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CP Violation from a Higher Dimensional Model

hep-h/0206187 PRD + α

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0. Introduction

Magnetic & Electric Dipole Moment

次元 $(\text{Mass})^5$

$$\frac{e}{2m} F_{\alpha\beta} \bar{\psi} \sigma^{\alpha\beta} \psi \sim -\mu \cdot B$$

$$\not{CP} \quad \frac{e}{2m} F_{\alpha\beta} \bar{\psi} \gamma^5 \sigma^{\alpha\beta} \psi \sim -d \cdot E$$

Kaluza-Klein W. Thirring '72 super weak

Randall-Sundrum ?

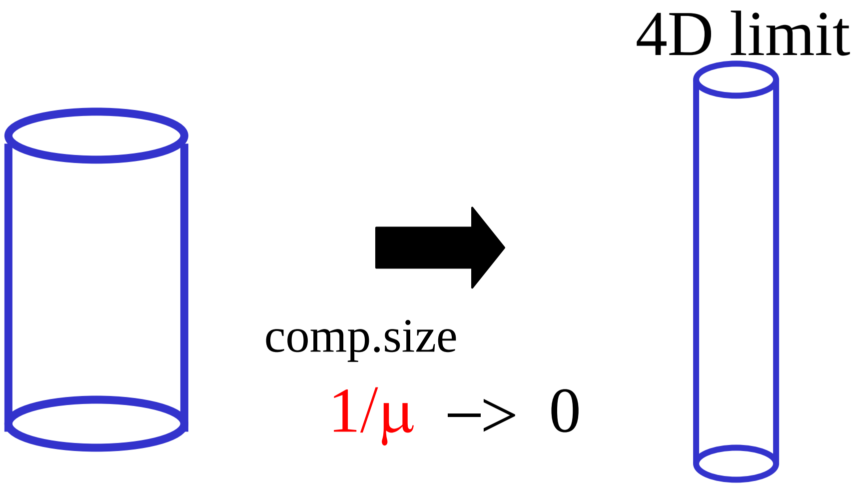
Experimental Bound

$d_n < 6.3 \times 10^{-26}$ e.cm '01

$d_e < (1.8 \pm 1.6) \times 10^{-27}$ e.cm '94

In SM, disregarding strong CP, quark EDM is generated only at 3-loop $\sim 10^{-31}$ e.cm

1. Kaluza-Klein Theory



5D Gravity
On S^1 x M_4
y $\{x^a\}$

4D Gravity
+Photon
+dilaton

This model gives us
the image that
the bulk world
shrinks to
some size which is
somewhat larger than
Planck length

Metric

a,b = 0,1,2,3

$$ds^2 = g_{ab}(x)dx^a dx^b + \mathrm{e}^{2\sigma(x)}(dy - f A_a(x)dx^a)^2$$

coupling

U(1) symmetry:

$$y \to y + \Lambda(x), \;\; A_a(x) \to A_a(x) + \frac{1}{f}\partial_a \Lambda$$

Periodicity

y → y + 1/μ   circle radius

funf-bein

$$(\hat{e}^{\mu}{}_{m}) = \begin{pmatrix} e^{\alpha}{}_{a} & 0 \\ -f e^{\sigma} A_a & e^{\sigma} \end{pmatrix}$$

General	m = 0,1,2,3,5	a = 0,1,2,3
Local Lorentz	μ = 0,1,2,3,5	α = 0,1,2,3

General 5D Dirac Equation

$$\left\{\gamma^\mu \hat{e}_\mu{}^m \frac{\partial}{\partial X^m} + \frac{1}{8}\underline{(\hat{\omega}^\sigma)_{\mu\nu}}\gamma_\sigma[\gamma^\mu, \gamma^\nu] + \underline{\hat{m}}\right\}\hat{\psi} = 0$$

connection

5D Mass

Switch off 4D gravity

$$\left\{\gamma^a(\partial_a + fA_a\partial_5) + \gamma^5\partial_5 - \frac{f}{16}F_{ab}\gamma^5[\gamma^a, \gamma^b] + \hat{m}\right\}\hat{\psi} = 0$$

(1) Massive Mode → Charged Fermion

$$\hat{\psi}(x, y) = \mathrm{e}^{i(\underline{\phi}\gamma^5 + \mu y)}\psi(x)$$

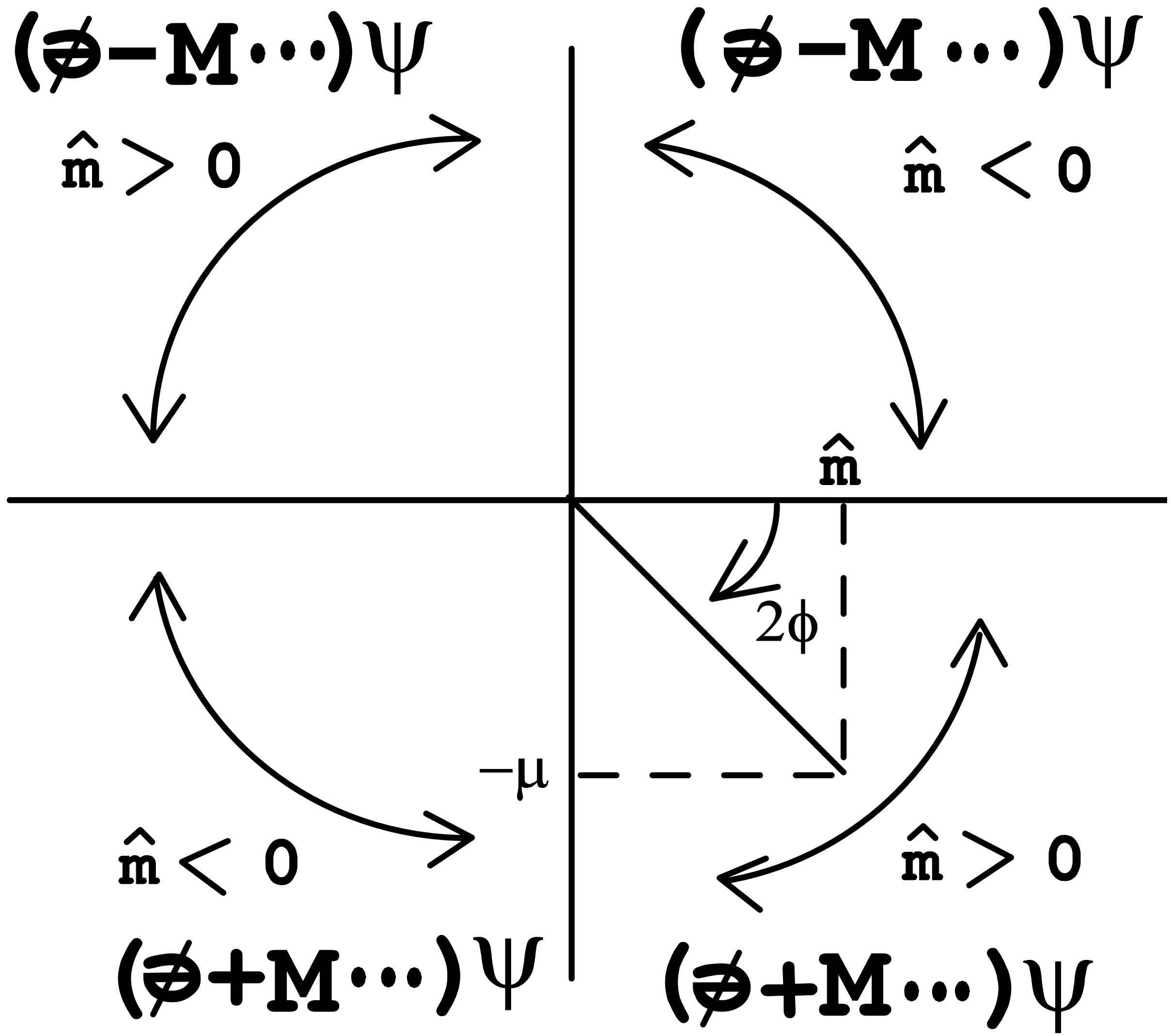
angle para.

$$(i\gamma^5\mu + \hat{m})\underline{\mathrm{e}^{2i\phi\gamma^5}} = \sqrt{\hat{m}^2 + \mu^2} \equiv M$$

⬇

$$e^{2i\phi\gamma^5} = \cos 2\phi + i\gamma^5 \sin 2\phi$$

$$\tan 2\phi = -\frac{\mu}{\hat{m}}\ ,(-\frac{\pi}{2} < 2\phi \leq 0 \text{ for } \hat{m} > 0;\ \pi \leq 2\phi < \frac{3\pi}{2} \text{ for } \hat{m} < 0)$$



$$\left\{\gamma^a(\partial_a + ieA_a) + M - \frac{1}{16M}(\hat{m}\frac{e}{\mu} - ie\gamma^5)F_{ab}\gamma^5[\gamma^a,\gamma^b]\right\}\psi = 0$$

MDM EDM

$$e \equiv f\mu$$

Electric Coupling Const.

(2) Zero Mode → Neutral Fermion

$$\hat{\psi}(x,y) = \psi(x) \ ,$$

$$\left\{\gamma^a\partial_a - \frac{f}{16}F_{ab}\gamma^5[\gamma^a,\gamma^b] + \hat{m}\right\}\psi(x) = 0$$

EDM

Order Estimation

$$S = -\frac{1}{2G_5}\int d^5X\sqrt{-\hat{g}}\hat{R}$$

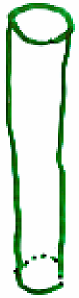
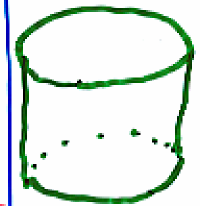
$$= -\frac{1}{2G_5}\frac{2\pi}{\mu}\int d^4x\sqrt{-g}\left(R + \frac{f^2}{4}F^{\alpha\beta}F_{\alpha\beta} + \cdots\right)$$

$$\frac{1}{G_5\mu} \sim \frac{1}{G} = (10^{19}\mathrm{GeV})^2\ , \ \ \frac{f^2}{G_5\mu} \sim 1\ , \ \ e = \mu f \sim 10^{-1}$$

$$\Downarrow$$

$$f = \frac{e}{\mu} \sim \sqrt{G} = 10^{-19}\mathrm{GeV}^{-1}. \quad \mu \sim 10^{-1}f^{-1} \sim 10^{18}\mathrm{GeV}$$

$1/M_{\mathrm{Planck}}$

	 Small Radius 4D Limit $\hat{m} \ll \mu$	Dual $\hat{m} \approx \mu$	 Large Radius 5D Limit $\hat{m} \gg \mu$
EDM $\frac{\lvert\hat{m}\rvert e}{M \mu}$	$\frac{\hat{m}}{\mu} \times 10^{-32}$	10^{-32}	10^{-32}
MDM $\frac{e}{M}$	10^{-32} ○	10^{-32} ×	$\frac{\mu}{\hat{m}} \times 10^{-32}$ × ×
	unit	$e \cdot cm$	$10^{-32} cm = L_{planc}$

5D Classical Gravity OK Condition

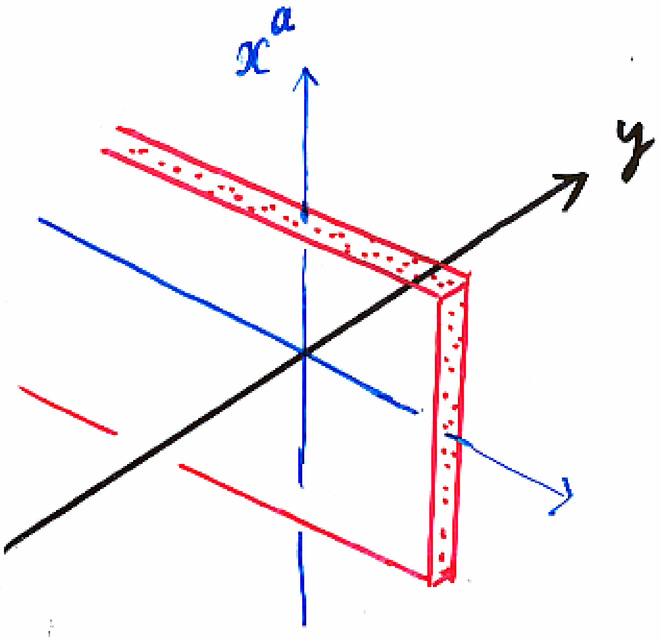
$$\frac{1}{\sqrt[3]{\alpha}} \sim \sqrt[3]{100}\,\mu \gg |\hat{m}|$$

2. Randall-Sundrum Theory

$$ds^2 = \mathrm{e}^{-2\sigma(y)}\eta_{ab}dx^a dx^b + dy^2$$

$$-\infty < y < +\infty \;,\; -\infty < x^a < +\infty$$

One Brane Model



AdS_5(thin wall limit): $\sigma(y) = \underline{\omega}|y|, \omega > 0$

bulk curvature

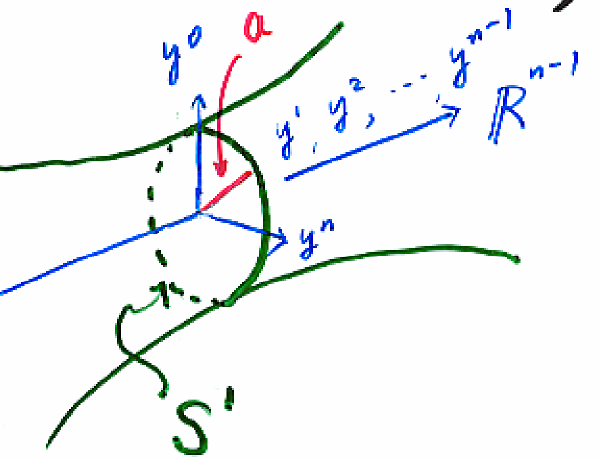
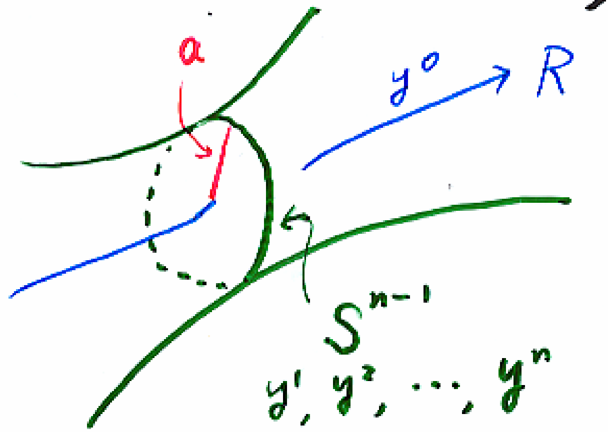
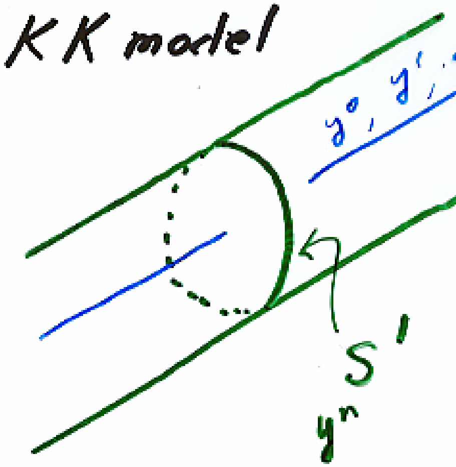
connection

$$\sqrt{-\hat{g}}\mathcal{L}^{Dirac} = \sqrt{-\hat{g}} i\bar{\hat{\psi}} \left\{ \gamma^\mu \hat{e}_\mu{}^m \partial_m + \frac{1}{8} \overline{(\hat{\omega}^\sigma)_{\mu\nu}\gamma_\sigma} [\gamma^\mu, \gamma^\nu] + \hat{m}(y) \right\} \hat{\psi}$$

$$= i\mathrm{e}^{-\frac{3}{2}\sigma} \bar{\hat{\psi}} \{ \gamma^a \partial_a - 2\mathrm{e}^{-\sigma} (\frac{1}{4}\sigma' - \frac{1}{2}\partial_y)\gamma^5 + \hat{m}(y) e^{-\sigma} \} (\mathrm{e}^{-\frac{3}{2}\sigma} \hat{\psi})$$

$$\hat{\psi}(x,y) = \sum_k (\psi_L^k(x)\xi_k(y) + \psi_R^k(x)\eta_k(y)),$$

$$\gamma^5 \psi_L(x) = -\psi_L(x) \ , \ \gamma^5 \psi_R(x) = +\psi_R(x)$$

	Anti de Sitter	de Sitter	Flat
	$AdS^n \ (\simeq S^1 \times R^{n-1})$	$dS^n \ (\simeq R \times S^{n-1})$	KK model
			
	$(y^0)^2 + (y^1)^2 + \cdots + (y^{n-1})^2 - (y^n)^2 = -a^2 < 0$	$-(y^0)^2 + (y^1)^2 + \cdots + (y^n)^2 = a^2 > 0$	
	isometry $SO(n-1, 2)$ Conformal	isometry $SO(n, 1)$	
...u- ...e	$-\frac{1}{a^2}$	$\frac{1}{a^2}$	0
...: ...t	$-\frac{1}{2}(n-1)(n-2)\frac{1}{a^2} < 0$	$\frac{1}{2}(n-1)(n-2)\frac{1}{a^2} > 0$	0

Let us cosider 5D Parity Even case

$$\gamma^5 \hat{\psi}(x,-y) = +\hat{\psi}(x,y)$$

Then

$$\xi_n(-y) = -\xi_n(y) \;,\;\; \eta_n(-y) = +\eta_n(y)$$

odd even

$$\xi_n(0) = 0 \text{ (Dirichlet)} \quad \partial_y \eta_n|_{y=0} = 0 \text{ (Neumann)}$$

$$\mathrm{e}^{-\sigma}(\frac{\sigma'}{2} - \partial_y)\tilde{\xi}_n + \mathrm{e}^{-\sigma}\hat{m}(y)\tilde{\xi}_n = m_n\tilde{\eta}_n$$

$$-\mathrm{e}^{-\sigma}(\frac{\sigma'}{2} - \partial_y)\tilde{\eta}_n + \mathrm{e}^{-\sigma}\hat{m}(y)\tilde{\eta}_n = m_n\tilde{\xi}_n$$

$$\tilde{\xi}_n \equiv \mathrm{e}^{-\frac{3}{2}\sigma}\xi_n \quad \tilde{\eta}_n \equiv \mathrm{e}^{-\frac{3}{2}\sigma}\eta_n$$

$$\int d^5X\sqrt{-\hat{g}}\mathcal{L}^{Dirac} =$$

$$i\int d^4x \sum_n \{\bar{\psi}_L^n(\gamma^a\partial_a\psi_L^n + m_n\psi_R^n) + \bar{\psi}_R^n(\gamma^a\partial_a\psi_R^n + m_n\psi_L^n)\}$$

System of 4D Free Fermions

Thin Wall Limit

sign func. delta func.

$$\sigma(y) = \omega|y| \ , \ \sigma'(y) = \omega\epsilon(y) \ , \ \sigma''(y) = 2\omega\delta(y)$$

$$\hat{m}(y) = \tilde{m}\epsilon(y) \ , \ \hat{m}'(y) = 2\tilde{m}\delta(y)$$

$$e^{-2\omega|y|}[(\omega + 2\tilde{m})\delta(y) - \frac{3}{4}\omega^2$$
$$+2\omega\epsilon(y)\partial_y - \tilde{m}\omega + \tilde{m}^2 - \partial_y{}^2]\tilde{\xi}_n = m_n{}^2\tilde{\xi}_n$$

Bessel Diff. Eq.

Solution for y>0

$$\tilde{\xi}_n(y) = \frac{1}{(\omega z)^{3/2}}\xi_n(z) = z\{J_\nu(m_n z) + c_n N_\nu(m_n z)\}$$
$$\nu = |\frac{\tilde{m}}{\omega} - \frac{1}{2}| \ , \ z \equiv \frac{1}{\omega}e^{\omega y} \ , \ c_n = -\frac{J_\nu(m_n/\omega)}{N_\nu(m_n/\omega)}$$

3. Bulk Higgs Mechanism and Fermion Localization

$$\sqrt{-\hat{g}}(\mathcal{L}^{grav}+\mathcal{L}^{S})\ ,\ \mathcal{L}^{grav}=\frac{-1}{2G_5}\hat{R}\ ,\ \mathcal{L}^{S}=-\frac{1}{2}\nabla_m\Phi\nabla^m\Phi-\underline{V(\Phi)}$$

Higgs(Bulk Scalar) Higgs Potential

$$ds^2=\mathrm{e}^{-2\sigma(y)}\eta_{ab}dx^a dx^b+dy^2$$

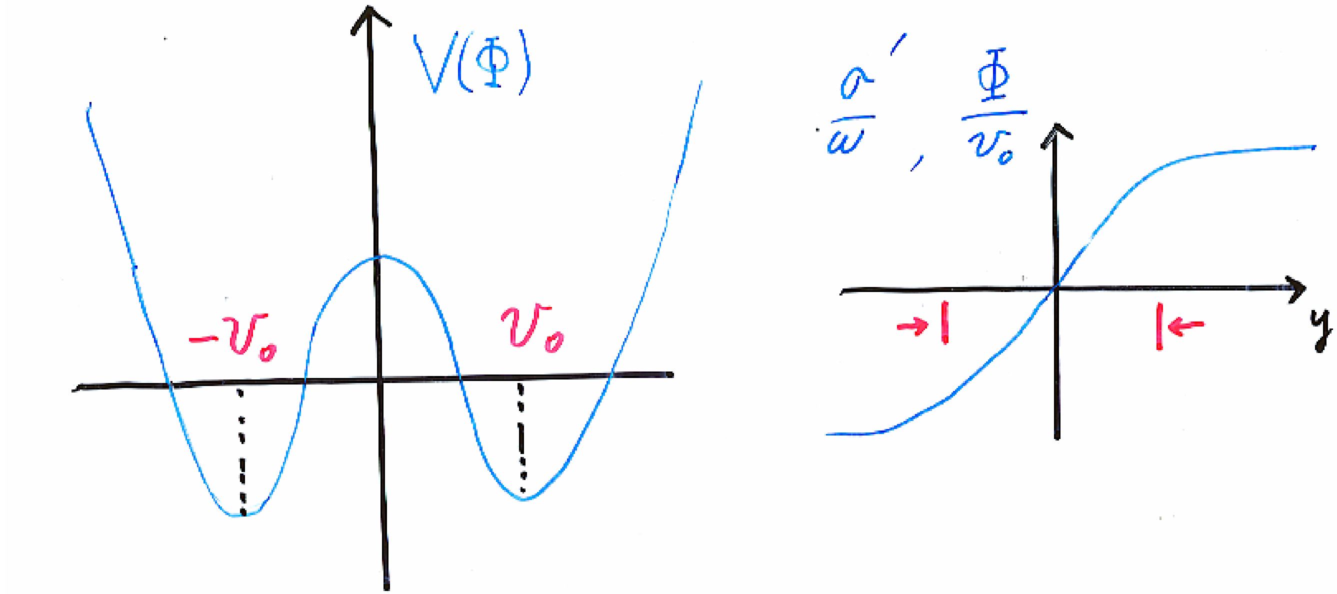
CQG18(01)421 S.I.
CQG18(01)5239 S.I.

Asymptotic behavior

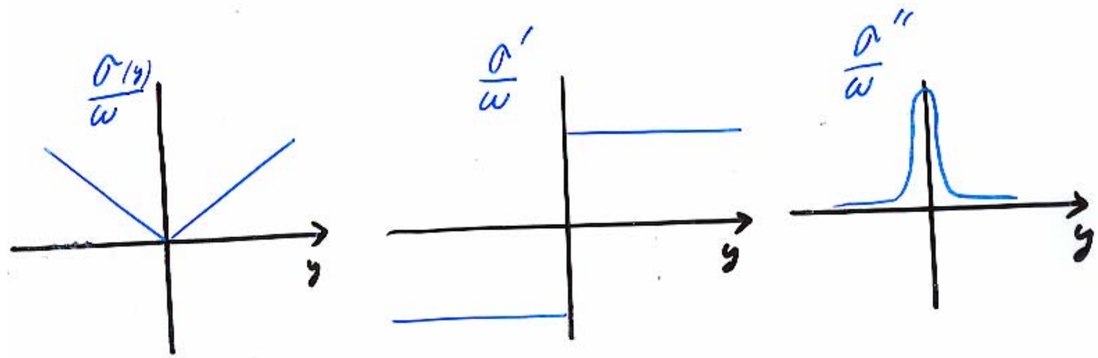
$$\sigma'(y)=\begin{cases}+\omega, & y\to+\infty\\ -\omega, & y\to-\infty\end{cases}\quad ,\quad \Phi(y)=\begin{cases}+v_0, & y\to+\infty\\ -v_0, & y\to-\infty\end{cases}$$

Near the origin

$$\sigma'(y)=\omega\tanh(y)\ ,\ \Phi(y)=v_0\tanh(y)$$



Thin Wall Limit



$$\sigma'(y) \to \omega\theta(y) \ , \ \Phi(y) \to v_0\theta(y)$$

$$\sqrt{-\hat{g}}\mathcal{L} = \sqrt{-\hat{g}}(\mathcal{L}^{Dirac} + \mathcal{L}^{Y}) \ , \ \mathcal{L}^{Y} = ig_Y\bar{\hat{\psi}}\hat{\psi}\Phi$$

Yukawa Int.

$$ie^{\sigma}\{\gamma^a\partial_a - 2e^{-\sigma}(\sigma' - \frac{1}{2}\partial_y)\gamma^5 + g_Y e^{-\sigma}\Phi\}\hat{\psi} = 0$$

Right-chirality zero mode

$$\hat{\psi}(x,y) = \psi_R^0(x)\eta(y) \ , \ \gamma^5\psi_R^0 = +\psi_R^0 \ , \ \gamma^a\partial_a\psi_R^0 = 0$$

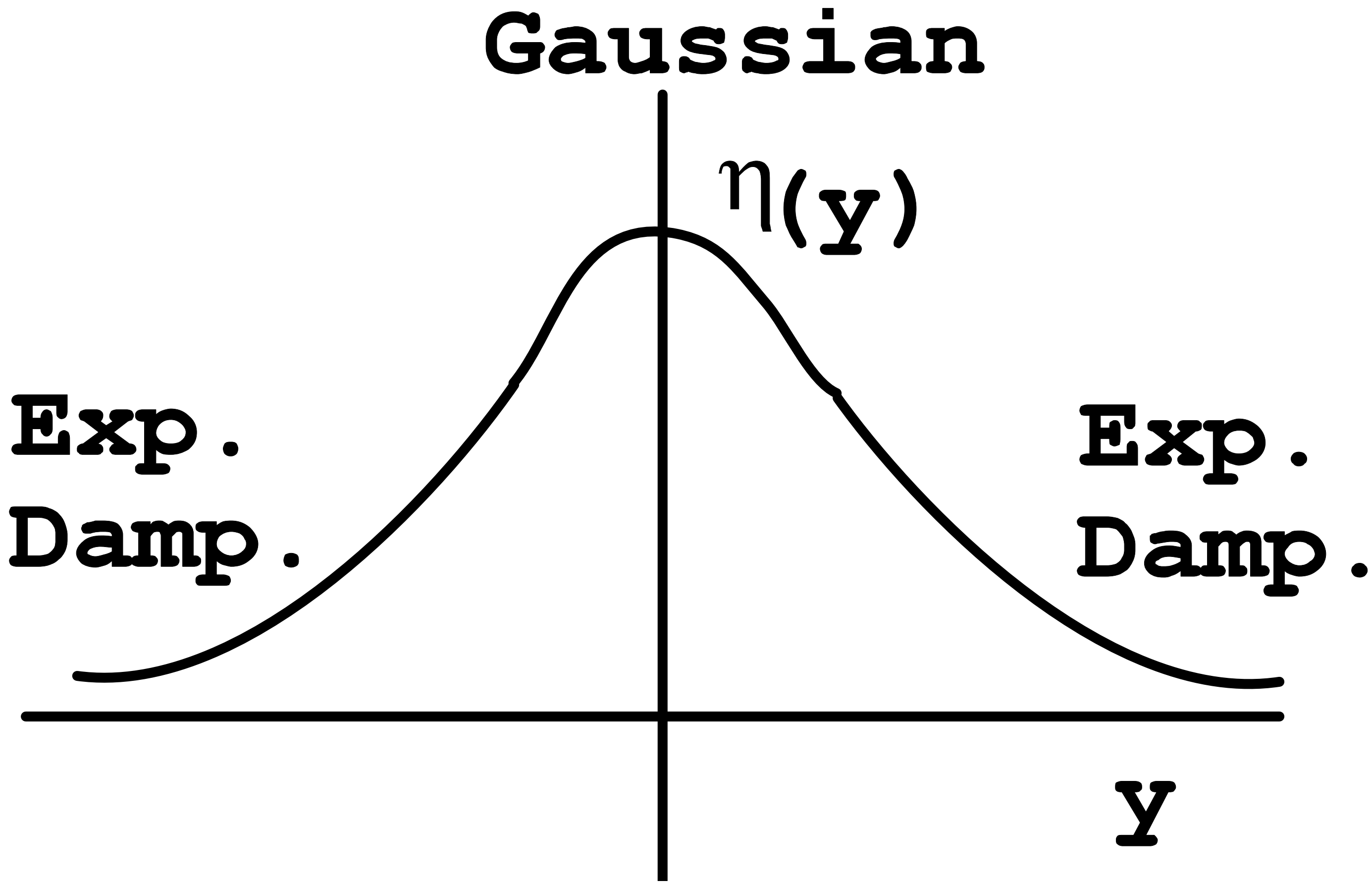
$$\partial_y\eta = (2\sigma' + g_Y\Phi(y))\eta$$

symptotic region

$$\eta(y) = \mathrm{const} \times e^{-(g_Y v_0 - 2\omega)|y|}$$

near the origin

$$\eta(y) = \mathrm{const} \times e^{-\frac{k}{2}(g_Y v_0 - 2\omega)y^2}$$


Gaussian
η(y)
Exp.
Damp.
Exp.
Damp.
y

4. Bulk Quantum Effect

$$\sqrt{-\hat{g}}(\mathcal{L}^{Dirac}+\mathcal{L}^{QED}+\mathcal{L}^{Y})$$

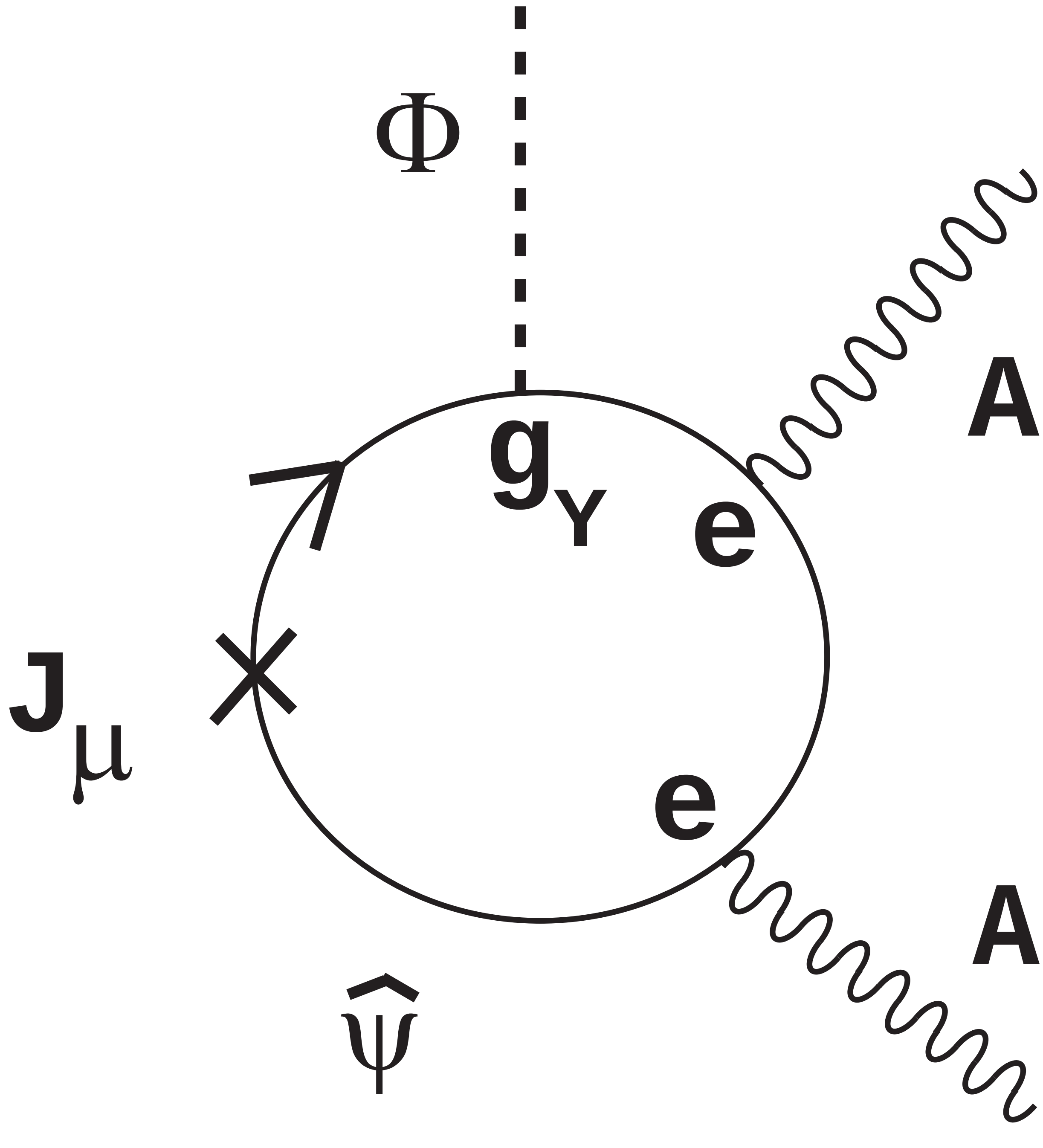
$$=\sqrt{-\hat{g}}\left[i\bar{\hat{\psi}}\left\{\gamma^{\mu}\hat{e}_{\mu}{}^{m}(\partial_m+ieA_m)+\frac{1}{8}(\hat{\omega}^{\sigma})_{\mu\nu}\gamma_{\sigma}[\gamma^{\mu},\gamma^{\nu}]\right\}\hat{\psi}+ig_Y\bar{\hat{\psi}}\hat{\psi}\Phi\right]$$

Induced Effective Action Method

Example 1

$$\frac{\delta S^{(1)}_{eff}}{\delta A^{\mu}(X)}\equiv< J_{\mu} >\sim e^2 g_Y \epsilon_{\mu\nu\lambda\sigma\tau}\Phi F^{\nu\lambda}F^{\sigma\tau}$$

$$S^{(1)}_{eff}\sim e^2 g_Y\int d^5X\epsilon_{\mu\nu\lambda\sigma\tau}\Phi A^{\mu}F^{\nu\lambda}F^{\sigma\tau}$$


Φ
A
g_Y
e
J_μ
e
A
$\hat{\psi}$

Variation for U(1) transformatoion $\delta A^{\mu} = \partial^{\mu}\Lambda$

thin wall limit

$$\delta_{\Lambda} S^{(1)}_{eff} \sim e^2 g_Y v_0 \int d^5X \epsilon_{\mu\nu\lambda\sigma\tau}\epsilon(y)\partial^{\mu}\Lambda F^{\nu\lambda}F^{\sigma\tau}$$

$$= e^2 g_Y v_0 \int d^5X \{\partial^{\mu}(\epsilon_{\mu\nu\lambda\sigma\tau}\epsilon(y)\Lambda F^{\nu\lambda}F^{\sigma\tau}) - \epsilon_{5\nu\lambda\sigma\tau}\delta(y)\Lambda F^{\nu\lambda}F^{\sigma\tau}\}$$

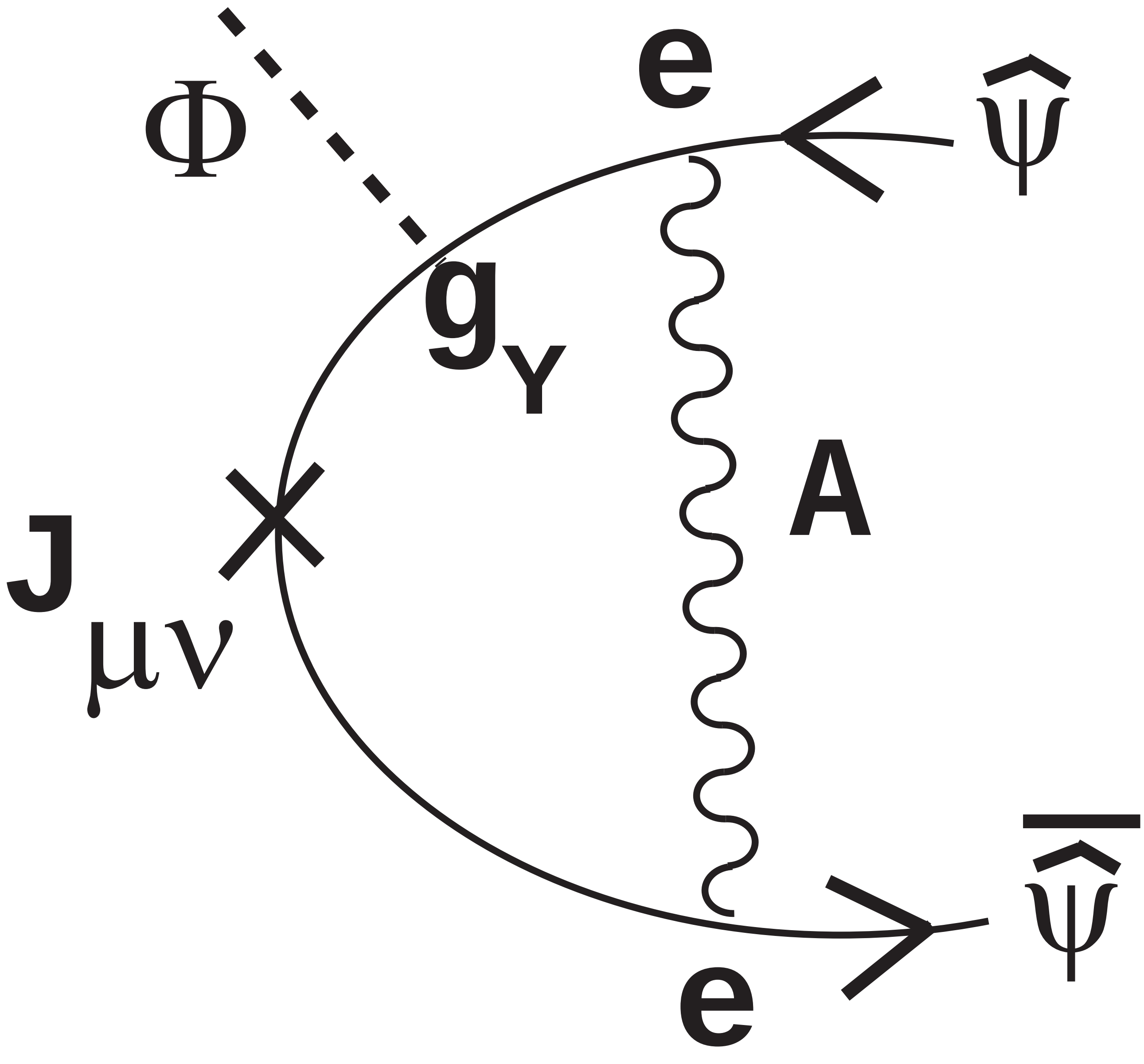
Chiral Anomaly

$$= -e^2 g_Y v_0 \int d^4x \Lambda(x) F^{\alpha\beta}\tilde{F}_{\alpha\beta}$$

’85 Callan & Harvey

Example 2

$$\frac{\delta S^{(2)}_{eff}}{\delta F^{\mu\nu}} \equiv < J_{\mu\nu} > \sim e^2 g_Y \epsilon_{\mu\nu\lambda\sigma\tau}\partial^{\lambda}\Phi \bar{\hat{\psi}}\Sigma^{\sigma\tau}\hat{\psi}$$


Φ
e
$\hat{\psi}$
g_Y
A
$J_{\mu\nu}$
e
$\bar{\hat{\psi}}$

$$S_{eff}^{(2)} \sim e^2 g_Y \epsilon_{\mu\nu\lambda\sigma\tau} \int d^5 X \partial^\lambda \Phi \, F^{\mu\nu} \bar{\hat{\psi}} \Sigma^{\sigma\tau} \hat{\psi}$$
$$= e^2 g_Y v_0 \epsilon_{\alpha\beta\gamma\delta} \int d^4 x F^{\alpha\beta} \bar{\psi} \sigma^{\gamma\delta} \psi$$
$$= -ie^2 g_Y v_0 \int d^4 x F^{\alpha\beta} \bar{\psi} \gamma^5 \sigma_{\alpha\beta} \psi$$

EMD

5. Conclusion

We have shown that Kaluza-Klein and Randall-Sundrum Models give MDM and CP-violating term EDM.