from
the broken Friedberg-Lee symmetry
to
the broken mu-tau symmetry

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IHEP, Beijing

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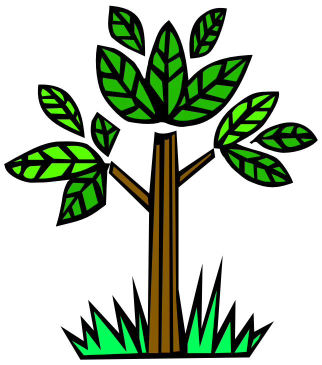
From 1997 to 2006

Outline

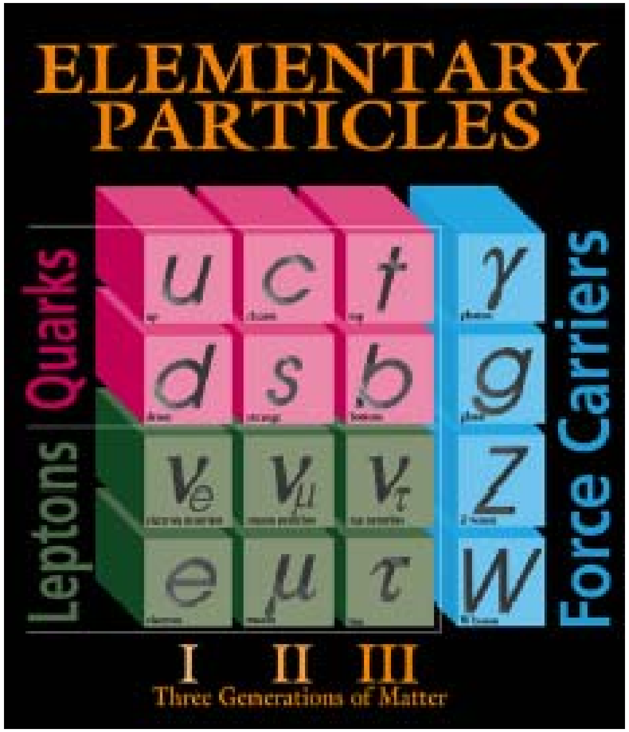
- Motivation: Masses, Flavors and Symmetry
- Mu-Tau Symmetry: Very Brief Comments
- Friedberg-Lee Model with a New Symmetry
- Oblique Symmetry Breaking: CP-conserving
- Oblique Symmetry Breaking: CP-violating
- Realization in the Minimal Seesaw Model
- Concluding Remarks: Neutrino Telescopes

Motivation



Fundamental Blocks

point

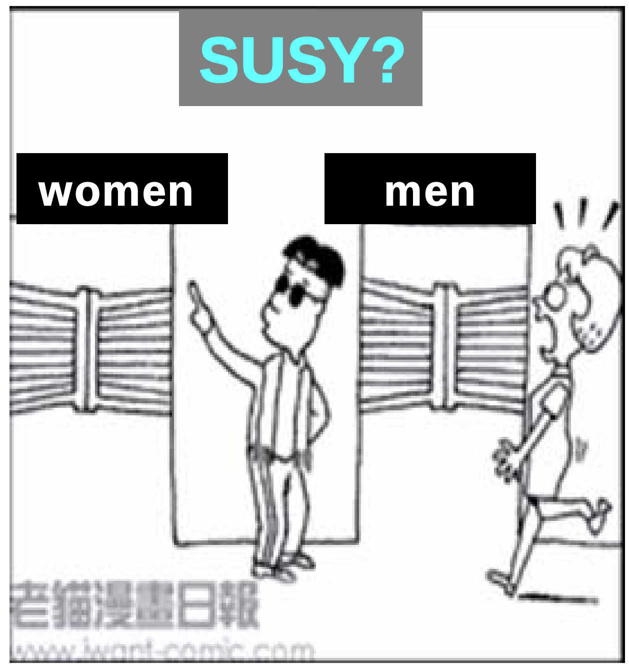


string

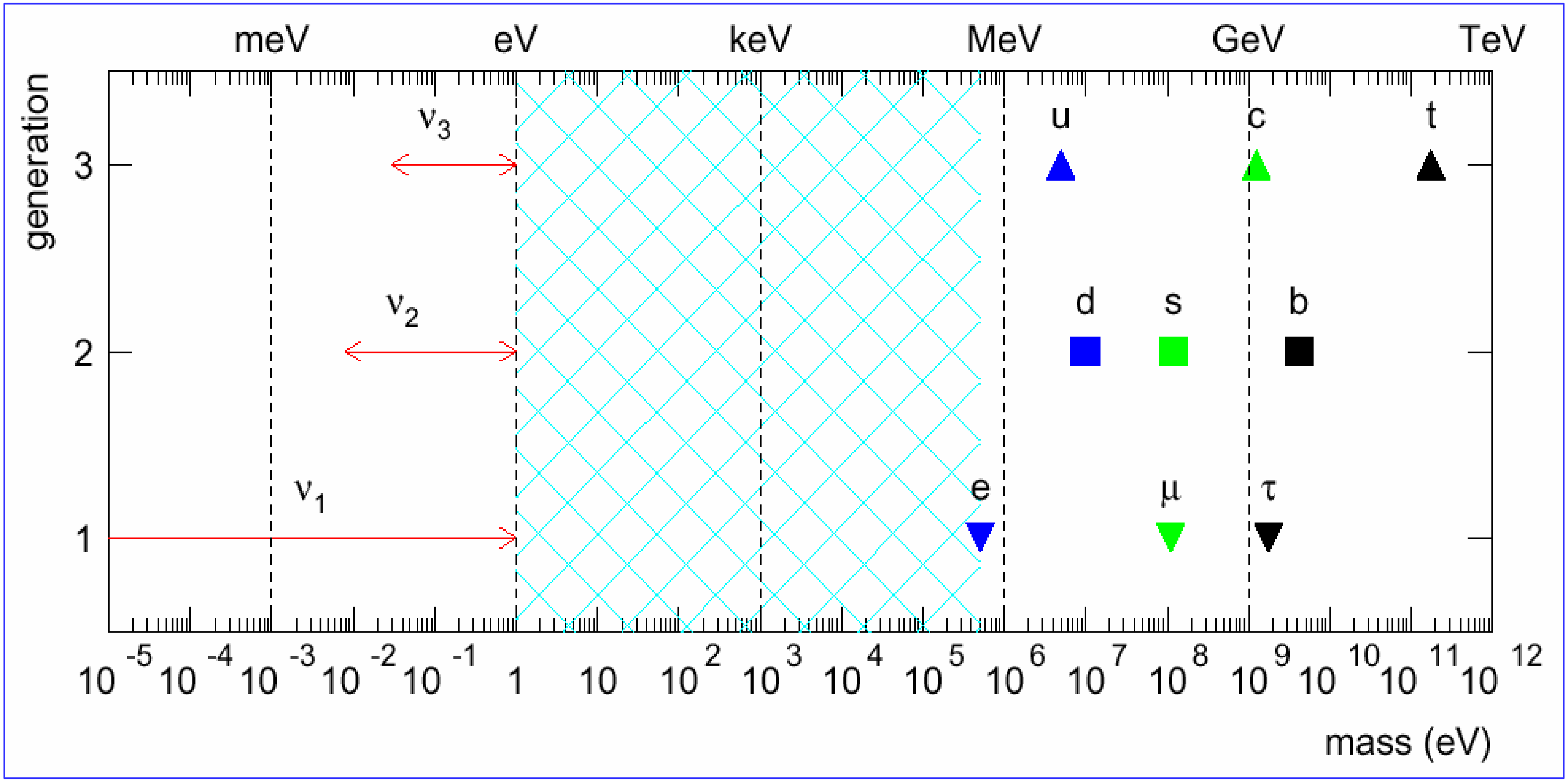
Origin of Masses

Where is the Higgs particle? (LHC)

$$L_{SM} = L(f,G) + L(f,H) + L(G,H) + L(G) - V(H)$$



Fermion Masses



a strange spectrum!

Who Ordered That?

What distinguishes different generations of leptons or quarks?

----- they have the same gauge quantum numbers,

yet they are quite different from one another.

Who ordered that?

Hidden flavor quantum numbers / flavor symmetries

Who Ordered This?

Koide Relation for Pole Masses of Charged Leptons:

$$Q_l^{\rm pole} \equiv \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}$$

$$-0.00001 \leq Q_l^{\rm pole} - 2/3 \leq +0.00002$$

Xing & Zhang, hep-ph/0611360 (PLB)

For example: **S(3)** **Koide, hep-ph/0612058**

Hidden flavor quantum numbers/flavor symmetries

Lesson

Guiding Principle

Flavor Symmetry

S_3 , S_4 , A_4 , Z_2 , $U(1)_F$, $SU(2)_F$, ……

Symmetry Breaking

Observed patterns of fermion masses and flavor mixing

Examples

(A)

$$V = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3} \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix}$$

Fritzsch, Xing 1996

(B)

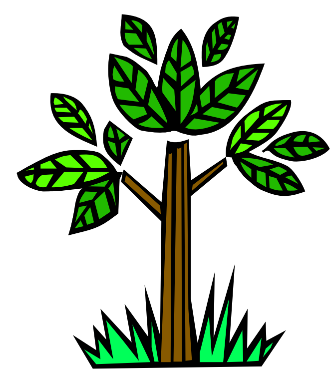
$$V = \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

Harrison, Perkins, Scott 2002

Zee's argument:

Simple numbers imply underlying flavor symmetries

Mu-Tau Symmetry



Mu-Tau Permutation

$$M_\nu = \frac{1}{2}\begin{pmatrix} C & D & D \\ D & A & B \\ D & B & A \end{pmatrix}$$

$$\mu \leftrightarrow \tau$$

Symmetry limit

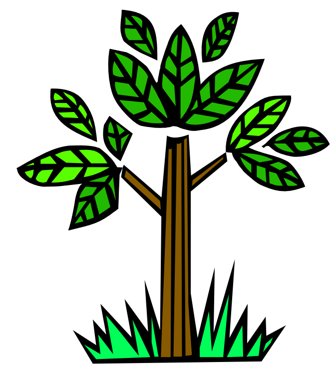
$$\theta_{23} = 45^\circ, \quad \theta_{13} = 0^\circ$$

Symmetry Breaking

$$V \Rightarrow \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \otimes \begin{cases} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & is \\ 0 & -is & c \end{bmatrix} \\ \text{or} \\ \begin{bmatrix} c & 0 & is \\ 0 & 1 & 0 \\ -is & 0 & c \end{bmatrix} \end{cases}$$

$$\begin{cases} \theta_{23} = 45^\circ \\ \theta_{13} \neq 0^\circ \\ \delta_{CP} = 90^\circ \end{cases}$$

F-L Model



T.D.'s 80th Birthday

hep-ph/0606071

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A Possible Relation between the Neutrino Mass Matrix and the Neutrino Mapping Matrix

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Abstract We explore the consequences of assuming a simple 3-parameter form, first without T-violation, for the neutrino mass matrix M in the basis ν_e, ν_μ, ν_τ with a new symmetry. This matrix determines the three neutrino masses m_1, m_2, m_3, as well as the mapping matrix U that diagonalizes M. Since U, without T-violation, yields three measurable parameters s_{12}, s_{23}, s_{13}, our form expresses six measurable quantities in terms of three parameters, with results in agreement with the experimental data. More precise measurements can give stringent tests of the model as well as determining the values of its three parameters. An extension incorporating T-violation is also discussed.

F-L Flavor Symmetry

The mass operator of Dirac neutrinos:

$$L_{\text{mass}} = a\left(\overline{\nu_\tau} - \overline{\nu_\mu}\right)\left(\nu_\tau - \nu_\mu\right) + b\left(\overline{\nu_\mu} - \overline{\nu_e}\right)\left(\nu_\mu - \nu_e\right) + c\left(\overline{\nu_e} - \overline{\nu_\tau}\right)\left(\nu_e - \nu_\tau\right)$$

Translational symmetry

$$\nu_\alpha \to \nu_\alpha + z$$

$\alpha = e, \mu, \tau$

The neutrino mass matrix:

$$L_{\text{mass}} = (\overline{\nu_e} \quad \overline{\nu_\mu} \quad \overline{\nu_\tau}) M_\nu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & c+a \end{pmatrix}$$

$\mathrm{Det}(M_\nu) \propto m_1 m_2 m_3 = 0$ **One neutrino massless!**

F-L Symmetry Breaking

Add a diagonal term---all masses are now non-zero:

$$L_{\text{mass}} = a\left(\overline{\nu_\tau} - \overline{\nu_\mu}\right)\left(\nu_\tau - \nu_\mu\right) + b\left(\overline{\nu_\mu} - \overline{\nu_e}\right)\left(\nu_\mu - \nu_e\right)$$
$$+ c\left(\overline{\nu_e} - \overline{\nu_\tau}\right)\left(\nu_e - \nu_\tau\right) + m_0\left(\overline{\nu_e}\nu_e + \overline{\nu_\mu}\nu_\mu + \overline{\nu_\tau}\nu_\tau\right)$$

$$L_{\text{mass}} = \begin{pmatrix} \overline{\nu_e} & \overline{\nu_\mu} & \overline{\nu_\tau} \end{pmatrix} M_\nu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

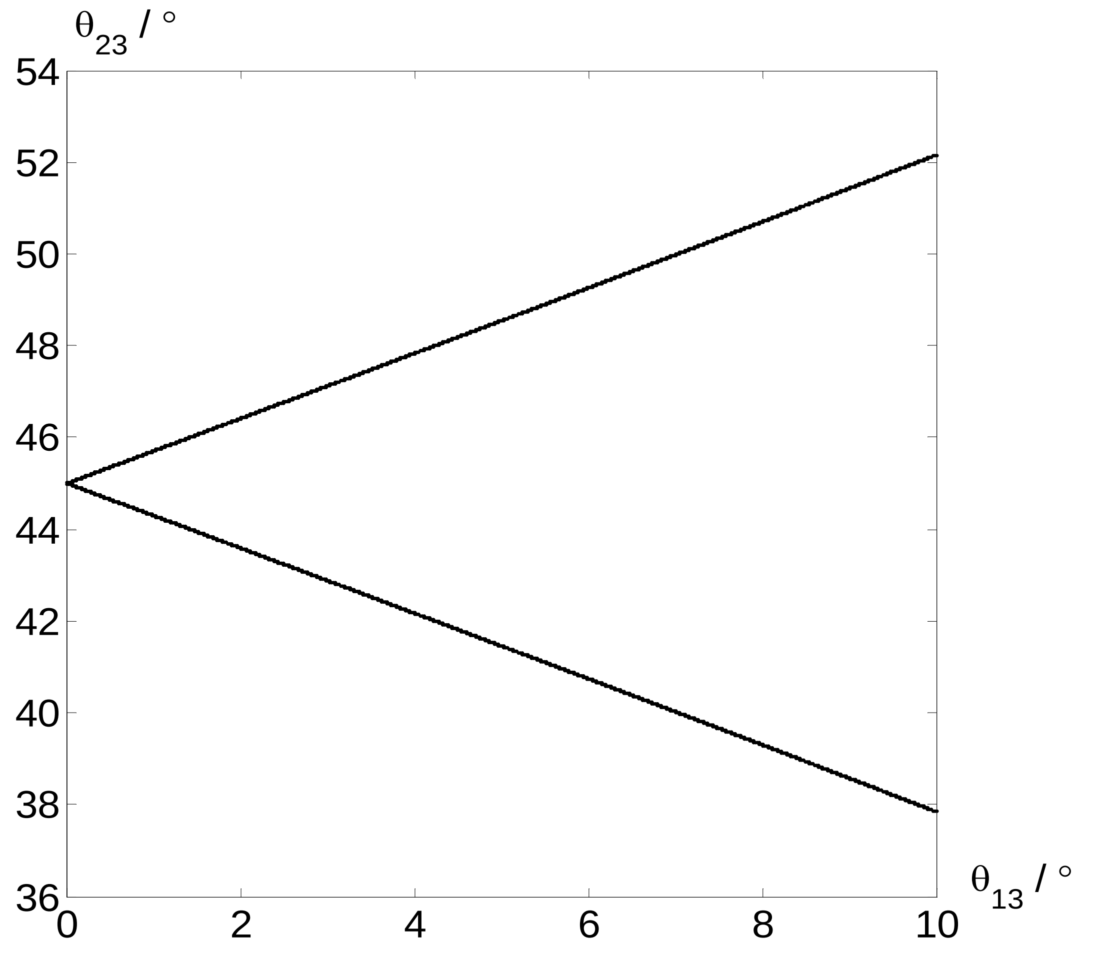
$$M_\nu = \begin{pmatrix} m_0 + b + c & -b & -c \\ -b & m_0 + a + b & -a \\ -c & -a & m_0 + c + a \end{pmatrix}$$

μ-τ symmetry breaking

$$\tan\theta = \frac{\sqrt{3}(c-b)}{(b+c)-2a}$$

$$V = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} & 0 & -\sin\frac{\theta}{2} \\ 0 & 1 & 0 \\ \sin\frac{\theta}{2} & 0 & \cos\frac{\theta}{2} \end{pmatrix}$$

Numerical Result



Comments

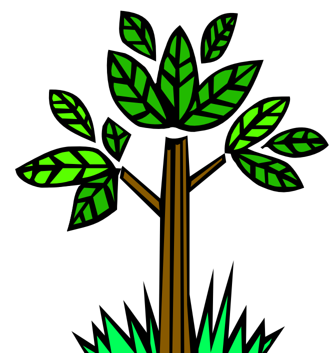
Straightforward extension of the F-L Model

Taking *a*, *b*, *c* or (and) *m*_0 to be complex:

CP violation

For example: Xing, Zhang & Zhou, hep-ph/0607091 (PLB)

Oblique SB (I)



Motivation

A nontrivial way to break the F-L symmetry:

★ the lightest neutrino remains massless

★ the observed MNS pattern is achievable

oblique

Luo & Xing, hep-ph/0611360

Oblique SB

Majorana neutrinos:

$$\nu_e \to \kappa^* \nu_e$$

$$\mathcal{L}'_{\rm mass} = \frac{1}{2}\Big[a(\overline{\nu_{\tau L}} - \overline{\nu_{\mu L}})(\nu^c_{\tau L} - \nu^c_{\mu L}) + b(\overline{\nu_{\mu L}} - \kappa\overline{\nu_{eL}})(\nu^c_{\mu L} - \kappa\nu^c_{eL}) + c(\kappa\overline{\nu_{eL}} - \overline{\nu_{\tau L}})(\kappa\nu^c_{eL} - \nu^c_{\tau L})\Big] + {\rm h.c.}$$

Mass matrix:

$$M'_\nu = \begin{pmatrix} \kappa^2(b+c) & -\kappa b & -\kappa c \\ -\kappa b & a+b & -a \\ -\kappa c & -a & a+c \end{pmatrix}$$

$$\mathrm{Det}(M'_\nu) = \kappa^2 \mathrm{Det}(M_\nu) = 0$$

This conclusion is true even for $\nu_\alpha \to \kappa^*_\alpha \nu_\alpha$

Masses and Mixing

$$m_2 = a + \frac{1}{2}(b+c)\left(|\kappa|^2 + 1\right) - \frac{1}{2}\sqrt{\left[2a - (b+c)|\kappa|^2\right]^2 + (b-c)^2\left(2|\kappa|^2+1\right)}$$

$$m_3 = a + \frac{1}{2}(b+c)\left(|\kappa|^2 + 1\right) + \frac{1}{2}\sqrt{\left[2a - (b+c)|\kappa|^2\right]^2 + (b-c)^2\left(2|\kappa|^2+1\right)}$$

$$V = \begin{pmatrix} \frac{1}{\sqrt{2|\kappa|^2+1}} & \frac{\sqrt{2}\,\kappa\cos\theta}{\sqrt{2|\kappa|^2+1}} & \frac{\sqrt{2}\,\kappa\sin\theta}{\sqrt{2|\kappa|^2+1}} \\ \frac{\kappa^*}{\sqrt{2|\kappa|^2+1}} & -\frac{1}{\sqrt{2}}\left(\frac{\cos\theta}{\sqrt{2|\kappa|^2+1}} + \sin\theta\right) & \frac{1}{\sqrt{2}}\left(\cos\theta - \frac{\sin\theta}{\sqrt{2|\kappa|^2+1}}\right) \\ \frac{\kappa^*}{\sqrt{2|\kappa|^2+1}} & -\frac{1}{\sqrt{2}}\left(\frac{\cos\theta}{\sqrt{2|\kappa|^2+1}} - \sin\theta\right) & -\frac{1}{\sqrt{2}}\left(\cos\theta + \frac{\sin\theta}{\sqrt{2|\kappa|^2+1}}\right) \end{pmatrix}$$

$$\tan 2\theta = \frac{(b-c)\sqrt{2|\kappa|^2+1}}{(b+c)|\kappa|^2 - 2a}$$

(*a, b, c* are all real)

***b* = *c* is of μ-τ symmetry**

Predictions

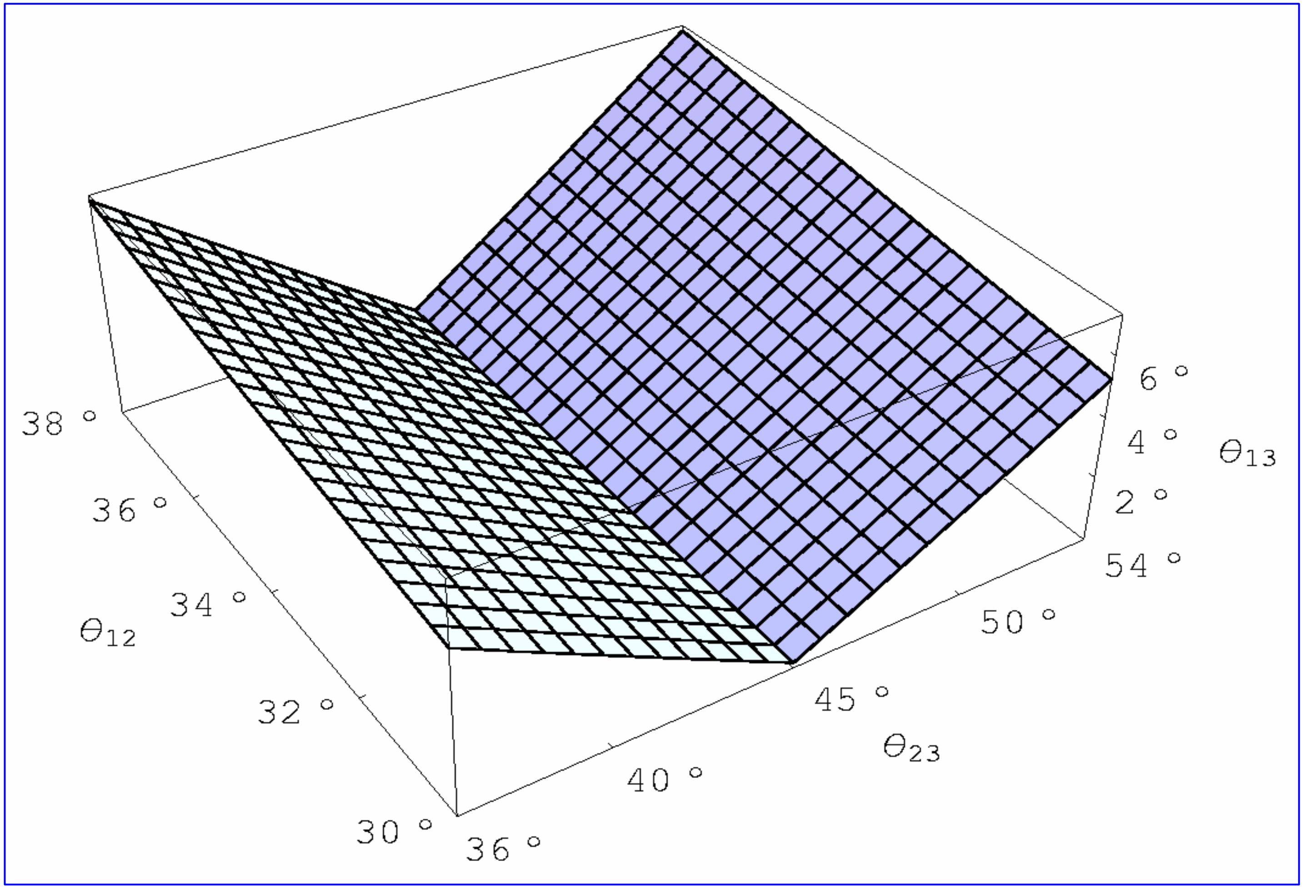
Current neutrino oscillation data:
(Strumia & Vissani, hep-ph/0606054)

Oscillation parameter	central value			99% CL range
solar mass splitting	Δm^2_{12}	$=$	$(8.0 \pm 0.3)\, 10^{-5}\,\mathrm{eV}^2$	$(7.2 \div 8.9)\, 10^{-5}\,\mathrm{eV}^2$
atmospheric mass splitting	$\lvert\Delta m^2_{23}\rvert$	$=$	$(2.5 \pm 0.2)\, 10^{-3}\,\mathrm{eV}^2$	$(2.1 \div 3.1)\, 10^{-3}\,\mathrm{eV}^2$
solar mixing angle	$\tan^2\theta_{12}$	$=$	0.45 ± 0.05	$30^\circ < \theta_{12} < 38^\circ$
atmospheric mixing angle	$\sin^2 2\theta_{23}$	$=$	1.02 ± 0.04	$36^\circ < \theta_{23} < 54^\circ$
'CHOOZ' mixing angle	$\sin^2 2\theta_{13}$	$=$	0 ± 0.05	$\theta_{13} < 10^\circ$

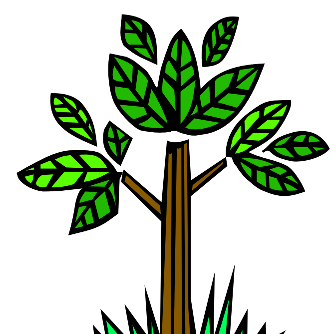
$|\kappa| = 1/2$ Tri-Bimax

$$\theta_{13} \leq 7.1^\circ$$

Angle Correlation



Oblique SB (II)



CP Violation

Majorana neutrinos: $\nu_e \rightarrow \kappa^* \nu_e$

$$\mathcal{L}'_{\rm mass} = \frac{1}{2}\left[a(\overline{\nu_{\tau L}} - \overline{\nu_{\mu L}})(\nu^{\rm c}_{\tau L} - \nu^{\rm c}_{\mu L}) + b(\overline{\nu_{\mu L}} - \kappa\overline{\nu_{e L}})(\nu^{\rm c}_{\mu L} - \kappa\nu^{\rm c}_{e L}) + c(\kappa\overline{\nu_{e L}} - \overline{\nu_{\tau L}})(\kappa\nu^{\rm c}_{e L} - \nu^{\rm c}_{\tau L})\right] + {\rm h.c.}$$

A simple example: ***a*** **real and** ***b*** **=** ***c*****^* complex**

soft μ-τ symmetry breaking

A CP-violating phase is from $b - c = 2i{\rm Im}(b)$

Flavor Mixing

$$V = \begin{pmatrix} \frac{1}{\sqrt{2|\kappa|^2+1}} & i\frac{\sqrt{2}\,\kappa\cos\theta}{\sqrt{2|\kappa|^2+1}} & i\frac{\sqrt{2}\,\kappa\sin\theta}{\sqrt{2|\kappa|^2+1}} \\ \frac{\kappa^*}{\sqrt{2|\kappa|^2+1}} & -\frac{1}{\sqrt{2}}\left(i\frac{\cos\theta}{\sqrt{2|\kappa|^2+1}}+\sin\theta\right) & \frac{1}{\sqrt{2}}\left(\cos\theta - i\frac{\sin\theta}{\sqrt{2|\kappa|^2+1}}\right) \\ \frac{\kappa^*}{\sqrt{2|\kappa|^2+1}} & -\frac{1}{\sqrt{2}}\left(i\frac{\cos\theta}{\sqrt{2|\kappa|^2+1}}-\sin\theta\right) & -\frac{1}{\sqrt{2}}\left(\cos\theta + i\frac{\sin\theta}{\sqrt{2|\kappa|^2+1}}\right) \end{pmatrix}$$

$$\tan 2\theta = -\mathrm{Im}(b)\sqrt{2|\kappa|^2+1}/\left[a+\mathrm{Re}(b)\left(|\kappa|^2+1\right)\right]$$

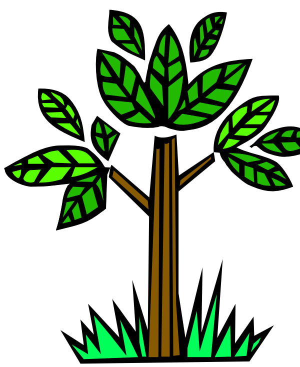
CP-violating phase:

$$\mathcal{J} = |\kappa|^2|\sin 2\theta|/[2(2|\kappa|^2+1)^{3/2}] \qquad \mathcal{J} \lesssim 0.041$$

Mixing angles:

$$\sin\theta_{13} = \sqrt{2}|\kappa||\sin\theta|/\sqrt{2|\kappa|^2+1} \quad \text{and} \quad \tan\theta_{12} = \sqrt{2}|\kappa|\cos\theta$$

Seesaw Scenario



The Minimal Seesaw

SM

+

two heavy right-handed Majorana neutrinos

Frampton, Glashow & Yanagida 2002
Endoh, Kaneko, Kang, Morozumi & Tanimoto 2002
……
Guo, Xing & Zhou, hep-ph/0612033 ----- A Review

$$-\mathcal{L}_{\rm MSM} = \frac{1}{2}\overline{(\nu_{\rm L}, \ N^{\rm c}_{\rm R})} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M^T_{\rm D} & M_{\rm R} \end{pmatrix} \begin{pmatrix} \nu^{\rm c}_{\rm L} \\ N_{\rm R} \end{pmatrix} + {\rm h.c.}$$

$$M'_\nu = M_{\rm D} M^{-1}_{\rm R} M^T_{\rm D}$$

$$m_1 = 0 \ (\text{or } m_3 = 0) \text{ is guaranteed}$$

A Simple Realization

$$M_{\rm D} = \Lambda_{\rm D} \begin{pmatrix} \kappa & 0 \\ -1 & -1 \\ 0 & 1 \end{pmatrix} ,$$

$$M_{\rm R} = \frac{\Lambda_{\rm D}^2}{ab + bc + ca} \begin{pmatrix} a + c & c \\ c & b + c \end{pmatrix} ,$$

$$M'_\nu = M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T$$

$$M'_\nu = \begin{pmatrix} \kappa^2 (b + c) & -\kappa b & -\kappa c \\ -\kappa b & a + b & -a \\ -\kappa c & -a & a + c \end{pmatrix}$$

There are, of course, many other possibilities for sale

Illustration

If *a*, *b* and *c* are real,

$$M_1 = \frac{a+b+2c-\sqrt{(a-b)^2+4c^2}}{2\,(ab+bc+ca)}\,\Lambda_{\rm D}^2$$

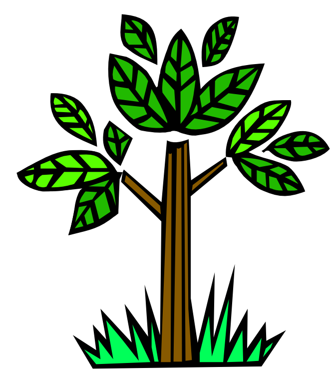
$$M_2 = \frac{a+b+2c+\sqrt{(a-b)^2+4c^2}}{2\,(ab+bc+ca)}\,\Lambda_{\rm D}^2$$

$\Lambda_{\rm D} \sim 174$ GeV | $a \sim 0.022$ eV | $b \sim c \sim 0.006$ eV

$M_1 \sim 1 \times 10^{15}$ GeV and $M_2 \sim 3 \times 10^{15}$ GeV

with CP violation, Leptogenesis is possible!

Conclusion



On F-L Symmetry

- F-L symmetry is a NEW flavor symmetry
- It can be extended to the quark sector
- It has a lot of implications on neutrinos
- Physics behind this symmetry is unclear

Symmetry is a guideline of model building

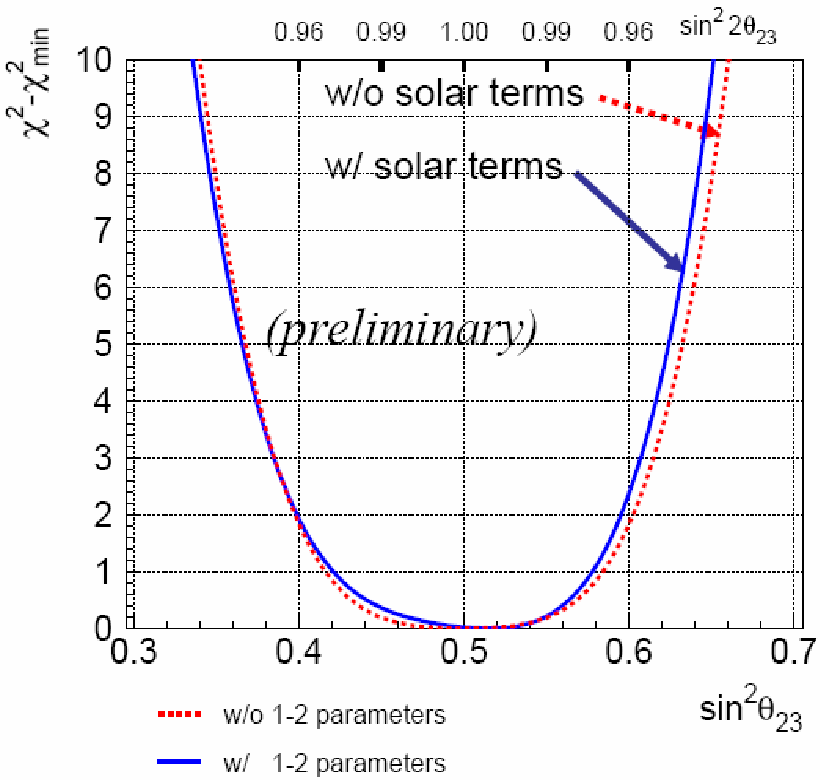
But there are sooooooo many possibilities

How to identify the unique / correct one?

On μ-τ Symmetry

Constraint on $\sin^2\theta_{23}$ with and without the solar terms

T. Kajita / ISVHECRI 06



Solar terms off :

best-fit : $\sin^2\theta_{23} = 0.50$

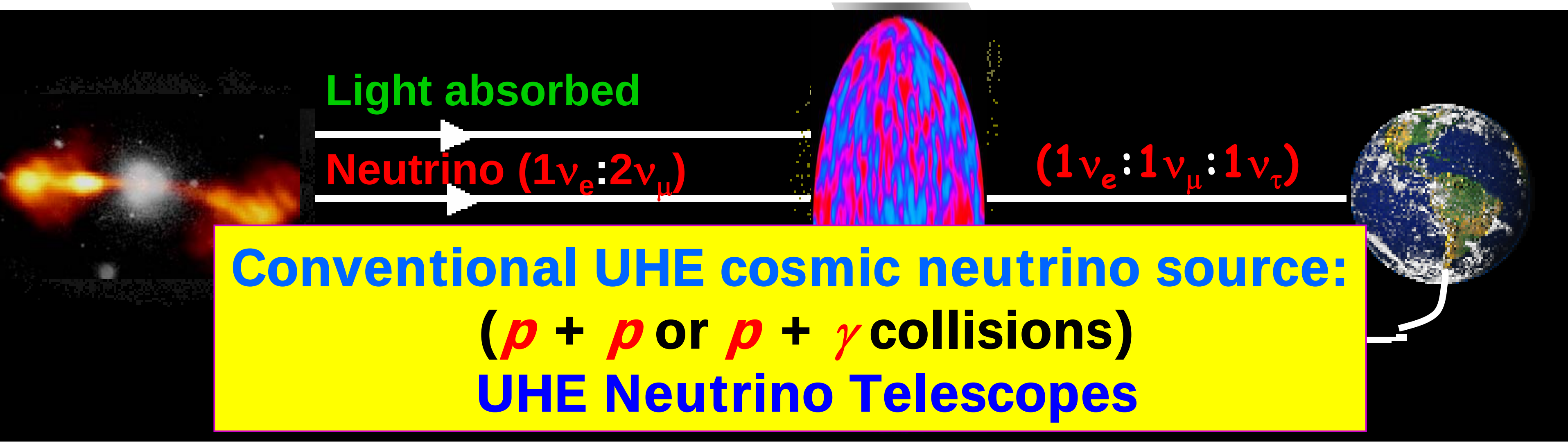
Solar terms on :

best-fit : $\sin^2\theta_{23} = 0.52$

($\sin^2 2\theta_{23} = 0.9984$)

Still (almost) maximum mixing is most favored.

A Signal of μ-τ SB



$$\{\phi_e:\phi_\mu:\phi_\tau\}=\{1:2:0\} \;\Rightarrow\; \{\phi_e:\phi_\mu:\phi_\tau\}=\{(1-2\Delta):(1+\Delta):(1+\Delta)\}$$

$$\Delta = \frac{1}{4}\left(2\varepsilon\sin^2 2\theta_{12} - \epsilon\sin 4\theta_{12}\cos\delta\right)$$

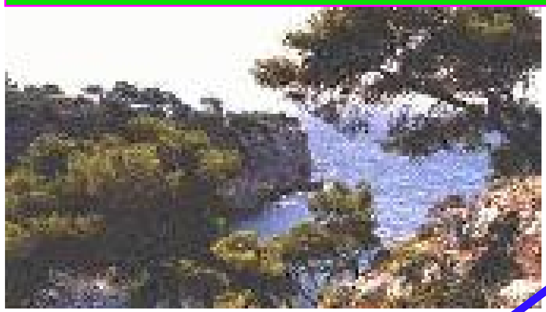
$$\varepsilon \equiv \theta_{23} - \frac{\pi}{4}$$

$$\epsilon \equiv \theta_{13}$$

Xing, hep-ph/0605219 (PRD)

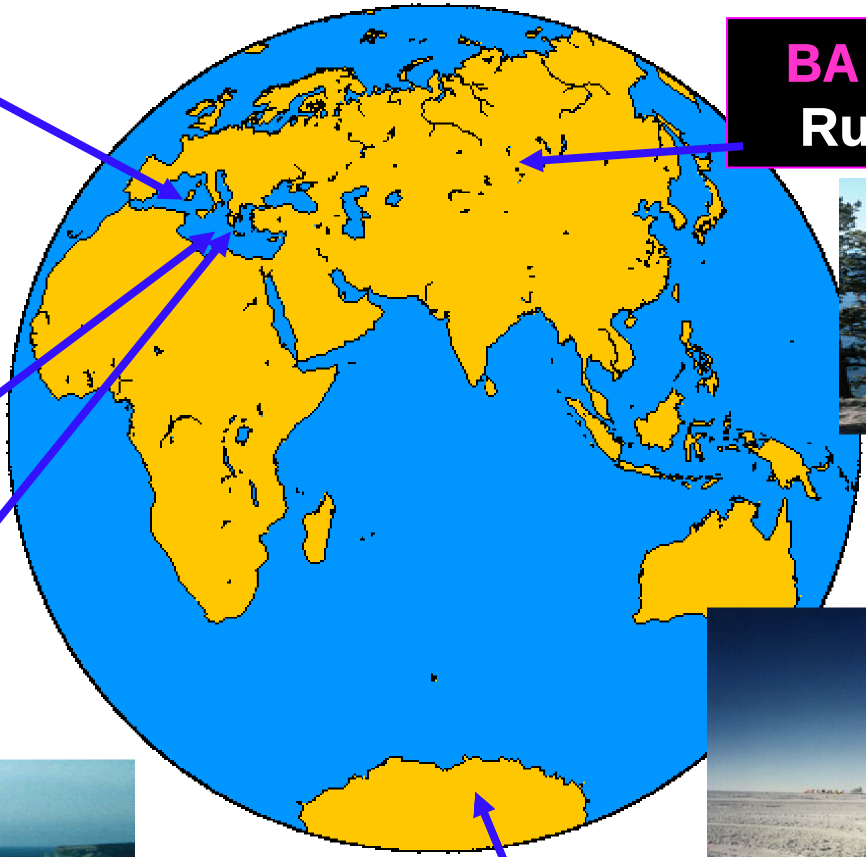
Neutrino Telescopes

ANTARES
La-Seyne-sur-Mer, France

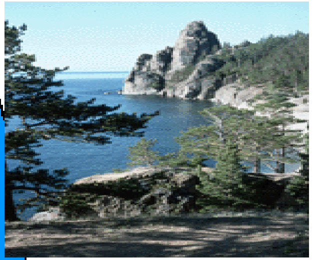


NEMO
Catania, Italy

NESTOR
Pylos, Greece



BAIKAL
Russia



AMANDA and IceCube
South Pole, Antarctica

From Water to Ice

The South Pole

Hunting for θ_13

Muss es sein --- 非如此不可?

Es muss sein --- 非如此不可!

Thank You

Yoshio

Best Wishes