & $\partial V / \partial m_\nu|_{m_\nu = m_\nu^0} \sim -\mathcal{O}(10^{-13})$ eV^3
- Present acceleron mass : $m^0_{\phi_a} \leq 10^{-4}$ eV
- Present neutrino mass : $m^0_\nu \sim \mathcal{O}(10^{-2})$ eV
- Stability of $\nu - \phi_a$ fluid $(c_s^2 > 0)$

$\Rightarrow$ Realization of small $\partial m_\nu / \partial z$

- Small power-law scalar potential ($V(\phi_a) \propto \phi_a^k$, $k \ll 1$)
- Constant dominant m_ν ($m_\nu = c' + f(\phi_a)$, $f(\phi_a) \ll c'$)

$\Rightarrow$ The lightest (relativistic) ν_i are the source of the DE

$\Downarrow$

- Stationary condition
- Equation of state

$\Downarrow$

$m_\nu(z)$ & $w(z)$

3. Supersymmetric Mass Varying Neutrinos Model

♠A chiral superfield is assumed to be in a dark sector.

One Superfield Model R.T., M. Tanimoto, (2006)

Superpotential

$$W = \frac{\lambda}{3}A^3 + m_D L A$$

A; Chiral superfield, Gauge singlet
Scalar component : ϕ_a ("Acceleron")
Spinor component : ψ_a (Sterile neutrino)
L; Left-handed lepton doublet

Scalar potential

$$V(\phi_a) = \lambda^2 |\phi_a|^4 + m_D^2 |\phi_a|^2$$

Lagrangian

$$\mathcal{L} = 2\lambda \phi_a \psi_a \psi_a + m_D \bar{\nu}_L \psi_a + h.c.$$

Neutrino mass matrix

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D & 2\lambda\phi_a \end{pmatrix} \begin{matrix} \nu_L \\ \psi_a \end{matrix}$$

$$\quad \nu_L \quad \psi_a$$

♠ **From two constraints on the scalar potential,**

$$\Rightarrow \left. \frac{V(\phi_a(m_\nu))}{\frac{\partial V(\phi_a(m_\nu))}{\partial m_\nu}} \right|_{m_\nu = m_\nu^0} \simeq \frac{2.96 \times 10^{-11}}{-8.82 \times 10^{-13}} = -33.6 \text{ eV}$$

$\Downarrow$ Putting $m_\nu^0 = 10^{-2}\,\text{eV}$

$$\frac{m_\nu^0}{4} \cdot \frac{1 - \frac{m_D^4}{(m_\nu^0)^4}}{1 + \frac{m_D^4}{(m_\nu^0)^4}} \neq -33.6\,\text{eV} \quad \left(\because \text{ LHS} > -\frac{m_\nu^0}{4} \right)$$

$\Downarrow$

× **This model cannot simultaneously lead to the dark energy and the neutrino mass scale.**

One Superfield + Right-handed Neutrino

Superpotential

$$W = \frac{\lambda}{6}A^3 + m_D LA + M_D LR + \frac{M_A}{2}AA + \frac{M_R}{2}RR$$

R; Right-handed neutrino superfield

Scalar potential

$$V(\phi_a) = \frac{\lambda^2}{4}|\phi_a|^4 + m_D^2|\phi_a|^2 + M_A^2|\phi_a|^2$$

Lagrangian

$$\mathcal{L} = \lambda\phi_a\psi_a\psi_a + m_D\bar{\nu}_L\psi_a + M_D\bar{\nu}_L\nu_R + M_A\psi_a\psi_a + M_R\nu_R\nu_R + h.c.$$

Neutrino mass matrix

$$\mathcal{M}_\nu \simeq \begin{pmatrix} c & m_D \\ m_D & \lambda\phi_a + M_A \end{pmatrix}, \quad c \equiv -\frac{M_D^2}{M_R}, \quad M_D \ll M_R$$

Case of vanishing mixing between ν_L & ψ_a ($m_D = 0$)

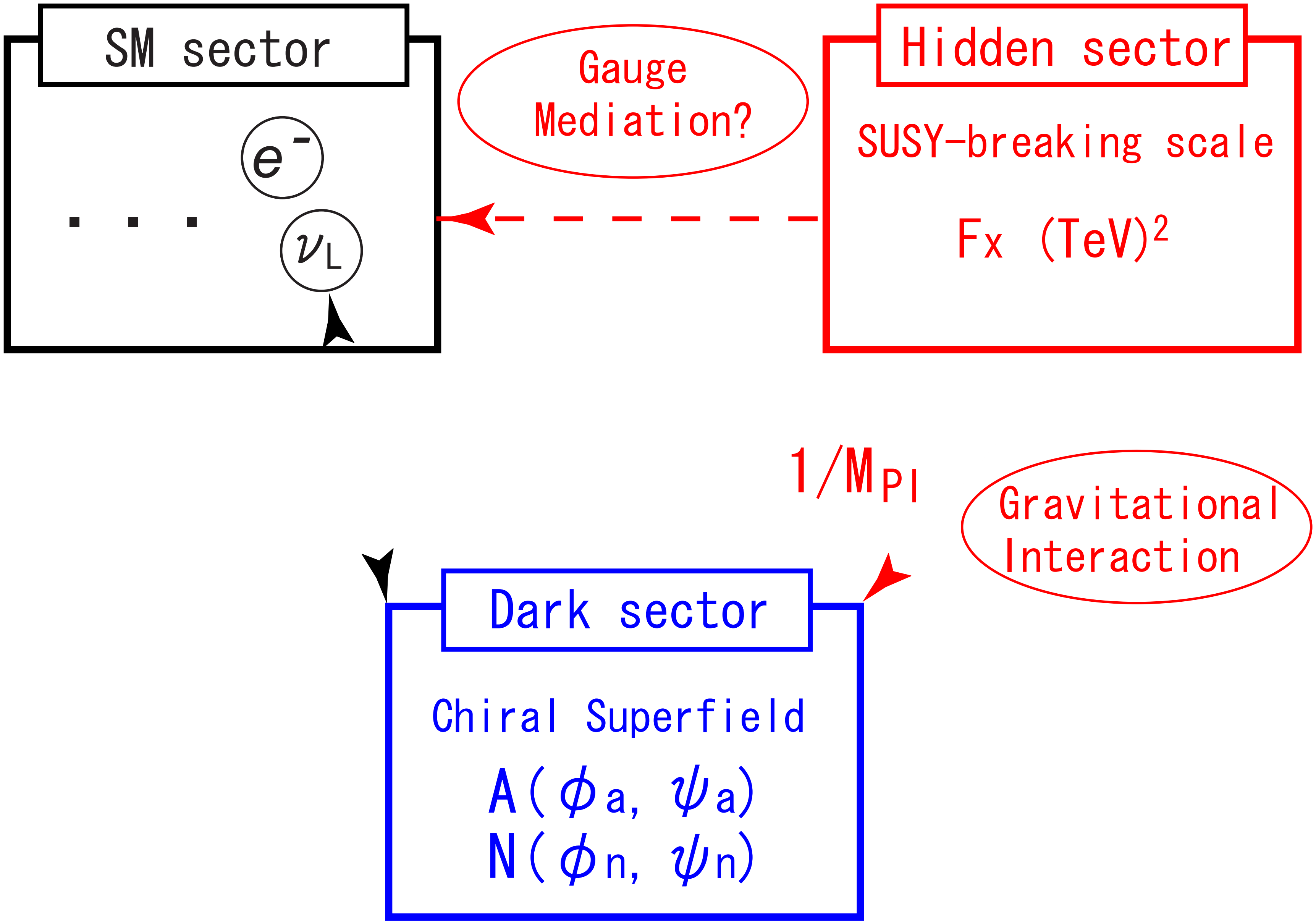
$$\begin{cases} m_{\nu_L} = c & : \text{Constant} \\ m_{\psi_a} = M_A + \lambda\phi_a & : \text{Variable} \end{cases}$$

Case of finite mixing between ν_L & ψ_a ($m_D \neq 0$)

$$\begin{cases} m_{\nu_L} = \frac{c+M_A}{2} + \frac{\lambda\phi_a}{2} + f(\phi_a, m_D) & : \text{Variable} \\ m_{\psi_a} = \frac{c+M_A}{2} + \frac{\lambda\phi_a}{2} - f(\phi_a, m_D) & : \text{Variable} \end{cases}$$

4. Effects of the SUSY-breaking

Our Scheme



Operators

$$\int d^4\theta \frac{X^\dagger X}{M_{\rm Pl}^2} A^\dagger A, \quad \int d^4\theta \frac{X^\dagger + X}{M_{\rm Pl}} A^\dagger A$$

Scale of soft terms

$$\frac{F_X}{M_{\rm Pl}} \sim \frac{({\rm TeV})^2}{10^{18}{\rm GeV}} \sim \mathcal{O}(10^{-2}\text{-}10^{-3})\,({\rm eV})$$

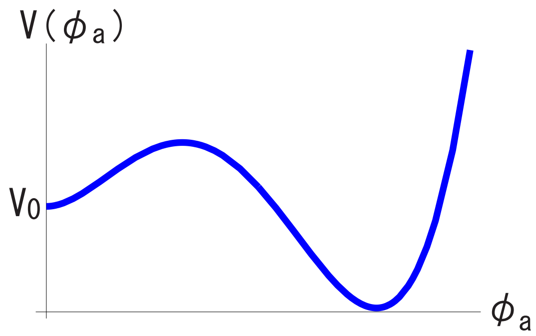
(Chacko, Hall, Nomura, JCAP 0410, 011 (2004))

Scalar potential

$$V(\phi_a) = \frac{\lambda^2}{4}|\phi_a|^4 - \frac{\kappa}{3}(\phi_a^3 + h.c.) + m_D^2|\phi_a|^2 + M_A^2|\phi_a|^2 - m^2|\phi_a|^2 + V_0$$

κ , $m \sim \mathcal{O}(10^{-2}\text{-}10^{-3})\,({\rm eV})$

$V_0 : V(\phi_a) = 0$ @ True vacuum



Case of vanishing mixing between ν_L & ψ_a ($m_D = 0$)

$$\mathcal{M}_\nu \simeq \begin{pmatrix} c & 0 \\ 0 & \lambda\phi_a + M_A \end{pmatrix}$$

$$\begin{cases} m_{\nu_L} = c & : \text{Constant} \\ m_{\psi_a} = M_A + \lambda\phi_a & : \text{Variable} \end{cases}$$

Prameters

$$\begin{cases} \lambda = 1 \\ m^0_{\nu_L} = c = 2 \times 10^{-2}\,\text{eV} \\ m^0_{\psi_a} = 10^{-2}\,\text{eV} \\ m_D = 0 \end{cases}$$

$\phi^0_a \simeq -1.31 \times 10^{-5}\,\text{eV}$

$M_A \simeq 10^{-2}\,\text{eV}$

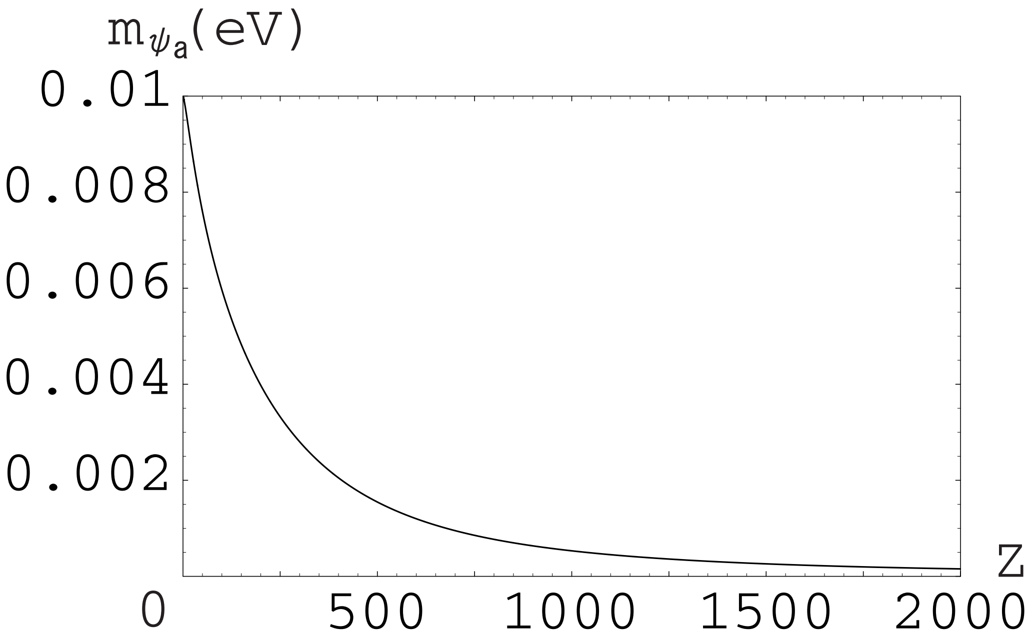
$m \simeq 10^{-2}\,\text{eV}$

$\kappa \simeq 4.34 \times 10^{-3}\,\text{eV}$

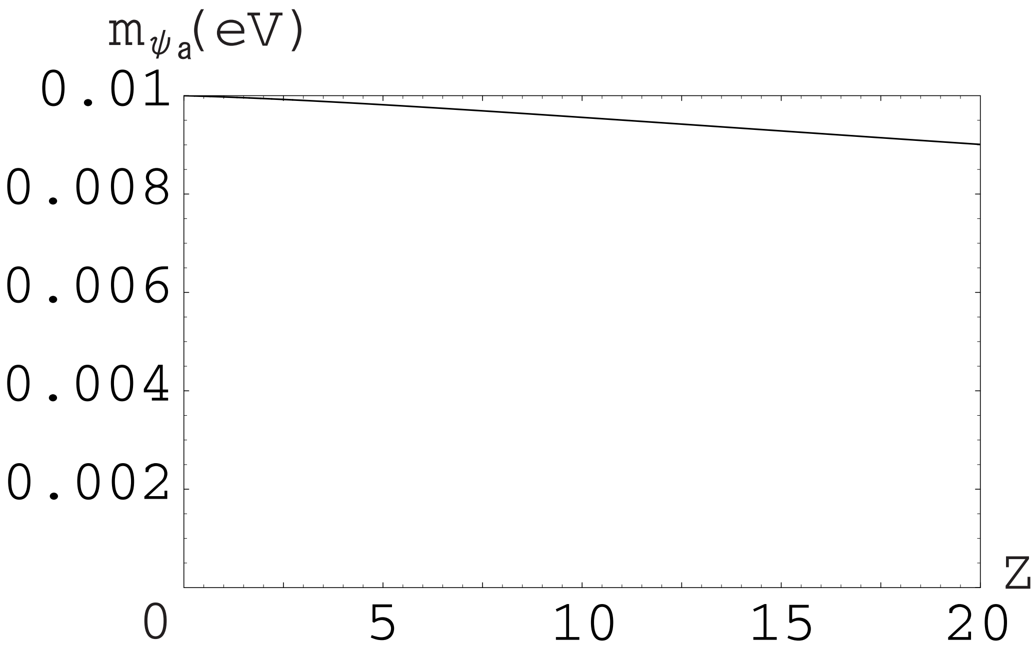
♠ We need fine-tuning between M_A and m to find $m^0_{\phi_a} = 10^{-4}$ eV.

Time evolution of m_{ψ_a}

$0 \le z \le 2000$



$0 \le z \le 20$



♠ m_{ψ_a} **has varied slowly.** $\Rightarrow$ $c_s^2 > 0$

Case of finite mixing between ν_L & ψ_a ($m_D \neq 0$)

$$\mathcal{M}_\nu \simeq \begin{pmatrix} c & m_D \\ m_D & \lambda\phi_a + M_A \end{pmatrix}$$

$$\begin{cases} m_{\nu_L} = \frac{c+M_A}{2} + \frac{\lambda\phi_a}{2} + f(\phi_a, m_D) & : \text{Variable} \\ m_{\psi_a} = \frac{c+M_A}{2} + \frac{\lambda\phi_a}{2} - f(\phi_a, m_D) & : \text{Variable} \end{cases}$$

Parameters

$$\begin{cases} \lambda = 1 \\ m^0_{\nu_L} = 2 \times 10^{-2}\,\text{eV} \\ m^0_{\psi_a} = 10^{-2}\,\text{eV} \\ m_D = 10^{-3}\,\text{eV} \end{cases}$$

$c \simeq 1.99 \times 10^{-2}\,\text{eV}$

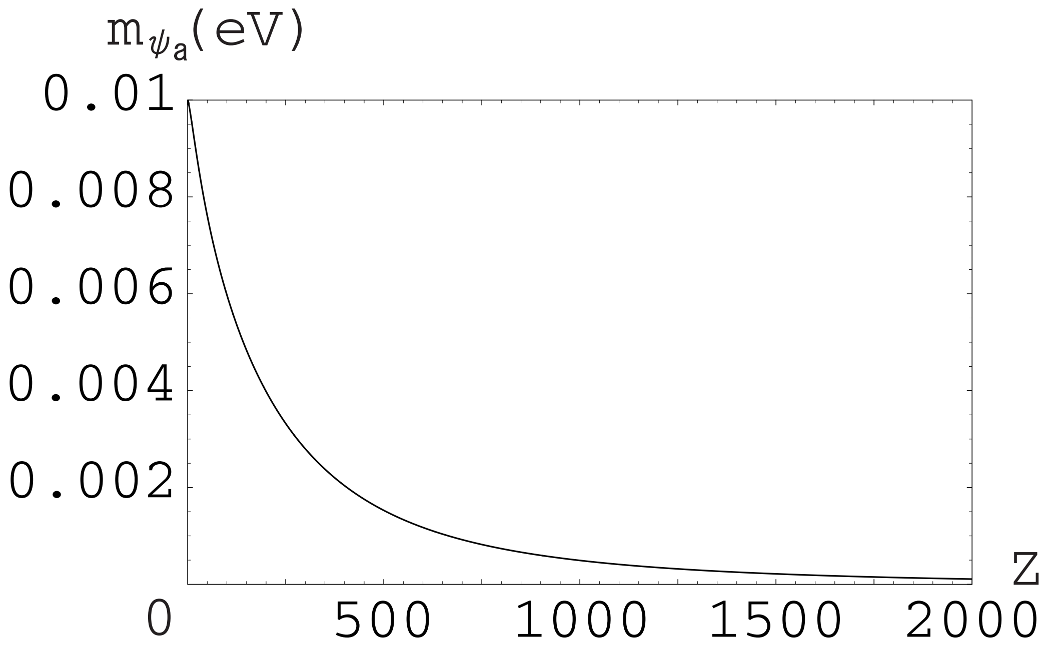
$\phi^0_a \simeq -1.31 \times 10^{-5}\,\text{eV}$

$M_A \simeq 1.01 \times 10^{-2}\,\text{eV}$

$m \simeq 1.02 \times 10^{-2}\,\text{eV}$

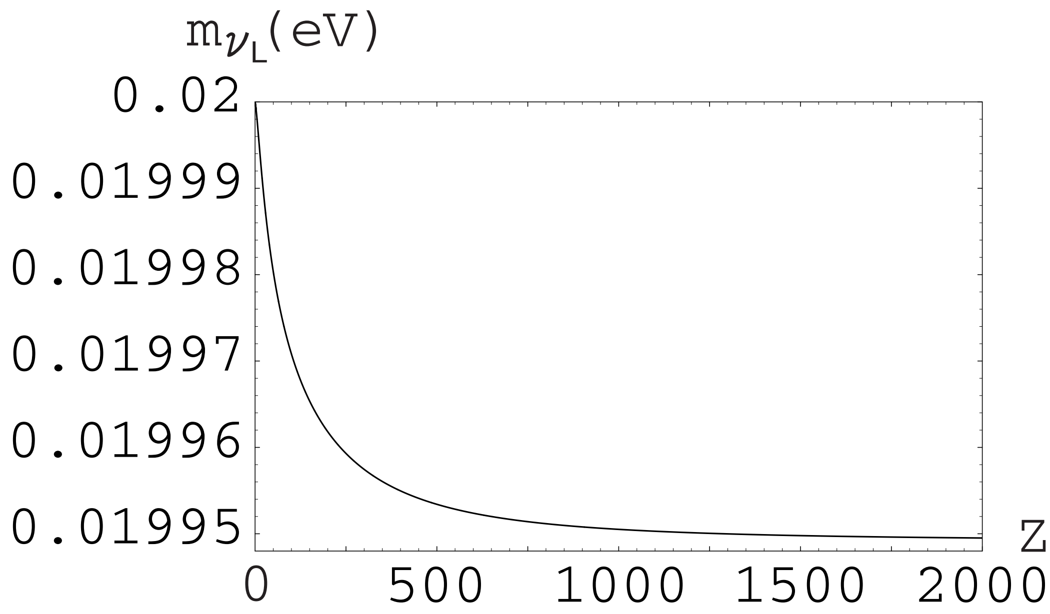
$\kappa \simeq 4.34 \times 10^{-3}\,\text{eV}$

Time evolution of m_{ψ_a}



- The mixing ($m_D \neq 0$) does not affect the evolution of m_{ψ_a}.

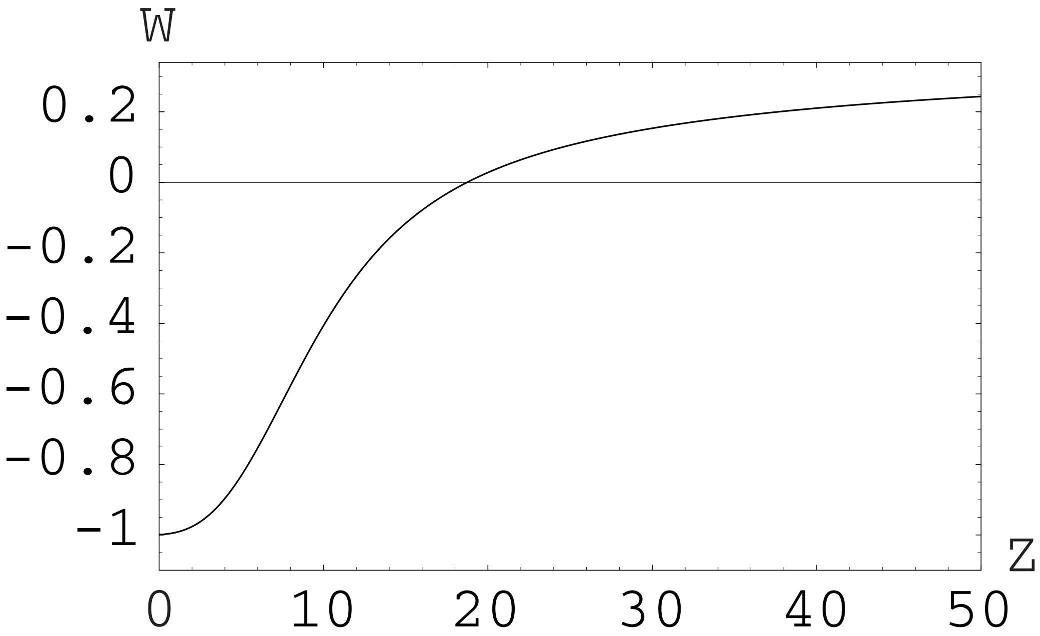
Time evolution of m_{ν_L}



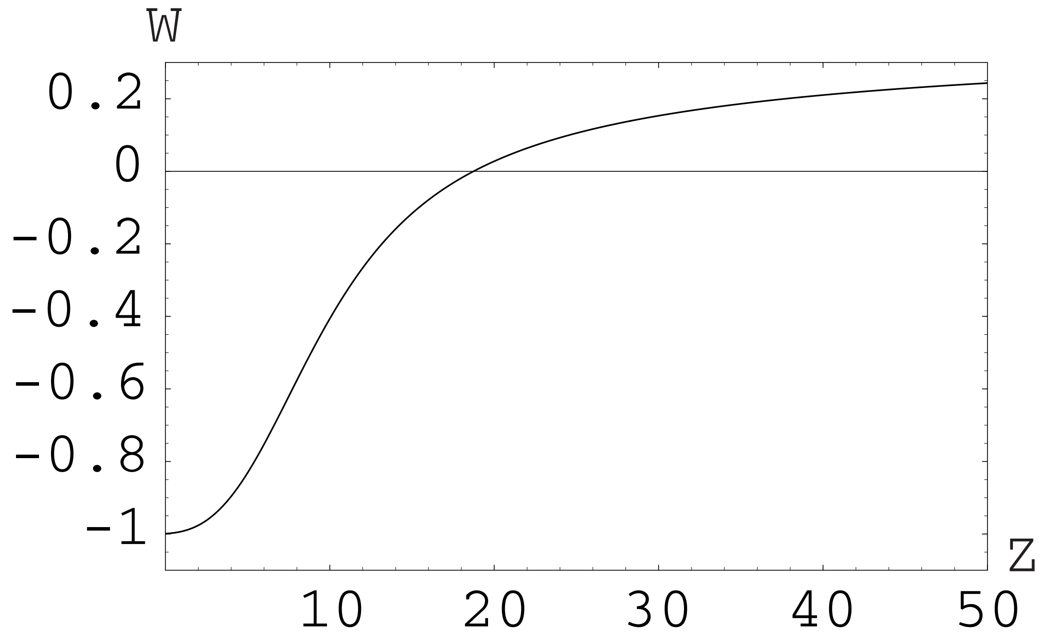
- m_{ν_L} has vaied slightly.

Time evolution of w

Case of vanishing mixing



Case of finite mixing



$\Downarrow$

♠ **The mixing ($m_D \neq 0$) does not affect the evolution of w.**

5. Summary

- Cosmological observations of type Ia supernovae suggest **accelerating expansion of the universe**.
- The most common explanation for the question of the origin of such an acceleration is to assume the existence of unknown **"Dark Energy"** component.

Candidates of Dark Energy

$w = -1$: Cosmological Constant (Vacuum Energy)

$w > -1$: Quintessence, Mass Varying Neutrinos...

$w < -1$: Phantom Energy...

$$\text{Mass Varying Neutrinos} : \Lambda_{\mathrm{DE}}(2 \times 10^{-3}\mathrm{eV}) \sim m_\nu$$

Constraints on a MaVaNs model

- $V(\phi_a^0) \sim \mathcal{O}(10^{-11})$ eV^4 & $\partial V/\partial m_\nu|_{m_\nu = m_\nu^0} \sim -\mathcal{O}(10^{-13})$ eV^3
- Present acceleron mass : $m_{\phi_a}^0 \leq 10^{-4}$ eV
- Present neutrino mass : $m_\nu^0 \sim \mathcal{O}(10^{-2})$ eV
- Stability of $\nu - \phi_a$ fluid $(c_s^2 > 0)$

  $\Rightarrow$ Realization of small $\partial m_\nu/\partial z$

  – Small power-law scalar potential ($V(\phi_a) \propto \phi_a^k$, $k \ll 1$)

  – Constant dominant m_ν ($m_\nu = c' + f(\phi_a)$, $f(\phi_a) \ll c'$)

  $\Rightarrow$ The lightest (relativistic) ν_i are the source of the DE

Supersymmetric MaVaNs model

♠ Acceleron mass : $m^0_{\phi_a} = 10^{-4}$ eV

$\Rightarrow (m^0_{\phi_a})^2 = 2(m_D^2 + M_A^2 - m^2)$ **Fine-tuning**

♠ Stability of $\nu - \phi_a$ fluid $(c_s^2 > 0)$

$\Rightarrow$ Constant dominant m_{ν_i}

$$m_D = 0 : \begin{cases} m_{\nu_L} = c & : \text{Constant} \\ m_{\psi_a} = M_A + \lambda\phi_a & : \text{Variable} \end{cases}$$

$$m_D \neq 0 : \begin{cases} m_{\nu_L} = \frac{c+M_A}{2} + \frac{\lambda\phi_a}{2} + f(\phi_a, m_D) & : \text{Variable} \\ m_{\psi_a} = \frac{c+M_A}{2} + \frac{\lambda\phi_a}{2} - f(\phi_a, m_D) & : \text{Variable} \end{cases}$$

♠ Effects of SUSY-breaking

$\Rightarrow$ Gravity mediation (Scale of soft terms $= \mathcal{O}(10^{-2}\text{-}10^{-3}\text{eV})$)