Tree FCNC and non-Unitarity of Mixing Matrix

2006-12-17

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- Exclusive $B \to M\nu\bar{\nu}$ and Leptophobic Z'
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Vector-like Iso-singlet Down-quark (VdQ)

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2006-12-17

III Vector-like-(isosinglet)-Downquark (VdQ)
[and 4-th generation SM (4SM)]

- low E limit of E_6 GUT

** models of L.E.D.

→ (KK) tower of vector-like fermions
for each of SM chiral quarks & leptons.

⇒ presence of tree-level FCNC
(NO tree-level FCNC within SM due to GIM)

For (SM) + n VdQ

$M_u = (3\times 3)$; $M_d = (3+n)\times(3+n)$

$M_u^{diag} = M_u$; $M_d^{diag} = U_{dL}^{\dagger} M_d U_{dR}$ ($U_{d,LR} = (3+n)\times(3+n)$

$V_{CKM} = (U_{dL})_{(3\times(3+n))}$ for C.C

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for N.C.

At low energies (i.e. only w/ 3-generations)

$$V_{CKM}^{+}\, Diag(1,1,1,0,0\cdots)\, V_{CKM} \neq I_{(3+n)\times(3+n)}$$

N.C. in terms of flavor eigenstates:

$$\mathcal{L}_Z = \frac{g}{2\cos\theta_W}\left[\bar{u}_{Li}\gamma^\mu u_{Li} - \bar{d}_{L\alpha} U_{\alpha\beta}\gamma^\mu d_{L\beta} - 2\sin^2\theta_W J^\mu_{em}\right] Z_\mu$$

$$\mathcal{L}_H = \frac{g}{2M_W}\left[\bar{u}_{Li}\, m_i^u\, u_{Li} - \bar{d}_{L\alpha} U_{\alpha\beta}\, m_\beta^d\, d_{L\beta}\right],$$

where $U_{\alpha\beta} = \sum_{i=u,c,t} V_{i\alpha}^{*} V_{i\beta} \neq \delta_{\alpha\beta}$

(effectively non-unitary)

$\Rightarrow$ tree-level FCNC.

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For (SM) + one isosinglet down quark b' :

$$V_{VdQ} \Rightarrow \begin{cases} 6 \text{ angles } \theta_{ij} & (1 \leq i < j \leq 4) \\ 3 \text{ phase } e^{i\delta_x} & (x \in \{ub, ub', cb'\}) \end{cases}$$

$$V_{VdQ} = K \cdot V_{4GSM}$$ ← just V_{4GSM} with 4-th row removed.

$$K = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$V_{4GSM} \equiv R_{34}(\theta_{34}) \cdot \Phi(0, -\delta_{cb'}, 0, 0) \cdot R_{24}(\theta_{24}) \cdot \Phi(-\delta_{ub'}, \delta_{cb'}, 0, 0) \cdot R_{14}(\theta_{14}) \cdot \Phi(\delta_{ub'}, 0, 0, 0) \cdot R_{23}(\theta_{23}) \cdot \Phi(-\delta_{ub}, 0, 0, 0) \cdot R_{13}(\theta_{13}) \cdot \Phi(0, 0, \delta_{ub}, 0) \cdot R_{12}(\theta_{12})$$

Wolfenstein-type expansion:

$$V_{us} = (\sin\theta_c) = \lambda, \quad V_{cb} = A\lambda^2, \quad V_{ub} = A\lambda^3(\rho + i\eta) = AC\lambda^3 e^{-i\delta_{ub}}$$

$$\left[V_{ub'} = p\lambda^3 e^{-i\delta_{ub'}}, \quad V_{cb'} = q\lambda^2 e^{-i\delta_{cb'}}, \quad V_{tb'} = r\lambda \right]$$

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PDG $\Rightarrow 0.216 < \lambda < 0.223, \quad 0.76 < A < 0.90 \quad 0.23 < C < 0.59$

(2002 – PDG)

1st 2-row unitarity $\Rightarrow \quad |V_{ub'}| < 0.094, \quad |V_{cb'}| < 0.147$

$\rightarrow \quad p < 9.0 \quad q < 3.05$ (90% CL)

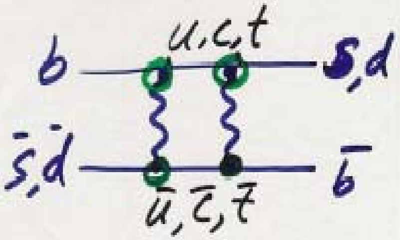
$Z \rightarrow b\bar{b} \Rightarrow (V^{+}V)_{bb} = 0.996 \pm 0.005 \rightarrow |V_{tb'}| < 0.11$ (90% CL)

$\rightarrow \quad r < 0.5$

Mixing Phases

$(B_d - \bar{B}_d) \sim \sin 2\beta$

$(B_s - \bar{B}_s) \sim \sin 2\chi$



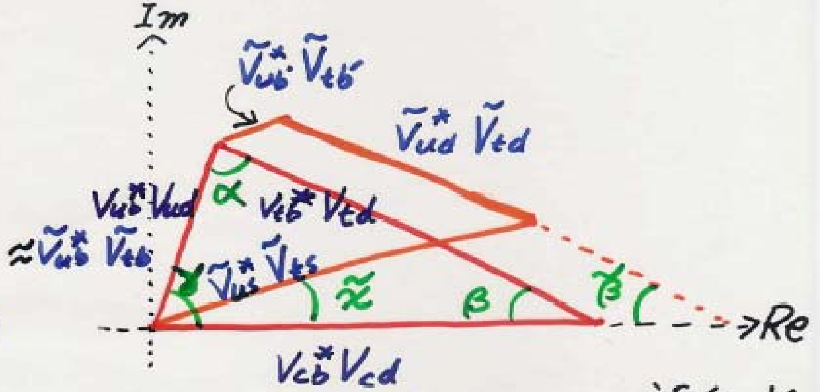
$$\beta \equiv Arg\left(-\frac{V_{cb}^{*} V_{cd}}{V_{tb}^{*} V_{td}}\right) \rightarrow (20 \sim 30)^{\circ} \quad \Rightarrow \beta_{vdQ} \equiv \tilde{\beta}\ ?$$

$$\chi \equiv Arg\left(-\frac{V_{cb}^{*} V_{cd}}{V_{tb}^{*} V_{ts}}\right) \rightarrow \mathcal{O}(\lambda^2) \simeq 0^{\circ} \text{ (SM)} \Rightarrow \chi_{vdQ} \equiv \tilde{\chi}\ ?$$

∘ Unitary relations

(SM) $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$

(VdQ) $\tilde V_{ud}^* \tilde V_{td} + \tilde V_{us}^* \tilde V_{ts} + \tilde V_{ub}^* \tilde V_{tb} + \tilde V_{ub'}^* \tilde V_{tb'} = 0$

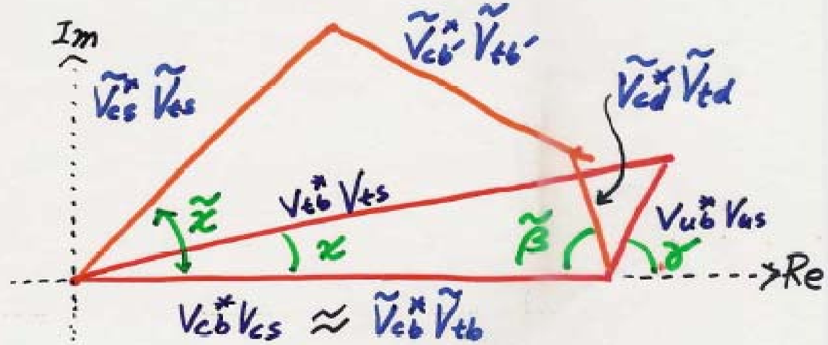


$$\Delta\beta \equiv \tilde\beta - \beta = Arg\left(\frac{\tilde V_{ud}^* \tilde V_{td}}{V_{ts}^* V_{td}}\right) = Arg\left(\frac{\tilde V_{td}}{V_{td}}\right) \approx Arg\left[1 + \frac{r\lambda}{A}\left(\frac{q e^{i\delta_{cb'}} - p e^{i\delta_{ub'}}}{1 - c e^{i\delta_{ub}}}\right)\right],$$

$$\lesssim \lambda \approx 0.04$$

(SM) $V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} = 0$

(VdQ) $\tilde V_{cd}^* \tilde V_{td} + \tilde V_{cs}^* \tilde V_{ts} + \tilde V_{cb}^* \tilde V_{tb} + \tilde V_{cb'}^* \tilde V_{tb'} = 0$



χ (also called as $\delta\phi$, ϕ_s, $2\delta\gamma$, β_s within SM)

$$\chi \equiv Arg\left(-\frac{V_{cb}^* V_{cs}}{V_{tb}^* V_{ts}}\right) \approx \mathcal{O}(\lambda^2)$$

$$\sin\chi \approx \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{\sin\beta \sin(\gamma+\chi)}{\sin(\beta+\gamma)}\left[1 + \mathcal{O}(\lambda^4)\right]$$

$$\Delta\chi \equiv \tilde\chi - \chi = Arg\left(\frac{\tilde V_{cs}^* \tilde V_{ts}}{V_{tb}^* V_{ts}}\right) = Arg\left(\frac{\tilde V_{ts}}{V_{ts}}\right) \approx Arg\left(1 + \frac{q r\lambda}{A} e^{i\delta_{cb'}}\right) \sim \frac{q r \lambda}{A}$$

$$\lesssim 0.4$$

With new physics, like VdQ, we can easily fit the data :

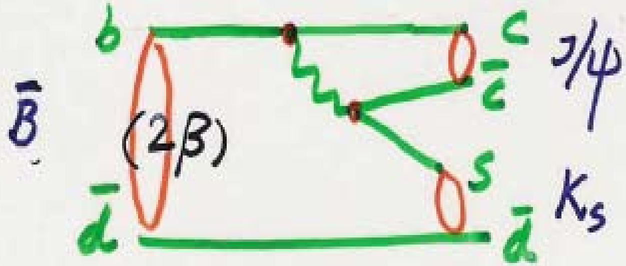
∘ Recent Belle & BaBar [2002] :

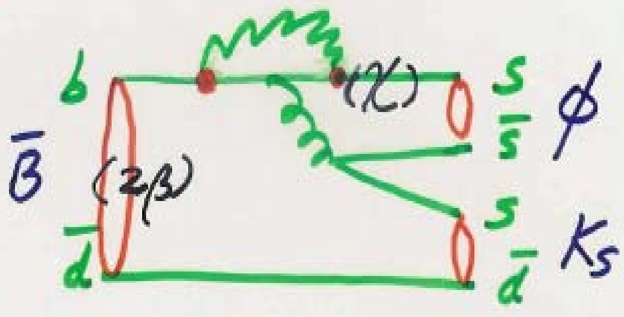
$$\sin(2\beta_{\psi K}) = \left[\begin{array}{ll} \sin(2\beta) & (SM) \\ \sin(2\tilde{\beta}) & (VdQ) \end{array}\right] = 0.734 \pm 0.054$$

$\longrightarrow \tilde{\beta} = \beta(SM) \sim 23^\circ$

$$\sin(2\beta_{\phi K}) = \left\{\begin{array}{ll} \sin(2\beta + 2\chi) \simeq \sin(2\beta) + O(\lambda^2) & (SM) \\ \sin(2\tilde{\beta} + 2\tilde{\chi}) & (VdQ) \end{array}\right] = -0.39 \pm 0.41$$

$\longrightarrow \tilde{\chi} = -0.6 \pm 0.4 \quad (-34^\circ \pm 23^\circ)$





Exclusive $B \to M\nu\bar{\nu}$ Decays and Leptophobic Z' Model

C. S. Kim (w/ J.H. Jeon, J. Lee, C. Yu)

PLB636(2006)270
hep-ph/0602156

2006-12-17

First observation of Direct CPV in B decays

$B^0 \to K^+\pi^-$

BABAR *hep-ex/0408057, submitted to PRL*

$A_{CP} = -0.133 \pm 0.030 \pm 0.009$

4.2σ

Belle **Confirmation at ICHEP04**

Signal (274M $B\bar{B}$ pairs): 2140 ± 53

$A_{CP} = -0.101 \pm 0.025 \pm 0.005$

3.9σ

Average $A_{CP} = -0.114 \pm 0.020$

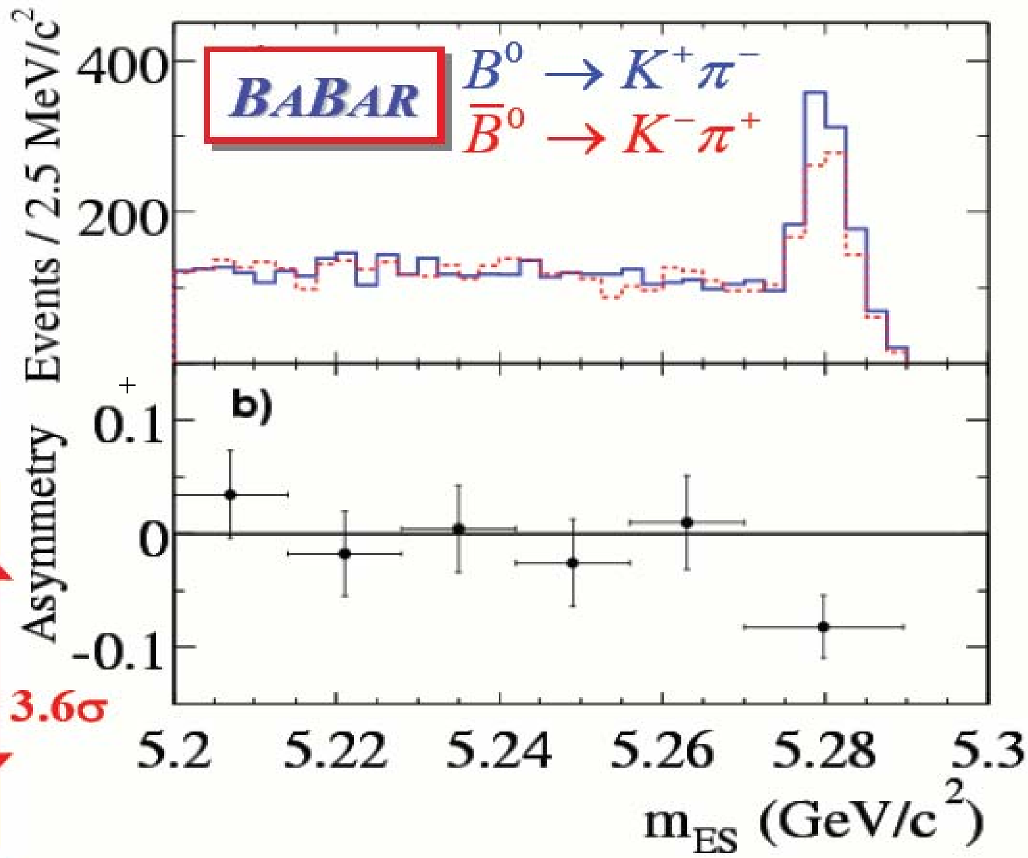
$B^+ \to K^+\pi^0$

$A_{CP} = +0.06 \pm 0.06 \pm 0.01$ *BABAR*
$A_{CP} = +0.04 \pm 0.05 \pm 0.02$ Belle

3.6σ

Average $A_{CP} = +0.049 \pm 0.040$

Signal (227M $B\bar{B}$ pairs): 1606 ± 51



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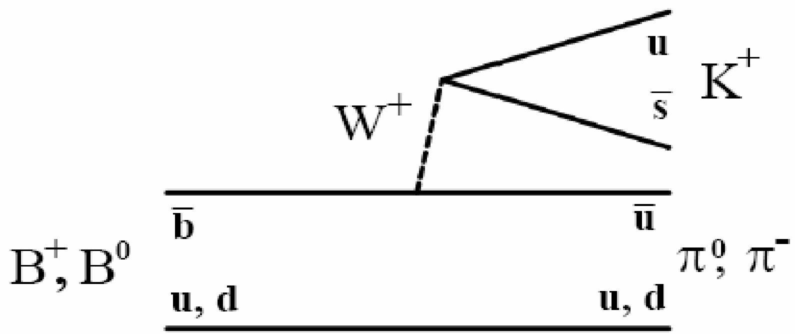
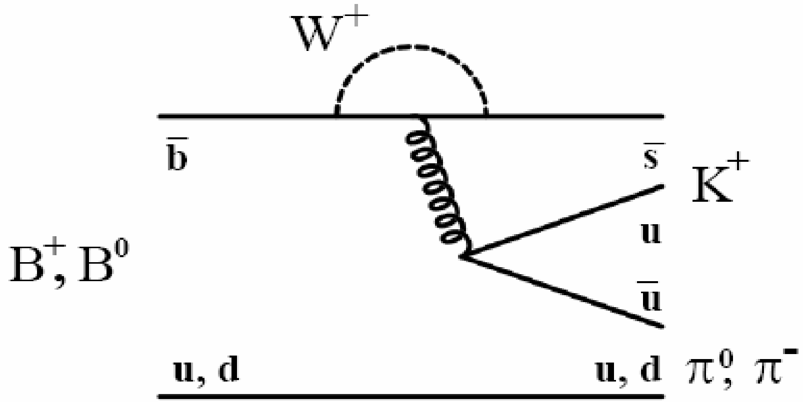
Interpretation

Both decay modes should show the same asymmetry



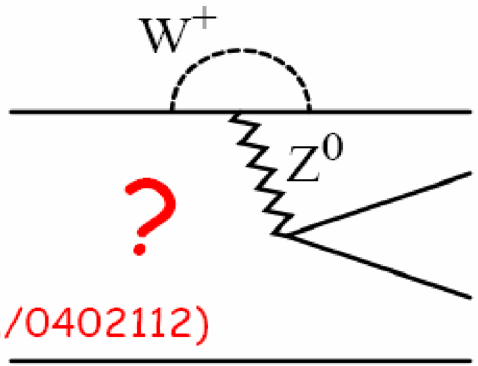


B->$K^+\pi^-$ A_{CP} = -0.101± 0.025±0.005

B->$K^+\pi^0$ A_{CP} = +0.04 ±0.05 ± 0.02

difference is 2.4 σ

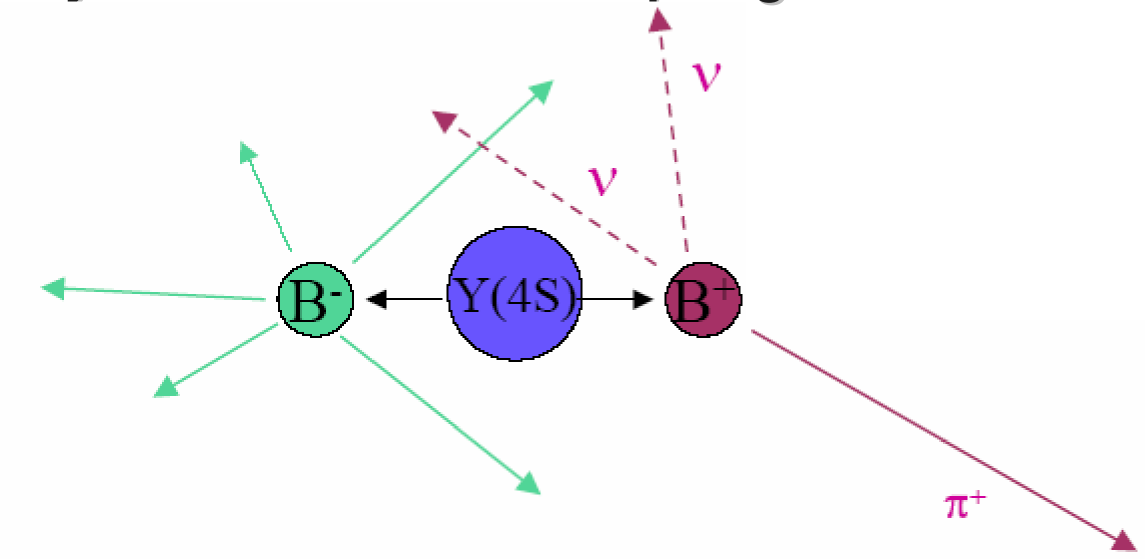
-> indication for ...?



Buras et al. (hep-ph/0402112)

$b \to s\nu\bar{\nu}$ and $b \to d\nu\bar{\nu}$

- Single meson + missing energy in final state.
- Analyzed through reconstruction of another B meson from $\Upsilon(4S) \to B^+B^-$ event.
- Although experiments are difficult, good channel for the study of Z-mediated EW penguin contribution.



Exp. Bounds from Belle and BaBar

TABLE II: Expected BRs in the SM and experimental bounds (90% C.L.) in units of 10^{-6}.

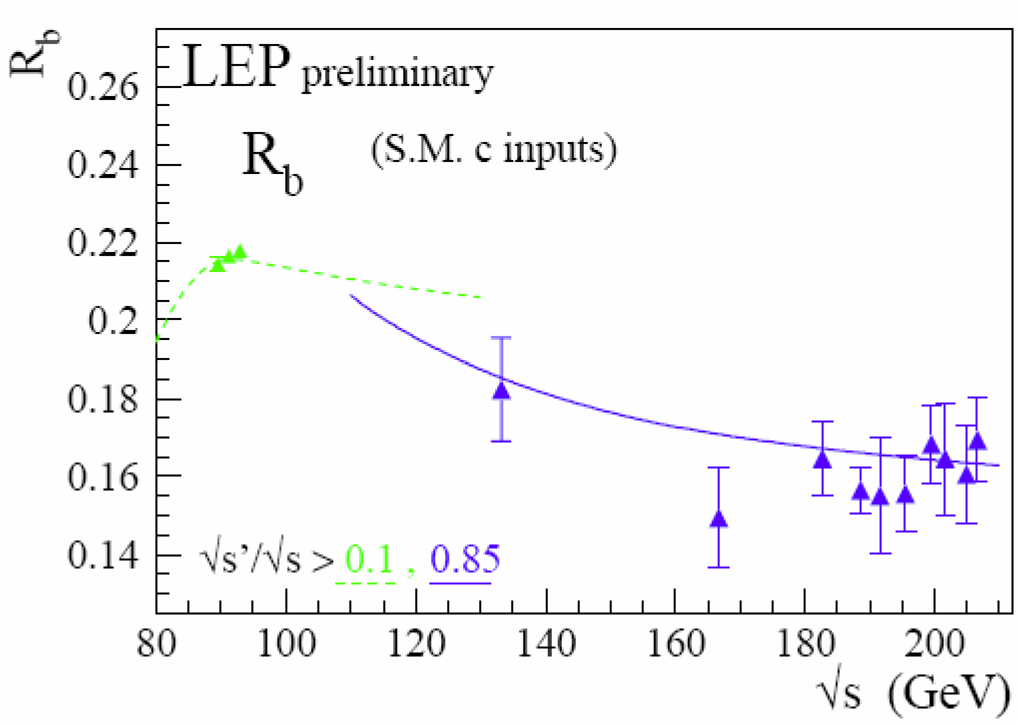
mode	BRs in the SM	Experimental bounds
$B \to K\nu\bar{\nu}$	$5.31^{+1.11}_{-1.03}$	$<$ 36 [30]
$B \to \pi\nu\bar{\nu}$	$0.22^{+0.27}_{-0.17}$	$<$ 100 [31]
$B \to K^*\nu\bar{\nu}$	$11.15^{+3.05}_{-2.70}$	$<$ 340 (Belle)
$B \to \rho\nu\bar{\nu}$	$0.49^{+0.61}_{-0.38}$	-

- SM predictions are highly dependent on form factors.

  ⇨ large theoretical uncertainty.

- $K\nu\nu$ sensitivity now <10X SM rate.
- $b \to d$ transition receive larger uncertainties from $|V_{td}|$.
- Vector boson production is 2 or 3 times larger than pseudoscalar production (because of polarization).

Fate of leptophobic U(1)'



However, it still remains as a viable candidate of physics beyond SM.

$M_{Z'}$

exclude !!! 365 GeV $\leq$ $M_{Z'}$ $\leq$ 615 GeV (D0)

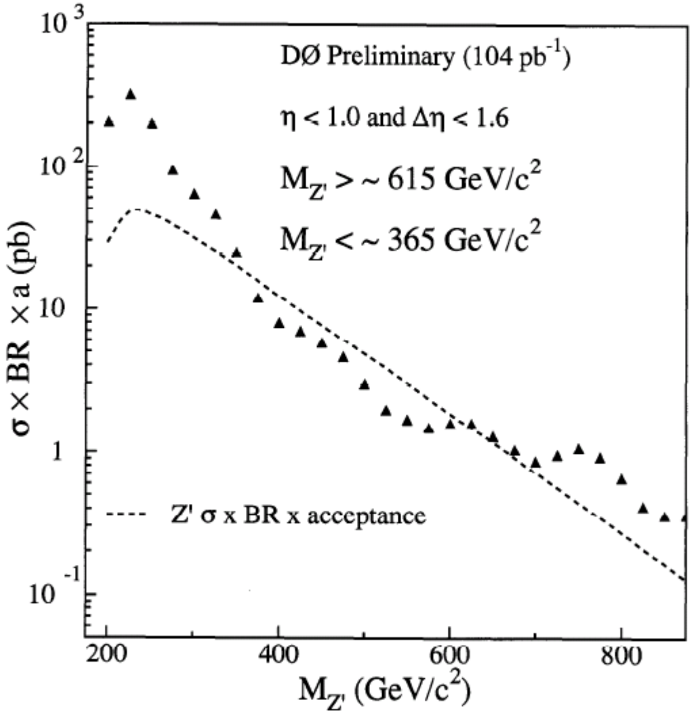


FIG. 9. The 95% CL on the production cross section ($\sigma_X \times \mathrm{BR} \times a$) for the Z' (solid triangles) compared with the predicted cross section (dashed line). Values of 365 < $M_{Z'}$ < 615 GeV/c^2 are excluded at the 95% CL.

Brief review of leptophobic Z' model

Leroux, London, PLB526

$SU(2)_L x U(1)_Y x U(1)'$: U(1)' arise from the breaking chain

$$E_6 \to SO(10) \times U(1)_\psi$$
$$\to SU(5) \times U(1)_\chi \times U(1)_\psi$$
$$\to SU(2)_L \times U(1)_Y \times U(1)'.$$

E_6

- Anomaly free GUT
- String theory motivated

with $Q'=Q_\psi \cos\Theta - Q_\chi \cos\Theta$, where Θ is the usual E_6 mixing angle

TABLE I: Charges of fermions contained in the **27** representation of E_6 within the conventional embedding [25].

Particle	$SU(3)_c$	Y	$2\sqrt{6}Q_\psi$	$2\sqrt{10}Q_\chi$
$Q=(u,d)^T$	**3**	1/6	1	−1
$L=(\nu,e)^T$	**1**	−1/2	1	3
u^c	$\bar{\mathbf{3}}$	−2/3	1	−1
d^c	$\bar{\mathbf{3}}$	1/3	1	3
e^c	**1**	1	1	−1
ν^c	**1**	0	1	−5
$H=(N,E)^T$	**1**	−1/2	−2	−2
$H^c=(N^c,E^c)^T$	**1**	1/2	−2	2
h	**3**	−1/3	−2	2
h^c	$\bar{\mathbf{3}}$	1/3	−2	−2
S^c	**1**	0	4	0

The most general $SU(2)_L \times U(1)_Y \times U(1)'$ invariant Lagrangian include a kinetic mixing term between $U(1)_Y$ and $U(1)'$ gauge bosons:

$$\mathcal{L}_{\rm kin} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} \widetilde{B}^{\mu\nu} \widetilde{B}_{\mu\nu} - \frac{1}{4} \widetilde{Z}'^{\mu\nu} \widetilde{Z}'_{\mu\nu} - \frac{\sin\chi}{2} \widetilde{B}_{\mu\nu} \widetilde{Z}'^{\mu\nu},$$

With the non-unitarity transformation

$$\widetilde{B}_\mu = B_\mu - \tan\chi Z'_\mu,$$
$$\widetilde{Z}'_\mu = \frac{Z'_\mu}{\cos\chi}.$$

Z'-fermion interaction term can be written (with $\delta \equiv \frac{g_{YQ'}}{g_{Q'}} = -\frac{\tilde{g}_Y \sin\chi}{\tilde{g}_{Q'}}$)

$$\mathcal{L}(Z')_{\rm int} = -\lambda \frac{g}{\cos\theta_W} \sqrt{\frac{5x_W}{3}}\, \bar{\psi} \gamma^\mu \left(Q' + \sqrt{\frac{3}{5}} \delta Y_{\rm SM} \right) \psi Z'_\mu$$

Leptophobic Z' if $\tan\theta = \sqrt{3/5}$ and $\delta = -1/3$ for conventional embedding.

$$(Q' + \sqrt{\tfrac{3}{5}} \delta Y_{SM}) = 0 \text{ for } L \text{ and } e^c \text{ simultaneously.}$$

Q_ψ and Q_χ quantum numbers of L and e^c for the six embeddings of charged leptons in the **27** representation of E_6, along with the values of θ and δ which produce a leptophobic Z' gauge boson

Model		$2\sqrt{6}\,Q_\psi$	$2\sqrt{10}\,Q_\chi$		$2\sqrt{6}\,Q_\psi$	$2\sqrt{10}\,Q_\chi$	$\tan\theta$	δ
1	L:	1	3	e^c:	1	-1	$\sqrt{3/5}$	$-1/3$
2	L:	-2	-2	e^c:	1	-1	$\sqrt{3/5}$	$-1/3$
3	L:	1	3	e^c:	1	-5	$\sqrt{15}$	$-\sqrt{10}/3$
4	L:	-2	-2	e^c:	1	-5	$\sqrt{5/27}$	$-\sqrt{5/12}$
5	L:	1	3	e^c:	4	0	$\sqrt{5/3}$	$-\sqrt{5/12}$
6	L:	-2	-2	e^c:	4	0	0	$-\sqrt{10}/3$

$U(1)'$ quantum numbers of d^c and h^c for each of the six models given in Table 2, calculated using $Q' = Q_\psi \cos\theta - Q_\chi \sin\theta$

Model		$2\sqrt{6}\,Q_\psi$	$2\sqrt{10}\,Q_\chi$	Q'		$2\sqrt{6}\,Q_\psi$	$2\sqrt{10}\,Q_\chi$	Q'
1	d^c:	1	3	$-1/2\sqrt{15}$	h^c:	-2	-2	$-1/2\sqrt{15}$
2	d^c:	1	3	$-1/2\sqrt{15}$	h^c:	-2	-2	$-1/2\sqrt{15}$
3	d^c:	1	-1	$1/2\sqrt{6}$	h^c:	-2	-2	$1/2\sqrt{6}$
4	d^c:	1	-1	$1/4$	h^c:	-2	-2	$-1/4$
5	d^c:	1	-1	$1/4$	h^c:	1	3	$-1/4$
6	d^c:	1	-1	$1/2\sqrt{6}$	h^c:	1	3	$1/2\sqrt{6}$

$U(1)'$ quantum numbers of ν^c and S^c for models 4 and 5 of Table 2, calculated using $Q' = Q_\psi \cos\theta - Q_\chi \sin\theta$

Model		$2\sqrt{6}\,Q_\psi$	$2\sqrt{10}\,Q_\chi$	Q'		$2\sqrt{6}\,Q_\psi$	$2\sqrt{10}\,Q_\chi$	Q'
4	ν^c:	1	-1	$1/4$	S^c:	4	0	$3/4$
5	ν^c:	1	-1	$1/4$	S^c:	1	-5	$3/4$

(Leptophobic) Z′-mediated FCNCs

K.Leroux, D.London (PLB526,97)(2002)

$$\mathcal{L}_{\text{int}} = -\lambda \frac{g}{\cos\theta_W}\sqrt{\frac{5\sin^2\theta_W}{3}}\bar{\psi}\gamma^{\mu}\left(Q' + \sqrt{\frac{3}{5}}\delta Y_{SM}\right)Z'_{\mu}\psi$$

- **neutrino** $\boxed{\mathcal{L}^{Z'} = -\frac{g}{2\cos\theta_W} Q^{Z'}_{\nu_R}\ \bar{\nu}_R\ \gamma^{\mu}\ \nu_R\ Z'_{\mu}}$ $\quad Q^{Z'}_{\nu_R} = \frac{1}{2}\lambda\sqrt{\frac{5\sin^2\theta_W}{3}} = 0.31$

- **quark** $\boxed{\mathcal{L}^{Z'}_{\text{FCNC}} = -\frac{g}{2\cos\theta_W} U^{Z'}_{qp}\ \bar{d}_{qR}\ \gamma^{\mu}\ \bar{d}_{pR}\ Z'_{\mu}}$ **Parameterize !!**

- **Two unknown parameters !!!**

$$\boxed{U^{Z'}_{qp}\ ,\ \mathrm{M}_{Z'}}$$

Parametrize Z'- mediated FCNC coupling as

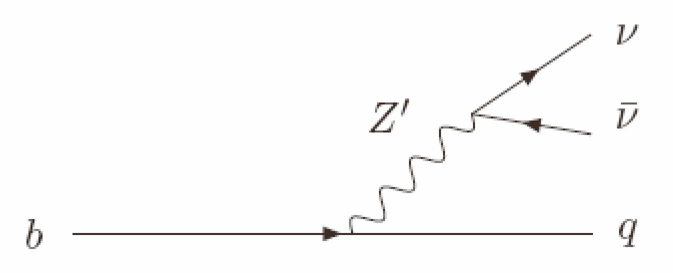
$$\mathcal{L}^{Z'}_{\rm FCNC} = -\frac{g}{2\cos\theta_W} U^{Z'}_{qp} \bar{d}_{qR}\gamma^{\mu} d_{pR} Z'_{\mu}.$$

Z' does not couple to SM neutrino because it is leptophobic.

However, it may couple to the right-handed neutrino (υ^c or S^c).

$$-\frac{g}{2\cos\theta_W} Q^{Z'}_{\nu_R} \bar{\nu}_R \gamma^{\mu} \nu_R Z'_{\mu}$$

Effectively, tree level FCNC in B decays



$$H_{\rm eff}(b \to q\nu_R\bar{\nu}_R) = \frac{\pi\alpha}{\sin^2 2\theta_W M^2_{Z'}} U^{Z'}_{qb} Q^{Z'}_{\nu_R} \bar{q}\gamma^{\mu}(1+\gamma_5) b\bar{\nu}\gamma_{\mu}(1+\gamma_5)\nu;$$

Exclusive $B \to M\nu\bar{\nu}$ $(M = \pi, K, \rho, K^*)$ Decays

- We investigate $B \to M\nu\bar{\nu}$ $(M = \pi, K, \rho, K^*)$ Decays in the leptophobic Z' model as a possible candidate of new physics in the EW penguin sector.

SM

$$H_{\text{eff}}(b \to q\nu_{\text{SM}}\bar{\nu}_{\text{SM}}) = \frac{G_F\alpha}{2\pi\sqrt{2}} V_{tb}V^*_{tq}C^{\nu}_{10}\bar{q}\gamma^{\mu}(1-\gamma^5)b\bar{\nu}\gamma_{\mu}(1-\gamma^5)\nu,$$

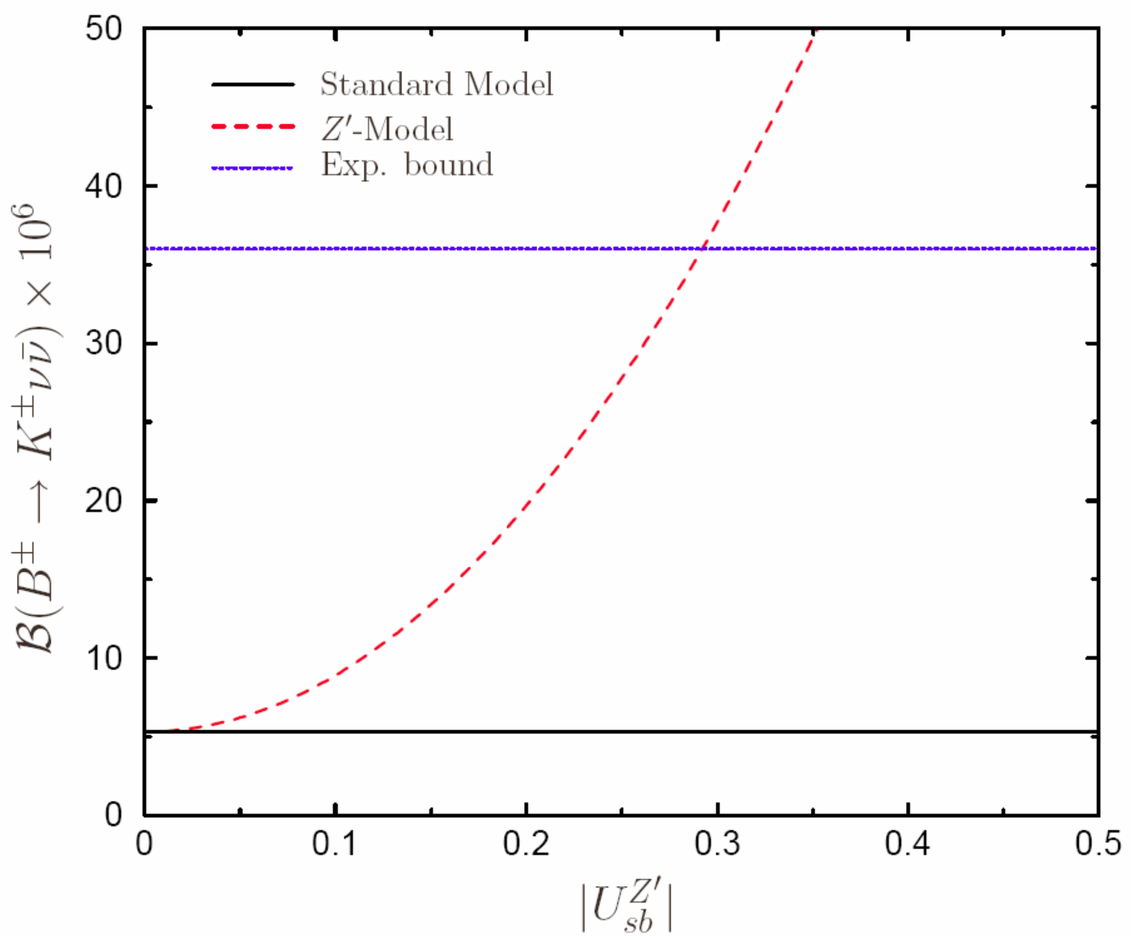
L-Z'

$$H_{\text{eff}}(b \to q\nu_R\bar{\nu}_R) = \frac{\pi\alpha}{\sin^2 2\theta_W M^2_{Z'}} U^{Z'}_{qb} Q^{Z'}_{\nu_R}\bar{q}\gamma^{\mu}(1+\gamma_5)b\bar{\nu}\gamma_{\mu}(1+\gamma_5)\nu,$$

- Additional right-handed neutrinos(ν^c, s^c) can contribute to the missing energy signal in $B \to M + \not{E}$ Decays

$$\mathcal{B}(B \to M\nu\bar{\nu}) = \mathcal{B}(B \to M\nu_{\text{SM}}\bar{\nu}_{\text{SM}}) + \mathcal{B}(B \to M\nu_R\bar{\nu}_R).$$

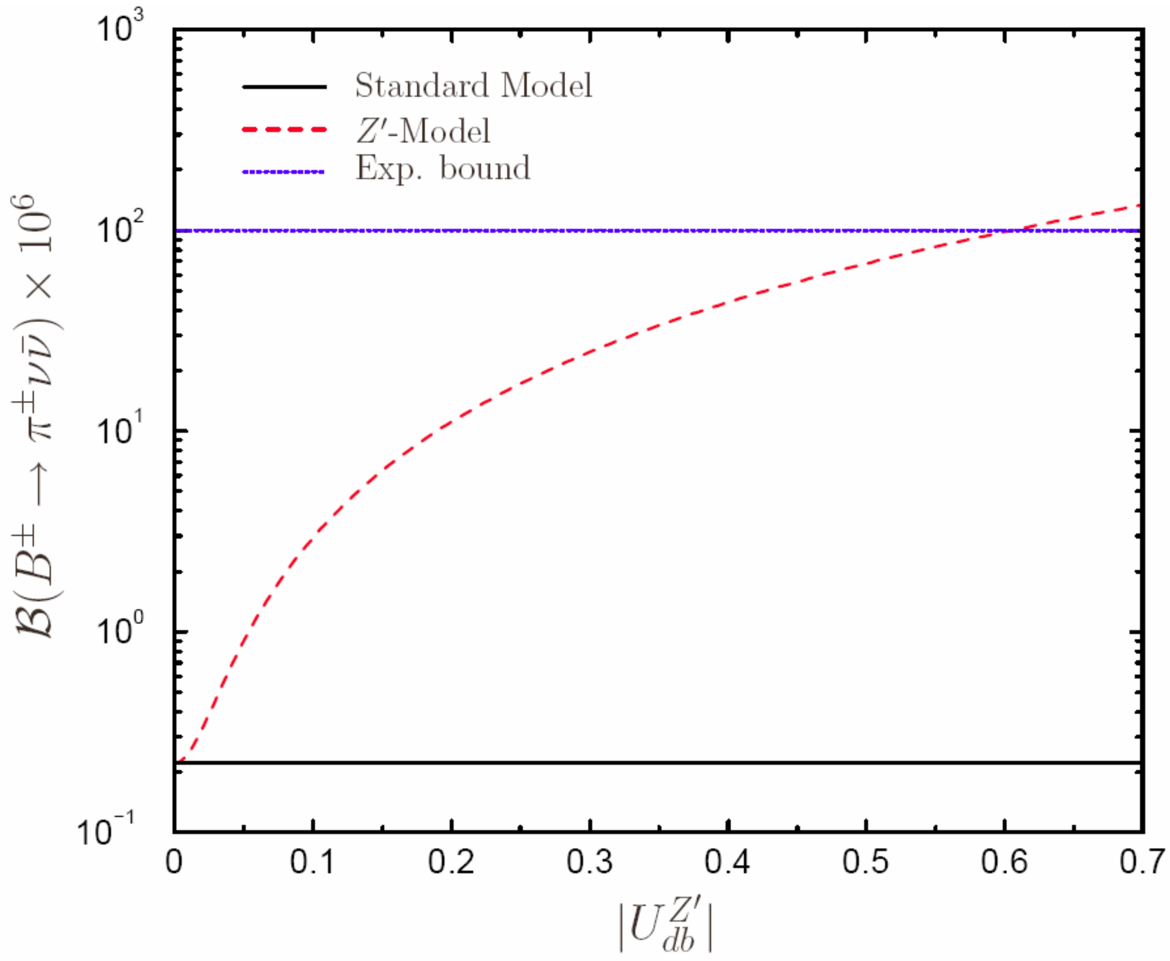
- Main theoretical uncertainties srise from the hadronic transition form factors for these Decays



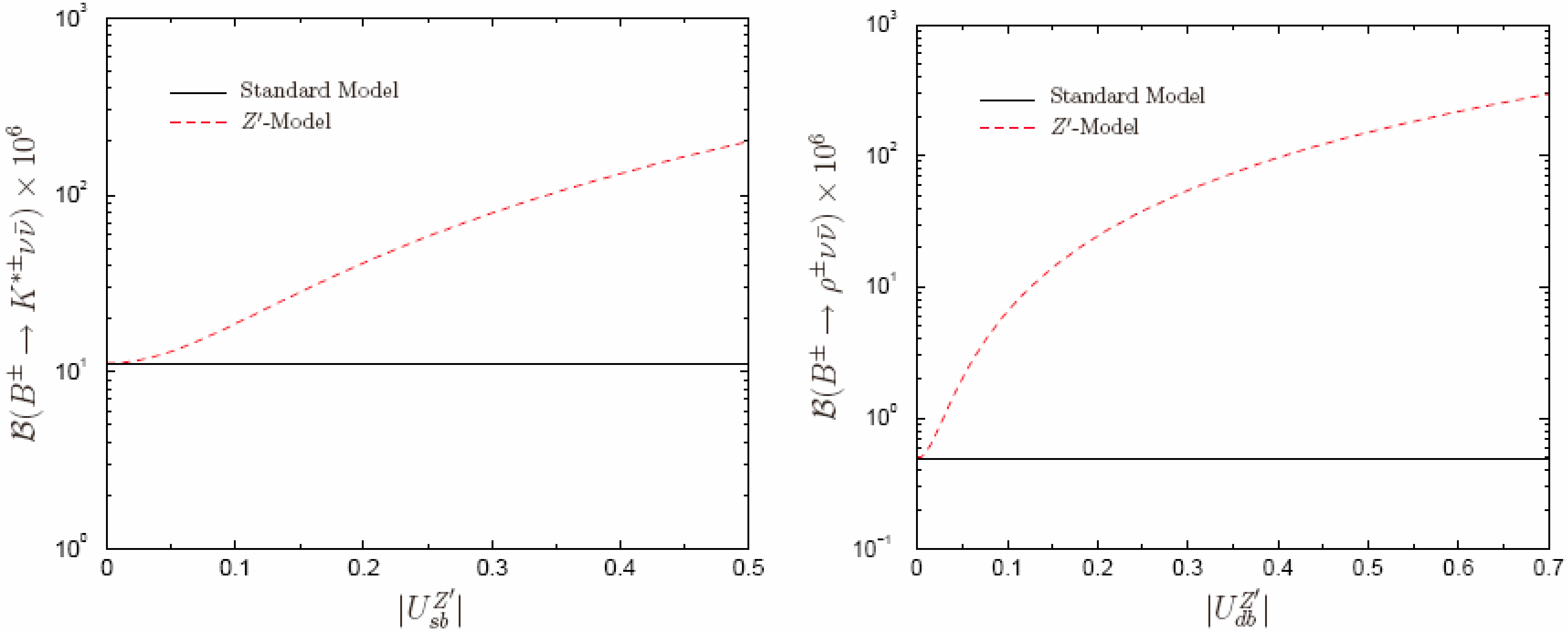
$M_{Z'} = 700$ GeV

$U_{sb\,\max}^{Z'} = 0.29$

Cf. from inclusive decay b→sυυ at ALEPH $U_{sb}^{Z'} < 0.83$


Standard Model
Z′-Model
Exp. bound
10^3
10^2
10^1
10^0
10^-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$\mathcal{B}(B^\pm \to \pi^\pm \nu\bar{\nu}) \times 10^6$
$|U_{db}^{Z'}|$

$$U_{db\,\max}^{Z'} = 0.61$$



Standard Model
Z'-Model
$\mathcal{B}(B^\pm \to K^{*\pm}\nu\bar{\nu}) \times 10^6$
10^0
10^1
10^2
10^3
0
0.1
0.2
0.3
0.4
0.5
$|U_{sb}^{Z'}|$
Standard Model
Z'-Model
$\mathcal{B}(B^\pm \to \rho^{\pm}\nu\bar{\nu}) \times 10^6$
10^{-1}
10^0
10^1
10^2
10^3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$|U_{db}^{Z'}|$

(a) **(b)**

Massive right-handed neutrino

- In principle, right-handed neutrino can be massive.
- Sharp rise near threshold
- If mass is lower than a few hundred MeV, it is hard to find difference from the massless case.

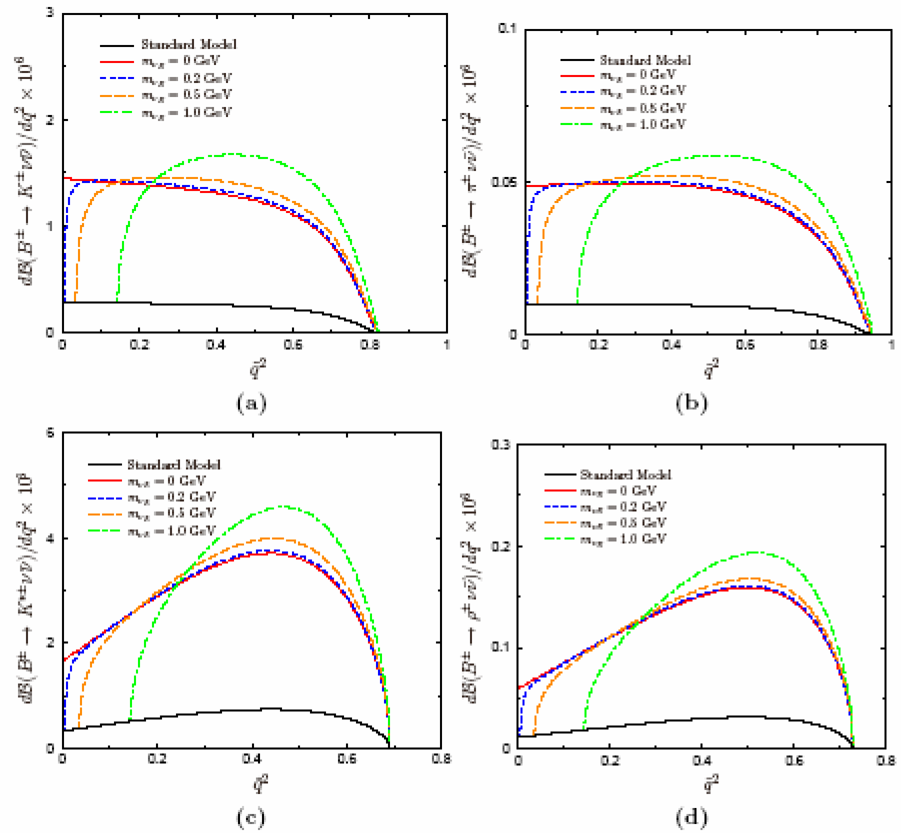


FIG. 3: Differential BRs as a function of the normalized momentum transfer square, $\tilde{q}^2 = q^2/M_B^2$, in units of 10^{-6}, for (a) $B^\pm \to K^\pm \nu\bar{\nu}$, (b) $B^\pm \to \pi^\pm \nu\bar{\nu}$, (c) $B^\pm \to K^{*\pm} \nu\bar{\nu}$, and (d) $B^\pm \to \rho^\pm \nu\bar{\nu}$. Here the decay rates in the leptophobic Z' model are normalized to be five times larger than those in the SM.

Review of Mixing-Matrix and SVD

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EPJC28(2003)55

Preview

(I) Review of CKM matrix.

- Diagonalization of quark mass matrices in Yukawa interaction

$$\mathcal{M}_u^{diag} = U_{uL}^{+} M_u U_{uR}$$
$$\mathcal{M}_d^{diag} = U_{dL}^{+} M_d U_{dR}$$

- CKM matrix in charged current interactions:

$$V_{CKM} \equiv U_{uL}^{+} U_{dL}$$

- Complex $\longrightarrow$ $2 \times n^2$ — $n=3$: 18

  unitary $\longrightarrow$ $-n^2$

  freedom in quark field $\longrightarrow$ $-(2n-1)$ $= (n-1)^2$

$\Rightarrow$ $n(n-1)/2$ rotational angles — 3

$(n-1)(n-2)/2$ phases — 1

- Historically; K, π decays $\begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$, θ_c = Cabibbo angle.

$\longrightarrow$ GIM $\rightarrow$ Charm $\longrightarrow$ $\begin{array}{c} \\ t \rightarrow \end{array}\begin{pmatrix} [\,\vdots \;\; \vdots\,] & 0 \\ & 0 \\ 0 \;\; 0 & 0 \end{pmatrix} \begin{array}{l} = V_{CKM} \\ KM, \end{array}$ δ_{KM}

$\uparrow b$

- hierarchy $\rightarrow$ Wolfenstein parametrization

$$V = \begin{pmatrix} \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda \\ -\lambda & 1-\frac{\lambda^2}{2} \end{bmatrix} & \begin{matrix} A\lambda^3(\rho - i\eta) \\ A\lambda^2 \end{matrix} \\ A\lambda^3(1-\rho-i\eta), \; -A\lambda^2, & 1 \end{pmatrix} + O(\lambda^4)$$

almost unitary

Preview

I Unitary parametrization of CKM matrix.

- Unitarity $\sum_\alpha V_{i\alpha}^* V_{j\alpha} = \delta_{ij}$, $\sum_i V_{i\alpha}^* V_{i\beta} = \delta_{\alpha\beta}$

$\Rightarrow$ (e.g.) $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ 6 identities to 1.
(exp.tally almost so!)

$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$ 6 triangles
$\hookrightarrow$ all same area $A = \frac{1}{2} J_{cp}$
$= \frac{1}{2} Im(V_{i\alpha} V_{j\beta} V_{i\beta}^* V_{j\alpha}^*)$

- PDG Standard parametrization:

$$V(\theta_{12}, \theta_{23}, \theta_{31}, \delta) = \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}\begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix}$$

$s_{12} = \lambda$, $s_{23} = A\lambda^2$, $s_{13} = A\lambda^3(\rho - i\eta)$

$\Rightarrow$ 9 possible ways of parametrization
[(e.g.) $V = R(\theta_{23}) R(\theta_{31}, \delta) R(\theta_{12})$
$= R(\theta_{31}) R(\theta_{12}, \delta) R(\theta_{23}) = \cdots$]

- all equivalent mathematically.
- Underlying physics maybe more transparent or less.

- Possible Problems in Unitary Parametrization:

- (fully & Completely) Unitary
- not flexible to extend (include) non-unitarity
- unknown parameter values $\longleftrightarrow$ unknown new physics
- no possibility of step-by-step test

Preview

III Step-by-step (re-) definition of Unitarity.

- Weak Unitarity (WUC) $\sum_\alpha |V_{i\alpha}|^2 = \sum_j |V_{j\beta}|^2 = 1$ (for all $i = u, c, t$; $\beta = d, s, b$.)

$\Rightarrow$ (looks) exp'tally well satisfied for $n=3$ generation.
$\hookrightarrow$ our starting point.

- Almost Unitarity (AUC) $\sum_{\alpha, i \neq j} V_{i\alpha}^* V_{j\alpha} = \sum_{i, \alpha \neq \beta} V_{i\alpha}^* V_{i\beta} = 0$

for part of $i = u, c, t$; $\beta = d, s, b$

and/or with Different area of triangles.

- Full Unitarity (FUC) $\leftarrow$ (usual) Unitarity

$\begin{cases} \text{6 identities to 1} \\ \text{6 triangles with same area} \end{cases}$

* New & alternative (w/o new physics uncertainty), and flexible (to test step-by-step unitarity) parametrization of CKM matrix?

* parameters maybe values not known precisely, however, sure of no new physics uncertainties, and flexible enough to extend step-by-step.

II Singular-Value-Decomposition method (SVD)

"If you use Unitary Parametrization (eg. CKM) {Wolfenstein, ...}, it is very difficult to make step-by-step test to check unitarity."

(1) Start with 3 inputs ($|V_{us}|, |V_{ub}|, |V_{cb}|$):

(Equivalently we can start with $|V_{ud}|, |V_{us}|, |V_{cd}|$, or etc.)

(2)
$$\begin{pmatrix} |V_{ud}| = [1-|V_{us}|^2-|V_{ub}|^2]^{1/2} & |V_{us}| & |V_{ub}| \\ \boxed{|V_{cd}| \quad\quad |V_{cs}|} & & |V_{cb}| \\ \boxed{|V_{td}| \quad\quad |V_{ts}|} & & |V_{tb}| = [1-|V_{ub}|^2-|V_{cb}|^2]^{1/2} \end{pmatrix}$$ from WUC.

(3)
$$R\cdot X = B : \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} |V_{cd}|^2 \\ |V_{cs}|^2 \\ |V_{td}|^2 \\ |V_{ts}|^2 \end{pmatrix} = \begin{pmatrix} 1-|V_{cb}|^2 \\ 1-|V_{tb}|^2 \\ 1-|V_{ud}|^2 \\ 1-|V_{us}|^2 \end{pmatrix}$$

(4) $\det(R) = 0 \rightarrow$ singular $\rightarrow$ not unique solution. $\Rightarrow$ SVD

$X = X_g + X_s$ $\quad\quad$ $X_s = \bar{R}B \quad (1 = R\bar{R} \neq \bar{R}R, \; R\bar{R}B = B)$

$R X_g = 0 \rightarrow X_g = a\begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \; a \in \mathbb{R}$

** $R = U\cdot W\cdot V^T$; U, V = orthogonal, W = diagonal

$\bar{R} = V(\frac{1}{W})U^T$; $\frac{1}{W} = \mathrm{diag}(\frac{1}{W_{ii}})$

$\longrightarrow X = (|V_{cd}|^2, |V_{cs}|^2, |V_{td}|^2, |V_{ts}|^2)^T$

$|V_{ij}|^2 \geq 0 \longrightarrow a \in (a_{min}, a_{max})$

(5) Now test FUC

$\rightarrow$ 6 more constraints $\left(\begin{array}{l} \sum V_{ij} V_{kj}^* = 0 \\ \sum V_{ji} V_{jk}^* = 0 \end{array} \right.$

(eg) $\sum_{j=u,c,t} V_{jd} V_{jb}^* = 0 \Longrightarrow$ Triangle

$|V_{cd}||V_{cb}| \le |V_{ud}||V_{ub}| + |V_{td}||V_{tb}|$ (Necessary condition)

$l_2 \le l_1 + l_3$

$\longrightarrow f(l_1, l_2, l_3) \equiv 2l_1^2 l_2^2 + 2l_2^2 l_3^2 + 2l_1^2 l_3^2 - l_1^4 - l_2^4 - l_3^4 \ge 0$

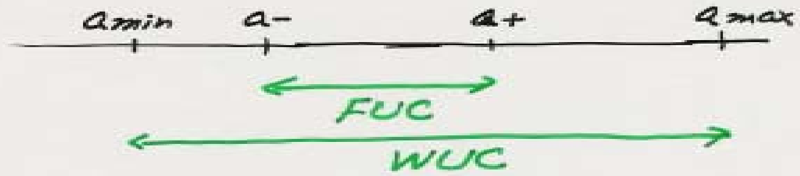
$\rightarrow$ Heron's theorem $A^2 = s(s-l_1)(s-l_2)(s-l_3) = \frac{1}{16} f(l_1, l_2, l_3)$

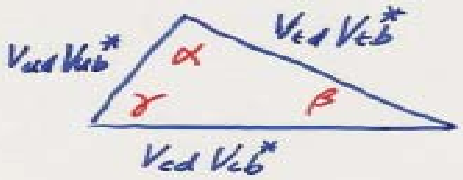
$s = (l_1 + l_2 + l_3)/2$

$\Rightarrow$ Jarlskog Invariant $J_{cp} = 2A = \frac{1}{2}\sqrt{f(l_1, l_2, l_3)}$

$\rightarrow$ expand f in terms of a: $f = f(a^2, a^1, a^0)$

2 roots $\rightarrow$ a_+, a_- $(a_+ > a_-)$





$$\sin\beta = \frac{2A}{|V_{td}||V_{tb}|\,|V_{cd}||V_{cb}|} \stackrel{?}{=} Arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\sin\gamma = \frac{2A}{|V_{ub}||V_{ud}|\,|V_{cd}|\,|V_{cb}|} \stackrel{?}{=} Arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\alpha = \pi - \beta - \gamma \stackrel{?}{=} Arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

TABLE I. Input values of the matrix elements and their sources referred from the PDG. The output values are allowed intervals (95% CL) for WUC and FUC.

	matrix elements	PDG values	Sources	
Input	$\lvert V_{us}\rvert$	0.2196 ± 0.0026	K_{e3} decays	
	$\lvert V_{ub}\rvert$	$(3.6 \pm 0.7) \times 10^{-3}$	B semileptonic decays	
	$\lvert V_{cb}\rvert$	$(41.2 \pm 2.0) \times 10^{-3}$	B semileptonic decays	
	matrix elements	WUC	FUC	PDG
Output	$\lvert V_{cd}\rvert$	$0.210 \sim 0.224$	$0.214 \sim 0.224$	$0.219 \sim 0.226$
	$\lvert V_{cs}\rvert$	$0.9735 \sim 0.9768$	$0.9735 \sim 0.9760$	$0.9732 \sim 0.9748$
	$\lvert V_{td}\rvert$	$0.004 \sim 0.045$	$0.004 \sim 0.014$	$0.004 \sim 0.014$
	$\lvert V_{ts}\rvert$	$0.001 \sim 0.045$	$0.035 \sim 0.045$	$0.037 \sim 0.044$

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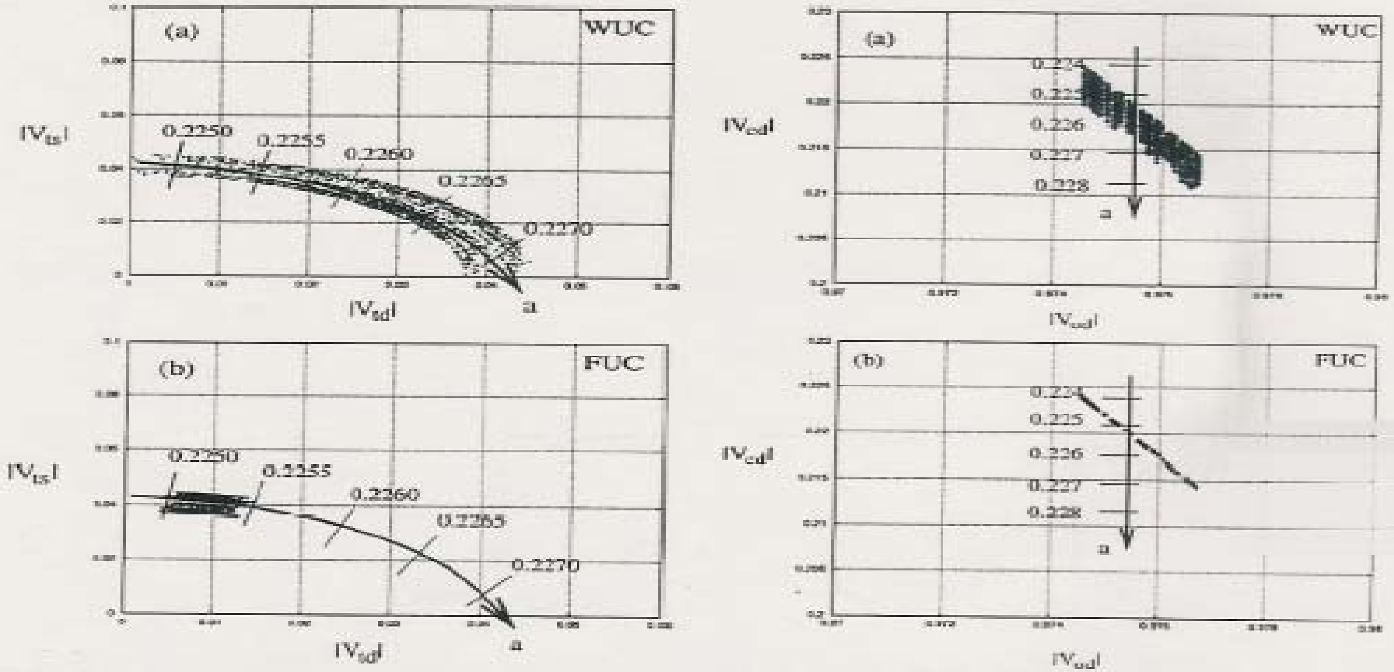
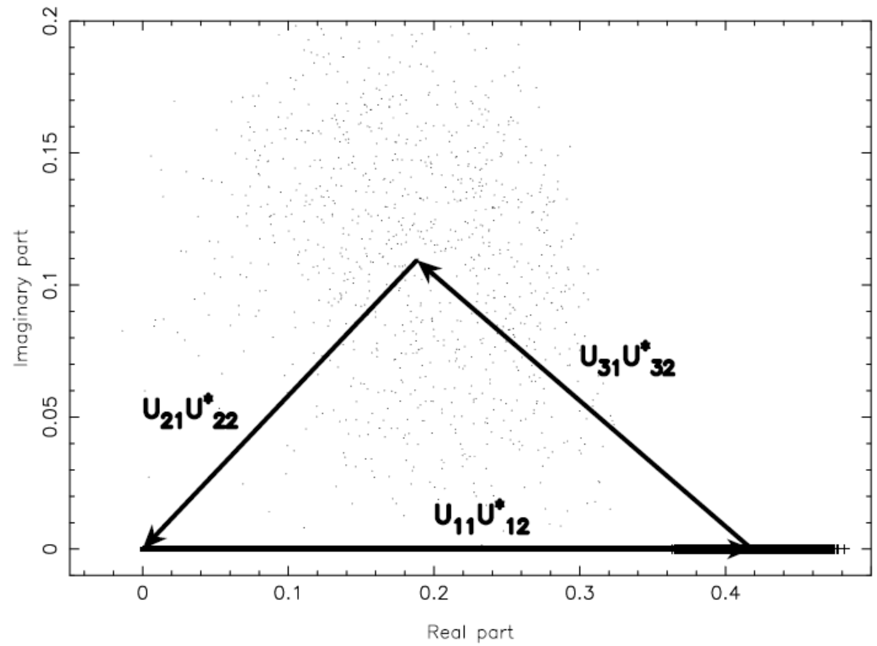
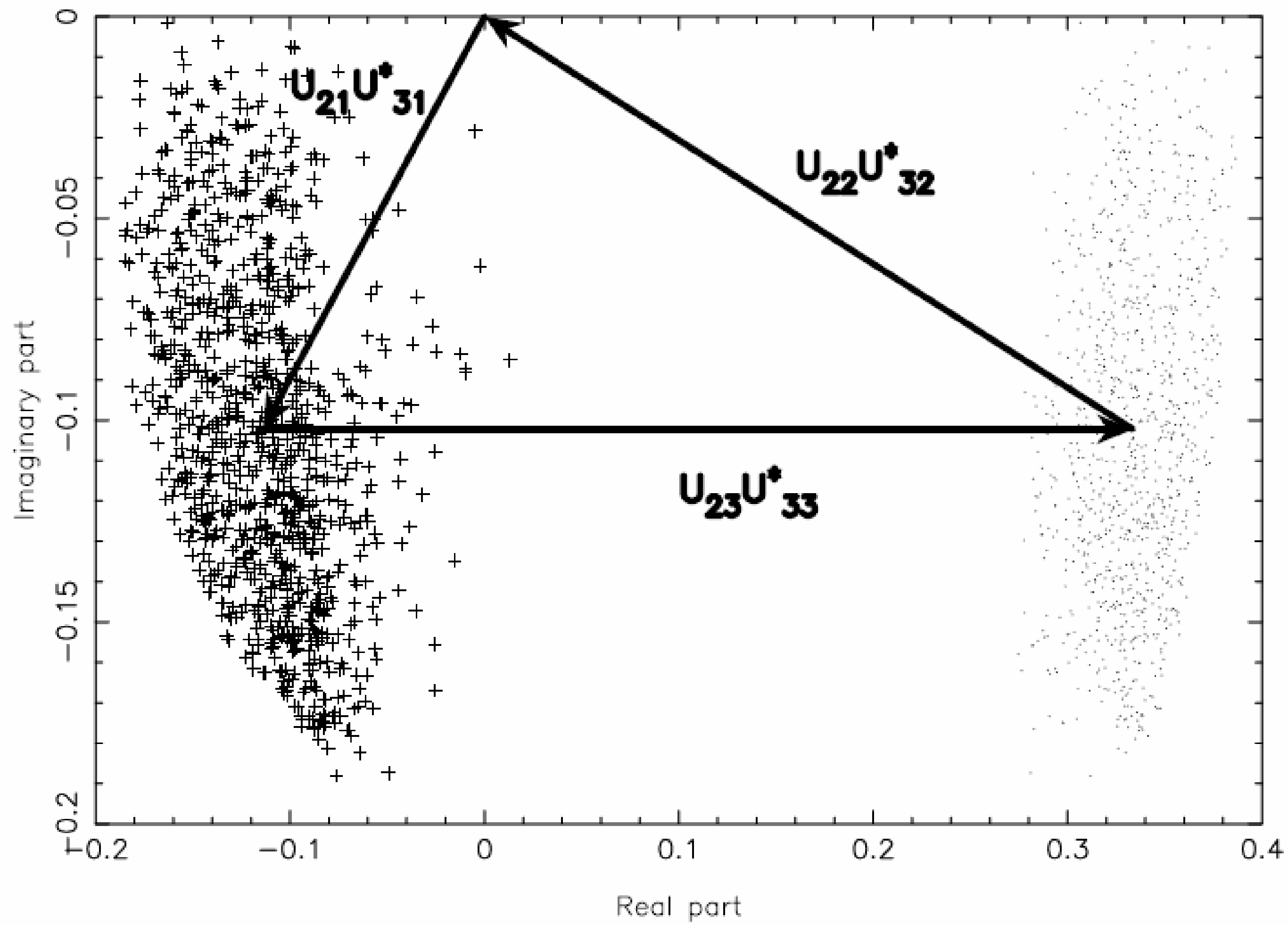


The elements of the modulus of the PMNS matrix are summaryized as follows:

$$\begin{pmatrix} 0.77-0.88 & 0.47-0.61 & <0.20 \\ 0.19-0.52 & 0.42-0.73 & 0.58-0.82 \\ 0.20-0.53 & 0.44-0.74 & 0.56-0.82 \end{pmatrix}.$$

Now we used the four input values of $|U_{11}|, |U_{12}|, |U_{22}|, |U_{23}|$. The unitarity condition with the first and second columns is shown in figure 7 to be on the real axis as that in quark case.





U21U*31
U22U*32
U23U*33
Imaginary part
Real part
0
−0.05
−0.1
−0.15
−0.2
−0.2
−0.1
0
0.1
0.2
0.3
0.4