Flavor Symmetry and Vacuum Aligned Mass Textures

arXiv:hep−ph/0609220

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Plan of this talk

1, Introduction **VEV=0**

2, S_3 invariant mass matrix on SUSY

$$M_D = \left(\begin{array}{cc|c} aH_1 & bH_S + cH_A & dH_2 \\ bH_S - cH_A & aH_2 & dH_1 \\ \hline eH_2 & eH_1 & fH_S \end{array}\right) \qquad M_R = \left(\begin{array}{cc|c} 0 & M & 0 \\ M & 0 & 0 \\ \hline 0 & 0 & M' \end{array}\right)$$

3, S_3 invariant Higgs scalar potential analysis

v_{uS}	v_{dS}	v_{uA}	v_{dA}	v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0	ø	ø	0	0	ø	ø

4, Higgs mass spectrum
and S_3 soft breaking in B-term

5,$K^0 - \bar{K}^0$ mixing

6, Summary

1, Introduction

Texture-zeros in quark–lepton mass matrix are successful to predict masses and mixings.

$$M_\nu = \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

However,

the origin of zero is not clear!

Discrete flavor symmetry approach

W. Grimus, A.S. Joshipura, L. Lavoura and M. Tanimoto,
Eur. Phys. J. C**36**, 227 (2004) [arXiv:hep-ph/0405016]

Points of our model :

Dynamical realization of Texture-zeros
in Discrete flavor symmetry approach

Mass matrix $M_{ij} = \underline{y_{ij}}\ \underline{\langle H \rangle}$

previous model

In order to derive Texture-zeros, certain Yukawa couplings are forbidden by discrete symmetry.

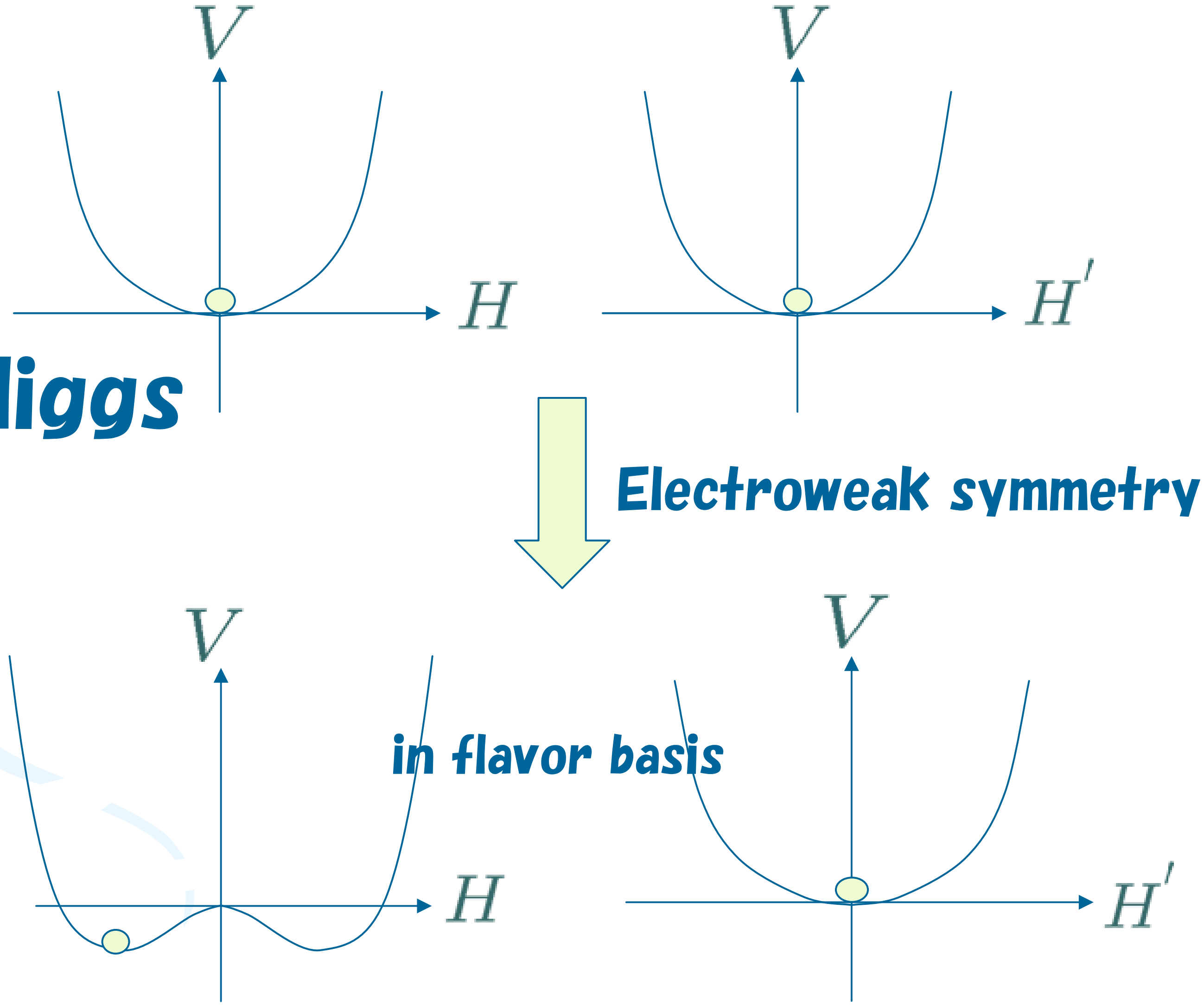
our model

In order to derive Texture-zeros, we consider

some of VEVs=0

in flavor basis.

We want to discuss whether texture-zeros can be realized as VEV = 0 or not!!



Multi-Higgs

Electroweak symmetry breaking

in flavor basis

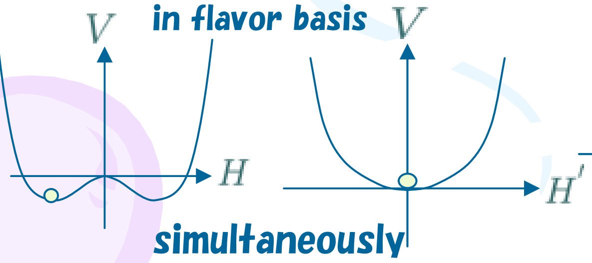
simultaneously

When we consider S_3 as discrete flavor symmetry, Texture-zeros can be derived in flavor basis dynamically??

S_3

- permutations of three objects
- the smallest group of non commutative discrete groups.

irreducible representation $\underline{2}$ $\underline{1}_S$ $\underline{1}_A$



Higgs : **All representations** of S_3 are taken into account.

Plan of this talk

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$$M_D = \left(\begin{array}{cc|c} aH_1 & bH_S + cH_A & dH_2 \\ bH_S - cH_A & aH_2 & dH_1 \\ \hline eH_2 & eH_1 & fH_S \end{array}\right) \qquad M_R = \left(\begin{array}{cc|c} 0 & M & 0 \\ M & 0 & 0 \\ \hline 0 & 0 & M' \end{array}\right)$$

3, S_3 invariant Higgs scalar potential analysis

v_{uS}	v_{dS}	v_{uA}	v_{dA}	v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0	Ø	Ø	0	0	Ø	Ø

4, Higgs mass spectrum
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5,$K^0 - \bar{K}^0$ mixing

6, Summary

2, S_3 invariant mass matrix on SUSY

N. Haba and K. Yoshioka,
Nucl. Phys. B **739**, 254 (2006),
hep-ph/0511108

Tensor product of S_3 doublet

$\phi, \quad \psi$: S_3 doublet (complex representation)

$$\phi^c \times \psi = \underset{\underline{2}}{(\phi_2\psi_2, \ \phi_1\psi_1)^T} + \underset{\underline{1}_A}{(\phi_1\psi_2 - \phi_2\psi_1)} + \underset{\underline{1}_S}{(\phi_1\psi_2 + \phi_2\psi_1)}$$

$\phi^c \equiv \sigma_1 \phi^*$: S_3 doublet

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

chiral superfield : Φ Ψ

ϕ_R , ψ_L : fermion

$\underline{3}$ (= $\underline{2} + \underline{1}_S$)

Third generation
(We reconfigure this
assignment later.)

Higgs

$\underline{2}$: (H_1, H_2)

$\underline{1}_S$: H_S

$\underline{1}_A$: H_A

Supersymmetric Dirac mass matrix

$$M_D = \left(\begin{array}{cc|c} aH_1 & bH_S + cH_A & dH_2 \\ bH_S - cH_A & aH_2 & dH_1 \\ \hline eH_2 & eH_1 & fH_S \end{array}\right)$$

$$W = \Phi M_D \Psi$$

Supersymmetric Majorana mass matrix

$$M_R = \left(\begin{array}{cc|c} 0 & M & 0 \\ M & 0 & 0 \\ \hline 0 & 0 & M' \end{array}\right)$$

$$W = \Phi M_R \Phi$$

▶

detail

For example: If VEV of H_1 remains zero ,

$$M_D = \left(\begin{array}{cc|c} 0 & bH_S + cH_A & dH_2 \\ bH_S - cH_A & aH_2 & 0 \\ \hline eH_2 & 0 & fH_S \end{array}\right).$$

However it is non trivial that VEV of H_1 = 0 is realized in the Higgs potential.

Texture-zeros is realized from Electroweak symmetry breaking patterns!

Plan of this talk

1, Introduction **VEV=0**

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$$M_D = \left(\begin{array}{cc|c} aH_1 & bH_S + cH_A & dH_2 \\ bH_S - cH_A & aH_2 & dH_1 \\ \hline eH_2 & eH_1 & fH_S \end{array}\right) \qquad M_R = \left(\begin{array}{cc|c} 0 & M & 0 \\ M & 0 & 0 \\ \hline 0 & 0 & M' \end{array}\right)$$

③, S_3 invariant Higgs scalar potential analysis

v_{uS}	v_{dS}	v_{uA}	v_{dA}	v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0	ø	ø	0	0	ø	ø

4, Higgs mass spectrum
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3, Potential analysis

In our model, we consider the following eight Higgses.

superfields

$\widehat{H}_{uS},\ \widehat{H}_{dS}$: S_3 symmetric singlet $\longrightarrow S$

$\widehat{H}_{uA},\ \widehat{H}_{dA}$: S_3 antisymmetric singlet $\longrightarrow A$

$\widehat{H}_{u1},\ \widehat{H}_{d1},\ \widehat{H}_{u2},\ \widehat{H}_{d2}$: S_3 doublet $\longrightarrow D$

hat : superfield

All the Higgses are gauge doublet.

Equations at vacuum

Lagrangian

$$S\left(\left(|\mu_S|^2 + m_{uS}^2\right) v_{uS} = \boxed{b_S} v_{dS} - X v_{uS}, \qquad (u \leftrightarrow d). \right.$$

$$A\left(\left(|\mu_A|^2 + m_{uA}^2\right) v_{uA} = \boxed{b_A} v_{dA} - X v_{uA}, \qquad (u \leftrightarrow d). \right.$$

$$D\left(\begin{aligned} \left(|\mu_D|^2 + m_{uD}^2\right) v_{u1} &= \boxed{b_D} v_{d2} - X v_{u1}, \qquad (u \leftrightarrow d \text{ and } 1 \leftrightarrow 2). \\ \left(|\mu_D|^2 + m_{uD}^2\right) v_{u2} &= \boxed{b_D} v_{d1} - X v_{u2}, \qquad (u \leftrightarrow d \text{ and } 1 \leftrightarrow 2). \end{aligned} \right.$$

$$v_{uS(dS)} \equiv \left|\left\langle H^0_{uS(dS)} \right\rangle\right|, \; v_{uA(dA)} \equiv \left|\left\langle H^0_{uA(dA)} \right\rangle\right|, \; v_{u1(d1)} \equiv \left|\left\langle H^0_{u1(d1)} \right\rangle\right|, \; v_{u2(d2)} \equiv \left|\left\langle H^0_{u2(d2)} \right\rangle\right|$$

$$X \equiv \frac{g_Y^2 + g_2^2}{4} \left\{ (v_{uS}^2 - v_{dS}^2) + (v_{uA}^2 - v_{dA}^2) + (v_{u1}^2 - v_{d2}^2) + (v_{u2}^2 - v_{d1}^2) \right\}$$

We can separate equations into three parts, symmetric singlet, antisymmetric single and doublet in terms of vanishing VEVs.

4.1, Classification of vacua about S_3 symmetric singlet H_S H_A

S_3 symmetric singlet

○ $v_{uS} = v_{dS} = 0$

This solution always exists as long as an instability condition at the origin of the other representations is satisfied.

$$\left(|\mu_S|^2 + m_{uS}^2\right) v_{uS} \;=\; b_S v_{dS} - X v_{uS}, \quad \left(|\mu_S|^2 + m_{dS}^2\right) v_{dS} \;=\; b_S v_{uS} + X v_{dS}$$

○ $v_{uS} = 0, \;\; v_{dS} \neq 0$ or $v_{uS} \neq 0, \;\; v_{dS} = 0$ ▶

$b_S = 0$ is necessary for this solution to exist.

However it is difficult to satisfy this condition at weak scale exactly. So we do not adopt this solution.

For antisymmetric singlet $S \rightarrow A$

4,2 Classification of vacua about S_3 doublet

Because b_D is common to v_{u1}, v_{d2} and v_{u2}, v_{d1},
it is necessary to discuss four VEVs simultaneously. ▶

Independent candidates of VEV=0 are below. ▶

b_D dependence

 # all of VEVs are zero

$$v_{u1}=0,\ \ v_{d2}=0,\ \ v_{u2}=0,\ \ v_{d1}=0$$

 # three of VEVs are zero

~~$v_{u1}=0,\ \ v_{d2}=0,\ \ v_{u2}=0,\ \ v_{d1}\neq 0$~~ $b_D=0$

 # two of VEVs are zero

$$v_{u1}=0,\ \ v_{d2}=0,\ \ v_{u2}\neq 0,\ \ v_{d1}\neq 0$$

~~$v_{u1}=0,\ \ v_{d2}\neq 0,\ \ v_{u2}=0,\ \ v_{d1}\neq 0$~~ $b_D=0$

~~$v_{u1}=0,\ \ v_{d2}\neq 0,\ \ v_{u2}\neq 0,\ \ v_{d1}=0$~~ $b_D=0$

▶

 # one of VEVs is zero

~~$v_{u1}\neq 0,\ \ v_{d2}\neq 0,\ \ v_{u2}\neq 0,\ \ v_{d1}=0$~~ $b_D=0$

 # all of VEVs are non zero

$$v_{u1}\neq 0,\ \ v_{d2}\neq 0,\ \ v_{u2}\neq 0,\ \ v_{d1}\neq 0$$

Summary of potential analysis

symmetric singlet

- $* \ v_{uS} = v_{dS} = 0$
- $* \ v_{uS} \neq 0, \ v_{dS} \neq 0$

antisymmetric singlet

- $* \ v_{uA} = v_{dA} = 0$
- $* \ v_{uA} \neq 0, \ v_{dA} \neq 0$

doublet

v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0	0	0
0	0		
		0	0

Consequently, there are 14 patterns in terms of Texture-zeros.

$$: (\underset{S}{\textcircled{2}} \times \underset{A}{\textcircled{2}} \times \underset{D}{\textcircled{4}}) - \textcircled{2}$$

All the VEVs are zero.

All the VEVs are non zero.

global minimum

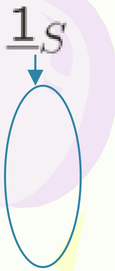
As results of potential analysis, there is an interesting VEV pattern in terms of Texture-zeros.

v_{uS}	v_{dS}	v_{uA}	v_{dA}	v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0	Ø	Ø	0	0	Ø	Ø

$$M_u = \begin{array}{cccc} \underline{1}_S & & & \\ 0 & b_u & 0 & \leftarrow \underline{1}_S \\ d_u & 0 & f_u & \\ 0 & -f_u & i_u & \end{array} \qquad M_d = \begin{array}{cccc} & & \underline{1}_S & \\ 0 & b_d & 0 & \leftarrow \underline{1}_S \\ d_d & e_d & 0 & \\ -e_d & 0 & i_d & \end{array} \qquad M_e = \begin{array}{cccc} \underline{1}_S & & & \\ 0 & d_d & 3e_d & \\ b_d & -3e_d & 0 & \\ 0 & 0 & i_d & \leftarrow \underline{1}_S \end{array}$$

This assignment is SU(5) grand unification like.

$$M_D = \begin{array}{cccc} & & \underline{1}_S & \\ 0 & b_\nu & c_\nu & \\ -b_\nu & e_\nu & 0 & \\ g_\nu & 0 & 0 & \leftarrow \underline{1}_S \end{array} \qquad M_R = \begin{array}{cc|cc} & & \underline{1}_S & \\ 0 & M & 0 & \\ M & 0 & 0 & \\ \hline 0 & 0 & M' & \leftarrow \underline{1}_S \end{array}$$



This means S_3 assignment of $\underline{1}_S$.

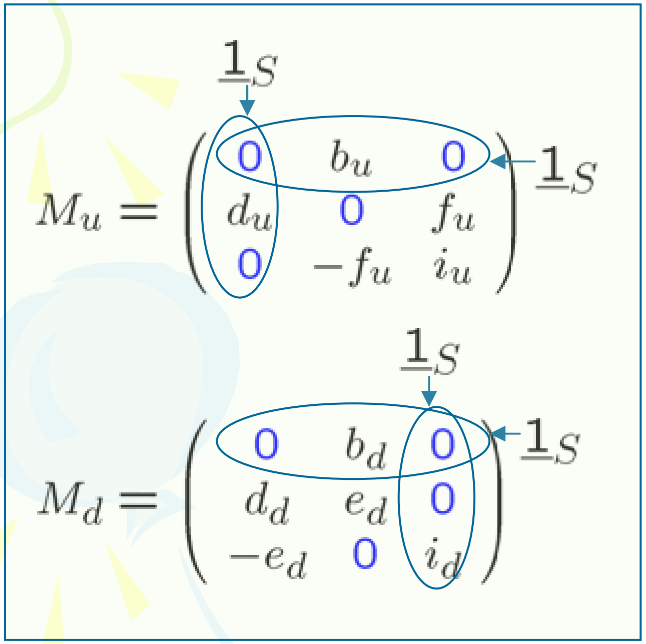
see-saw machanism

$$M_\nu = \frac{b_\nu^2}{M} \begin{array}{cccc} & & \underline{1}_S & \\ -z & 1 & -x & \\ 1 & 2y & -xy & \\ -x & -xy & 0 & \leftarrow \underline{1}_S \end{array}$$

$$x = \frac{g_\nu}{b_\nu}, \qquad y = \frac{e_\nu}{b_\nu}, \qquad z = \frac{c_\nu^2}{b_\nu^2}\frac{M}{M'},$$

$$y,\ z \ll 0$$

Cabibbo Kobayashi Maskawa (CKM) matrix



$$M_u = \begin{pmatrix} 0 & b_u & 0 \\ d_u & 0 & f_u \\ 0 & -f_u & i_u \end{pmatrix}$$

$$M_d = \begin{pmatrix} 0 & b_d & 0 \\ d_d & e_d & 0 \\ -e_d & 0 & i_d \end{pmatrix}$$

$$m_u = -\frac{b_u d_u i_u}{f_u^2} \qquad m_c = \frac{f_u^2}{i_u} \qquad m_t = i_u$$

$$m_d = -\frac{b_d d_d}{e_d} \qquad m_s = e_d \qquad m_b = i_d$$

$$V_{us} = -\frac{b_u i_u}{f_u^2} + \frac{b_d}{e_d} \qquad V_{cb} = -\frac{f_u}{i_u} \qquad V_{ub} = \frac{b_u}{f_u}$$

Prediction

$$V_{cb} = -\sqrt{\frac{m_c}{m_t}}$$

Experimental value
0.039 – 0.044

Theoretical value
0.05 – 0.06

Although it is necessary that V_{cb} is more than experimental bound a bit, there is a possibility that this prediction is satisfied by taking SUSY threshold corrections into account.

Maki Nakagawa Sakata(MNS) matrix

Charged lepton sector

$$M_e = \begin{pmatrix} 0 & d_d & 3e_d \\ b_d & -3e_d & 0 \\ 0 & 0 & i_d \end{pmatrix}$$

Derived from M_d^T
replacing $e_d \to -3e_d$

Georgi-Jarlskog parameter

$$U_e \simeq \begin{pmatrix} 1 & \frac{d_d}{3e_d} & 3\frac{e_d}{i_d} \\ -\frac{d_d}{3e_d} & 1 & -\frac{d_d}{i_d} \\ -3\frac{e_d}{i_e} & 0 & 1 \end{pmatrix}$$

Neutrino sector

$$M_\nu = \frac{b_\nu^2}{M} \begin{pmatrix} -z & 1 & -x \\ 1 & 2y & -xy \\ -x & -xy & 0 \end{pmatrix}$$

$x = \frac{g_\nu}{b_\nu}$, $y = \frac{e_\nu}{b_\nu}$, $z = \frac{c_\nu^2}{b_\nu^2}\frac{M}{M'}$,

$y,\ z \ll 0$

Prediction

$$U_{e2} \simeq \sin\theta_{12}^\nu - \frac{d_d}{3e_d}\cos\theta_{12}^\nu\cos\theta_{23}^\nu + \frac{3e_d}{i_d}\cos\theta_{12}^\nu\sin\theta_{23}^\nu$$

$$U_{\mu 3} \simeq \sin\theta_{23}^\nu + \frac{d_d}{3e_d}\sin\theta_{13}^\nu$$

$$\theta_{13}^\nu \ll 1$$

$$U_{e3} \simeq \sin\theta_{13}^\nu - \frac{d_d}{3e_d}\sin\theta_{12}^\nu - \frac{3e_d}{i_d}\cos\theta_{23}^\nu$$

lower bound of our model

$$U_{e3} \geq 0.04$$

This will be tested in the double CHOOZ experiment.

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3, S_3 invariant Higgs scalar potential analysis

v_{uS}	v_{dS}	v_{uA}	v_{dA}	v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0	ø	ø	0	0	ø	ø

4, Higgs mass spectrum
and S_3 soft breaking in B-term

5,$K^0 - \bar{K}^0$ mixing

6, Summary

4, Higgs mass spectrum and S_3 soft breaking in B-term

S_3 potential has the following global symmetries.

	H_{uS}	H_{dS}	H_{uA}	H_{dA}	H_{u1}	H_{d2}	H_{u2}	H_{d1}
$U(1)_1$	+1	-1	0	0	0	0	0	0
$U(1)_2$	0	0	+1	-1	0	0	0	0

$SU(2)_g$

$SU(2)_g$ doublet :

$(H_{u1}, H_{u2})^T \quad (-H_{d1}, H_{d2})^T$

$$
\begin{aligned}
V_{\rm soft} \;=\; & m^2_{uS} H^\dagger_{uS} H_{uS} + m^2_{dS} H^\dagger_{dS} H_{dS} + [b_S H_{uS} H_{dS} + h.c.] \\
& + m^2_{uA} H^\dagger_{uA} H_{uA} + m^2_{dA} H^\dagger_{dA} H_{dA} + [b_A H_{uA} H_{dA} + h.c.] \\
& + m^2_{uD} (H^\dagger_{u1} H_{u1} + H^\dagger_{u2} H_{u2}) + m^2_{dD} (H^\dagger_{d1} H_{d1} + H^\dagger_{d2} H_{d2}) \\
& + [b_D (H_{u1} H_{d2} + H_{u2} H_{d1}) + h.c.]
\end{aligned}
$$

We consider S_3 soft breaking terms which do not break phenomenological characters of exact S_3 model.

S_3 soft breaking terms in B-term

$$\begin{aligned} V_{\not S_3} &= b_{SD1} H_{uS} H_{d2} + b_{SD2} H_{u1} H_{dS} \\ &\quad + b_{AD1} H_{uA} H_{d1} + b_{AD2} H_{u2} H_{dA} + h.c. \end{aligned}$$

These S_3 soft breaking terms simultaneously satisfy the followings.

- No massless Higgs when taking the vev combination below.
- We can take the VEV combination below as global minimum with no parameter conditions.

v_{uS}	v_{dS}	v_{uA}	v_{dA}	v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0	$\not 0$	$\not 0$	0	0	$\not 0$	$\not 0$

- Flavor breaking effects in supersymmetry-breaking holomorphic mass terms do not propagate to other sectors.

Higgs mass spectrum of S_3 soft breaking model

●The ligtest Higgs mode : one neutral Higgs

$$h^0 = \frac{1}{v}(v_{uA}h^0_{uA} + v_{dA}bh^0_{dA} + v_{u2}h^0_{u2} + v_{d1}h^0_{d1})$$

$$v \equiv \sqrt{v_{uA}^2 + v_{dA}^2 + v_{u2}^2 + v_{d1}^2}$$

$h_i^0 (i = uA, dA, u2, d1)$: neutral Higgs in the flavor basis

The lightest Higgs has weak scale mass.

●Other neutral Higgs_masses are discussed at next section, K^0-$\bar{K}^0$ mixing.

Plan of this talk

1, Introduction **VEV=0**

2, S_3 invariant mass matrix on SUSY

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3, S_3 invariant Higgs scalar potential analysis

v_{uS}	v_{dS}	v_{uA}	v_{dA}	v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0	ø	ø	0	0	ø	ø

4, Higgs mass spectrum

and S_3 soft breaking in B-term

⑤, $K^0 - \overline{K}^0$ mixing

6, Summary

5, $K^0 - \bar{K}^0$ mixing

Tree level FCNC of S_3 soft breaking model ($K^0 - \bar{K}^0$ mixing)

$$\eta \equiv \frac{(y_d^S)_{22} b_d v_{d1}}{m_b^2} - \frac{(y_d^S)_{13} v_{d1} d_d}{m_s m_b}$$

$$\frac{1}{\bar{M}_1^2} \equiv \frac{1}{2}\left(\frac{1}{M_{H_1^0}^2} + \frac{1}{M_{H_4^0}^2}\right)$$

$$\frac{1}{\bar{M}_2^2} \equiv \frac{1}{2}\left(\frac{1}{M_{H_2^0}^2} + \frac{1}{M_{H_3^0}^2}\right)$$

$$M_H^2 \equiv \bar{M}_1^2 \cong \bar{M}_2^2$$

$$\begin{aligned}\Delta m_K^{\mathrm{tree}} &= 2\mathrm{Re}\left\langle K^0 \left|\mathcal{H}_{\mathrm{eff}}\right| \bar{K}^0 \right\rangle \\ &\sim \frac{m_b^2 m_K f_K^2}{6 v_{d1}^2 M_H^2}\left[\left(\frac{m_K}{m_s + m_d}\right)^2 \left(\frac{m_s}{m_b}\right)^2 - \eta^2 \left\{\left(\frac{m_K}{m_s + m_d}\right)^2 + \frac{1}{2}\right\}\right].\end{aligned}$$

Heavy neutral Higgs masses are bounded from below so as to suppress the extra Higgs contribution compared with the standard model one. ▶

η =0 and 0.03 are taken as typical values.

$$M_H \gtrsim \begin{cases} 3.8\mathrm{TeV} & (\eta = 0) \\ 1.4\mathrm{TeV} & (\eta = 0.03) \end{cases}$$

except for the lightest Higgs

Reference:
F. Gabbiani, E. Gabrielli,
A. Masiero and L. Silvestrini,
Nucl. Phys. B **477**, 321 (1996)
[arXiv:hep-ph/9604387]

input parameters

$m_K = 490\mathrm{MeV}$, $f_K = 132\mathrm{MeV}$

$m_d = 8\mathrm{MeV}$, $m_s = 150\mathrm{MeV}$

$m_b = 4.5\mathrm{GeV}$, $\Delta m_K < 3.521 \times 10^{-12}\mathrm{MeV}$

$v_{d1} = 100\mathrm{GeV}$

limit for calculation

$$\left|\mu_{S(D)}\right|^2 + m_{uS(D)}^2 \simeq \left|\mu_{S(D)}\right|^2 + m_{dS(D)}^2 \simeq \bar{m}^2,$$

$$b_D \simeq b_A \simeq b_S \simeq \bar{b}, \qquad b_{SD1} \simeq b_{SD2} \simeq b_{AD1} \simeq b_{AD2} \simeq \bar{b}_{\not{g}_3},$$

$$\bar{m}^2,\ \bar{b},\ \bar{b}_{\not{g}_3} \gg v^2.$$

$$b_d,\ d_d\ :\ e_d\ :\ i_d \sim \lambda^3\ :\ \lambda^2\ :\ 1 \qquad \lambda \sim 0.22$$

6, Summary

- We considered some of VEVs=0 in flavor basis in order to derive Teture-zeros.

- As discrete flavor symmetry we considered S_3 and we analyzed S_3 invariant potential. Then Higgses for all S_3 representations are taken into account.

- We found 14 VEV patterns in which VEV = 0 are allowed without parameter conditions.

  Among 14 VEVs the most favored VEV pattern

v_{uS}	v_{dS}	v_{uA}	v_{dA}	v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0	ø	ø	0	0	ø	ø

  gives quark/lepton mass matrices, which are consistent with experimental data.

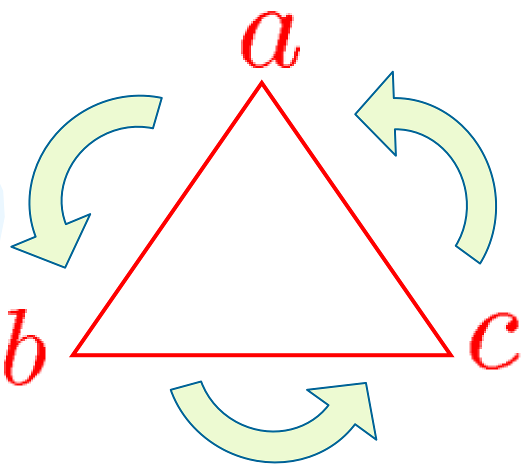
- We can construct the S_3 soft breaking model in B-term which has no massless Higgs and no parameter conditions.

- TeV scale typical Higgs mass is consistent with K^0-$\overline{K}^0$ mixing experimental bound.

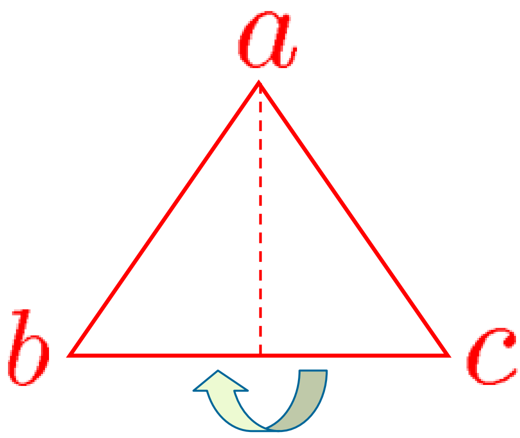
Appendix

S_3 : **permutations of three objects**

(a, b, c)



3 elements

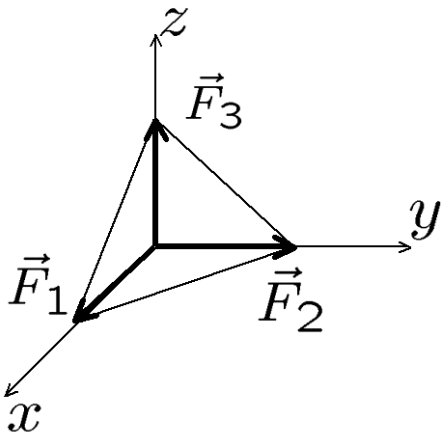


3 elements

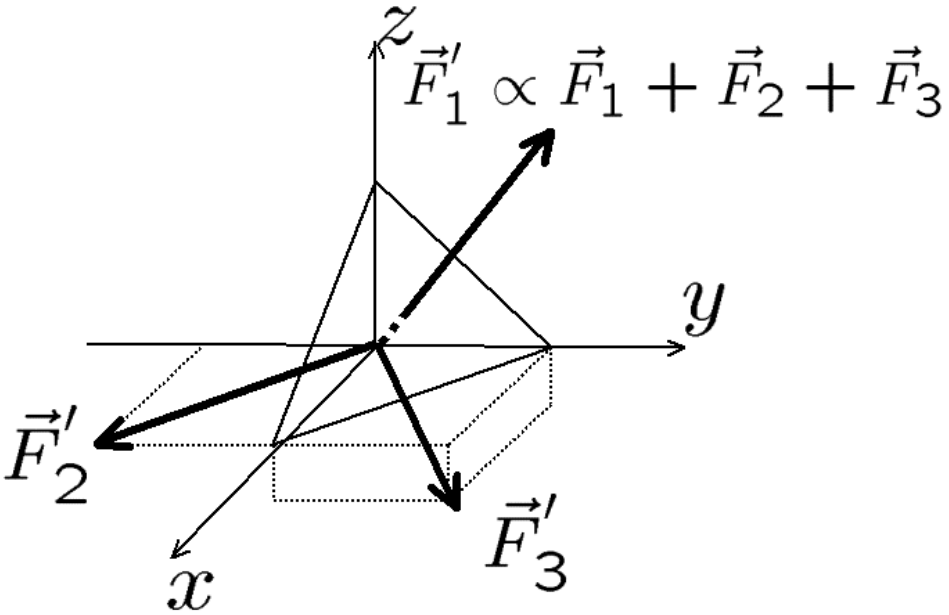
S_3 is the smallest group of non abelian finite groups.

S_3 irreducible representation

$$\underline{2} \quad \underline{1}_S \quad \underline{1}_A$$



$$\underline{3} = \underline{2} + \underline{1}_S$$



Supersymmetric S_3 mass matrix

invariant combination

$$\Phi, \quad H, \quad \Psi \quad : \quad \underline{2}$$

$$\Phi^c \times H \to (\Phi_2 H_2, \Phi_1 H_1)^T : \underline{2}$$

$$(\Phi_2 H_2, \Phi_1 H_1)\sigma_1 \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \Phi_1 H_1 \Psi_1 + \Phi_2 H_2 \Psi_2$$

$$M_D = \left(\begin{array}{cc|c} aH_1 & bH_S + cH_A & dH_2 \\ bH_S - cH_A & aH_2 & dH_1 \\ \hline eH_2 & eH_1 & fH_S \end{array} \right)$$

$$\phi^c \times \psi \;=\; \underset{\underline{2}}{(\phi_2\psi_2,\ \phi_1\psi_1)} \;+\; \underset{\underline{1}_A}{(\phi_1\psi_2 - \phi_2\psi_1)} \;+\; \underset{\underline{1}_S}{(\phi_1\psi_2 + \phi_2\psi_1)}$$

$$\phi^c \equiv \sigma_1 \phi^*$$

S_3 mass matrix on non-SUSY

$$M_D = \left(\begin{array}{cc|c} aH_S + a'H_S^* + bH_A + b'H_A^* & cH_2 + c'H_1^* & dH_1 + e'H_2^* \\ cH_1 + c'H_2^* & aH_S + a'H_S^* - bH_A - b'H_A^* & dH_2 + e'H_1^* \\ \hline eH_2 + d'H_1^* & eH_1 + d'H_2^* & fH_S + f'H_S^* \end{array} \right)$$

Lagrangian

$$\mathcal{L} \supset \int d^2\theta d^2\bar{\theta}\mathcal{K} + \left\{\frac{1}{4}\int d^2\theta(\mathcal{W}_{Y\alpha}\mathcal{W}_Y^{\alpha} + Tr\mathcal{W}_{2\alpha}\mathcal{W}_2^{\alpha}) + h.c.\right\}$$

$$+\left\{\int d^2\theta W + h.c.\right\}$$

$$\begin{aligned}
\mathcal{K} \;=\; & \hat{H}^{\dagger}_{uS}e^{\frac{1}{2}g_Y\hat{V}_Y+g_2\hat{V}_2}\hat{H}_{uS} + \hat{H}^{\dagger}_{dS}e^{-\frac{1}{2}g_Y\hat{V}_Y+g_2\hat{V}_2}\hat{H}_{dS} \\
& +\hat{H}^{\dagger}_{uA}e^{\frac{1}{2}g_Y\hat{V}_Y+g_2\hat{V}_2}\hat{H}_{uA} + \hat{H}^{\dagger}_{dA}e^{-\frac{1}{2}g_Y\hat{V}_Y+g_2\hat{V}_2}\hat{H}_{dA} \\
& +\hat{H}^{\dagger}_{u1}e^{\frac{1}{2}g_Y\hat{V}_Y+g_2\hat{V}_2}\hat{H}_{u1} + \hat{H}^{\dagger}_{u2}e^{\frac{1}{2}g_Y\hat{V}_Y+g_2\hat{V}_2}\hat{H}_{u2} \\
& +\hat{H}^{\dagger}_{d1}e^{-\frac{1}{2}g_Y\hat{V}_Y+g_2\hat{V}_2}\hat{H}_{d1} + \hat{H}^{\dagger}_{d2}e^{-\frac{1}{2}g_Y\hat{V}_Y+g_2\hat{V}_2}\hat{H}_{d2}
\end{aligned}$$

$$\begin{aligned}
W \;=\; & \mu_S\hat{H}_{uS}\hat{H}_{dS} + \mu_A\hat{H}_{uA}\hat{H}_{dA} \\
& +\mu_D(\hat{H}_{u1}\hat{H}_{d2} + \hat{H}_{u2}\hat{H}_{d1})
\end{aligned}$$

$$V = V_{\text{SUSY}} + V_{\text{soft}}$$

$$\begin{aligned}
V_{\text{SUSY}} &= \frac{1}{2}D_Y^2 + \frac{1}{2}\sum_{a=1,2,3}(D^a)^2 \\
&+ |\mu_S|^2\,(H_{uS}^\dagger H_{uS} + H_{dS}^\dagger H_{dS}) + |\mu_A|^2\,(H_{uA}^\dagger H_{uA} + H_{dA}^\dagger H_{dA}) \\
&+ |\mu_D|^2\,(H_{u1}^\dagger H_{u1} + H_{d1}^\dagger H_{d1}) + |\mu_D|^2\,(H_{u2}^\dagger H_{u2} + H_{d2}^\dagger H_{d2})
\end{aligned}$$

$$\begin{aligned}
V_{\text{soft}} &= m_{uS}^2 H_{uS}^\dagger H_{uS} + m_{dS}^2 H_{dS}^\dagger H_{dS} + [b_S H_{uS} H_{dS} + h.c.] \\
&+ m_{uA}^2 H_{uA}^\dagger H_{uA} + m_{dA}^2 H_{dA}^\dagger H_{dA} + [b_A H_{uA} H_{dA} + h.c.] \\
&+ m_{uD}^2 (H_{u1}^\dagger H_{u1} + H_{u2}^\dagger H_{u2}) + m_{dD}^2 (H_{d1}^\dagger H_{d1} + H_{d2}^\dagger H_{d2}) \\
&+ [b_D (H_{u1} H_{d2} + H_{u2} H_{d1}) + h.c.]
\end{aligned}$$

$$\begin{aligned} D_Y &= -\frac{1}{2} g_Y \left\{ (H_{uS}^\dagger H_{uS} - H_{dS}^\dagger H_{dS}) + (H_{uA}^\dagger H_{uA} - H_{dA}^\dagger H_{dA}) \right. \\ &\quad \left. + (H_{u1}^\dagger H_{u1} - H_{d1}^\dagger H_{d1}) + (H_{u2}^\dagger H_{u2} - H_{d2}^\dagger H_{d2}) \right\} \end{aligned}$$

$$\begin{aligned} D^a &= -g_2 \left\{ (H_{uS}^\dagger T^a H_{uS} + H_{dS}^\dagger T^a H_{dS}) + (H_{uA}^\dagger T^a H_{uA} + H_{dA}^\dagger T^a H_{dA}) \right. \\ &\quad \left. + (H_{u1}^\dagger T^a H_{u1} + H_{d1}^\dagger T^a H_{d1}) + (H_{u2}^\dagger T^a H_{u2} + H_{d2}^\dagger T^a H_{d2}) \right\} \end{aligned}$$

When $v_{uS} = 0$,$v_{dS} \neq 0$

Equations at vacuum about the symmetric singlet

$$\begin{aligned} \left(|\mu_S|^2 + m_{uS}^2\right) v_{uS} &= b_S v_{dS} - X v_{uS}, \\ \left(|\mu_S|^2 + m_{dS}^2\right) v_{dS} &= b_S v_{uS} + X v_{dS} \end{aligned}$$

$$\Downarrow$$

$b_S = 0$ **and** $\left(|\mu_S|^2 + m_{dS}^2\right) - X|_{v_{uS}=0} = 0$

The similar result is obtained when $v_{uS} \neq 0$ $v_{dS} = 0$

Therefore $b_S = 0$ is ***the necessary condition*** of $v_{uS} = 0$ $v_{dS} \neq 0$ or $v_{uS} \neq 0$ $v_{dS} = 0$.

The similar result is satisfied in the case of other representations.

Equations at vacuum

$$S\left(\left(|\mu_S|^2+m_{uS}^2\right)v_{uS} \;=\; b_S v_{dS}-Xv_{uS},\quad \left(|\mu_S|^2+m_{dS}^2\right)v_{dS} \;=\; b_S v_{uS}+Xv_{dS}\right.$$

$$A\left(\left(|\mu_A|^2+m_{uA}^2\right)v_{uA} \;=\; b_A v_{dA}-Xv_{uA},\quad \left(|\mu_A|^2+m_{dA}^2\right)v_{dA} \;=\; b_A v_{uA}+Xv_{dA}\right.$$

$$D\left(\begin{array}{l}\left(|\mu_D|^2+m_{uD}^2\right)v_{u1} \;=\; b_D v_{d2}-Xv_{u1},\quad \left(|\mu_D|^2+m_{dD}^2\right)v_{d2} \;=\; b_D v_{u1}+Xv_{d2}\\ \\ \left(|\mu_D|^2+m_{uD}^2\right)v_{u2} \;=\; b_D v_{d1}-Xv_{u2},\quad \left(|\mu_D|^2+m_{dD}^2\right)v_{d1} \;=\; b_D v_{u2}+Xv_{d1}\end{array}\right.$$

$$v_{uS(dS)} \equiv \left|\left\langle H^0_{uS(dS)}\right\rangle\right|,\quad v_{uA(dA)} \equiv \left|\left\langle H^0_{uA(dA)}\right\rangle\right|,\quad v_{u1(d1)} \equiv \left|\left\langle H^0_{u1(d1)}\right\rangle\right|,\quad v_{u2(d2)} \equiv \left|\left\langle H^0_{u2(d2)}\right\rangle\right|$$

$$X \equiv \frac{g_Y^2+g_2^2}{4}\left\{(v_{uS}^2-v_{dS}^2)+(v_{uA}^2-v_{dA}^2)+(v_{u1}^2-v_{d2}^2)+(v_{u2}^2-v_{d1}^2)\right\}$$

Independent candidates of S_3 doublet

$$v_{u1} = 0, \;\; v_{d2} = 0, \;\; v_{u2} = 0, \;\; v_{d1} \neq 0$$

All combinations in this case

$$v_{u1} = 0, \;\; v_{d2} = 0, \;\; v_{u2} = 0, \;\; v_{d1} \neq 0$$

$$v_{u1} = 0, \;\; v_{d2} = 0, \;\; v_{u2} \neq 0, \;\; v_{d1} = 0$$

$$v_{u1} = 0, \;\; v_{d2} \neq 0, \;\; v_{u2} = 0, \;\; v_{d1} = 0$$

$$v_{u1} \neq 0, \;\; v_{d2} = 0, \;\; v_{u2} = 0, \;\; v_{d1} = 0$$

⇓

Potential is invariant under $u \Leftrightarrow d \;\; 1 \Leftrightarrow 2$

⇓

An independent candidate

$$v_{u1} = 0, \;\; v_{d2} = 0, \;\; v_{u2} = 0, \;\; v_{d1} \neq 0$$

* Why are the followings not taken?

$$v_{u1} = 0, \;\; v_{d2} \neq 0, \;\; v_{u2} \neq 0, \;\; v_{d1} = 0$$

$$v_{u1} \neq 0, \;\; v_{d2} \neq 0, \;\; v_{u2} \neq 0, \;\; v_{d1} = 0$$

These solutions satisfy $2\,|\mu_D|^2 + m_{uD}^2 + m_{dD}^2 = 0$

$$b_D = 0$$

↓

The condition could be satisfied at certain energy scales but RGE (Renormalization Group Equation) dependent.

↓

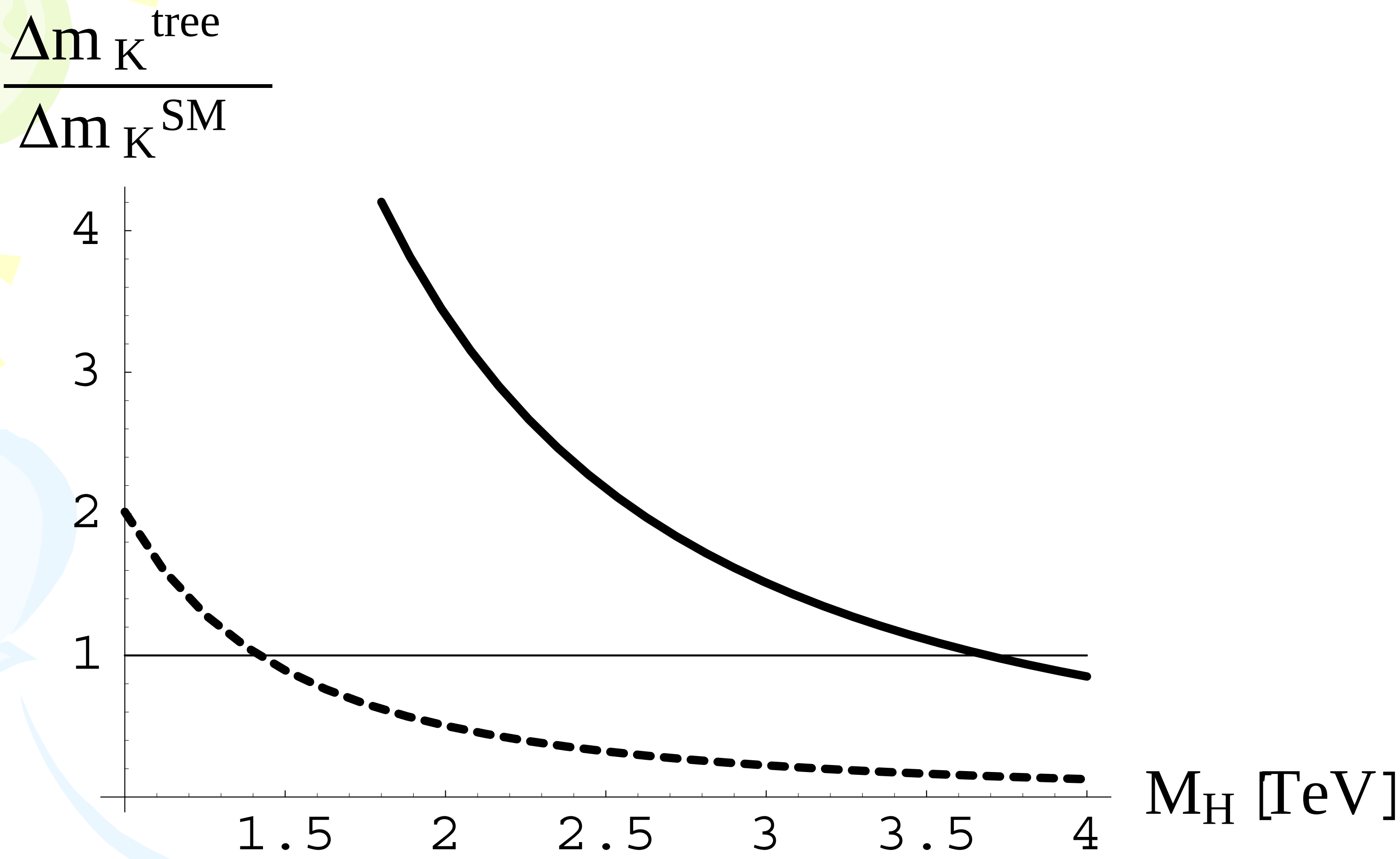
It is difficult to construct models that satisfy that condition.

Global minimum

The following relation is satisfied at the point which satisfies the equations at vacuum.

$$\mathrm{V}_{\mathrm{min}} = -\frac{4}{g_Y^2+g_2^2}\mathrm{X}^2 \leq 0$$

VEV can be **made the global vacuum** of the theory by controlling Higgs mass parameters in the Lagrangian.



$\Delta m_K^{\text{tree}}/\Delta m_K^{\text{SM}}$ as the function of neutral Higgs mass parameter M_H . The solid and dashed lines correspond to η =0 and 0.03, respectively.