Quark and lepton mass matrices with A_4 family symmetry

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Based on a paper

"Quark masses and mixing with A_4 family symmetry"
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～ Contents ～

- Introduction
- A4 symmetry
- A4 model for lepton sector
- A4 model for quark sector
- Summary

● Introduction (Motivation)

Quark and lepton mixing angles:

Lepton:	$\theta_{12} = 32^\circ \sim 35^\circ$, $\theta_{23} = 38^\circ \sim 52^\circ$, $\theta_{13} < 12^\circ$
Quark :	$\theta_{12} \sim 13^\circ$, $\theta_{23} \sim 2.3^\circ$, $\theta_{13} \sim 0.2^\circ$ (90% C.L.)

Neutrino mixings are near bimaximal !!

★ Why are CKM and PMNS matrices so different ?

★ In the lepton sector,
Why are θ_{12} and θ_{23} so large ? / Why is θ_{13} small ?
What is the origin of the maximal 2-3 mixing ?

☆ Flavor symmetry makes θ_{23} maximal.

Discrete symmetry can be a nice candidate !?

(Neutrino) mass matrix and mixing angle

bimaximal: $\theta_{12} = \theta_{23} = \pi/4, \ \theta_{13} = 0.$

$$M_\nu = \begin{pmatrix} A & B & B \\ B & C & A-C \\ B & A-C & C \end{pmatrix}, \quad U \sim \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \end{pmatrix}$$

tri-bimaximal: $\theta_{12} \fallingdotseq 35^\circ, \ \theta_{23} = \pi/4, \ \theta_{13} = 0.$

$$M_\nu = \begin{pmatrix} A & B & B \\ B & C & A-C+B \\ B & A-C+B & C \end{pmatrix},$$

$$U_{\mathrm{HPS}} \sim \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

[Harrison, Perkins, Scott ('02)]

Discrete non-Abelian group / its application

order (# of elements)	6	8	10	12	...	24	...
S_N : permutation group	S_3					S_4	
D_N : dihedral group	D_3	D_4	D_5	D_6	...		...
Q_N : quaternion groups		Q_4		Q_6	...		...
T : tetrahedral groups				$T(A_4)$			...

S3: Pakvasa, Sugawara('78); Kubo et al.('03,'04, ...); Koide('05); Haba, Yoshioka('05); Shingai et al.('06); ···

D4: Grimus, Lavoura('03) bimaximal

Q4: Frigerio, Kaneko, Ma, Tanimoto('04)

Q6: Babu, Kubo('05); Kajiyama, Itou, Kubo('06)

A4: Ma, Rajasekaran('01); Ma('02, ...); Babu, Ma, Valle('03); Hirsch et al.('04); Altarelli, Feruglio('05,'06); Zee('05); Babu, He('05); He, Keum, Volkus('06); ··· tri-bimaximal

A4 models ...

☆ A4 applied for leptons — very successful

tri-bimaximal mixing is easily realized !

★ A4 applied for quarks — not successful

$V_{\rm CKM} = 1$ is generally predicted !

- interactions beyond the SM (such as susy)
- explicit breaking the A4 symmetry

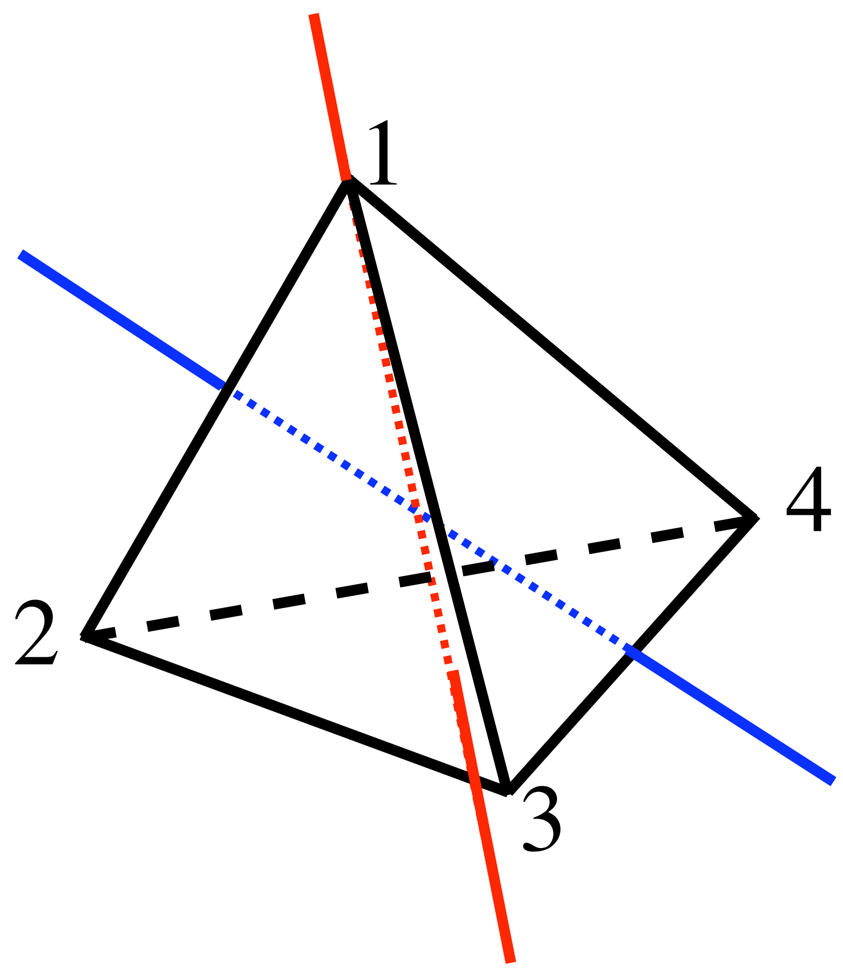
Are realistic quark masses and mixing angles obtained, entirely within the A4 context ?

● A4 symmetry

A4: the group of the even permutation of (1234)

- 4 classes of *A4* -

[C1]: (1234)
[C2]: (1342), (4213), (2431), (3124) ⋯ 120° rotation
[C3]: (1423), (3241), (4132), (2314) ⋯ 240° rotation
[C4]: (2143), (3412), (4321) ⋯ 180° rotation



- Character table -

	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C1	1	1	1	1	1	3
C2	4	3	1	ω	ω^2	0
C3	4	3	1	ω^2	ω	0
C4	3	2	1	1	1	-1

$$\omega = e^{2\pi i/3} = \frac{-1+i\sqrt{3}}{2}, \ \omega + \omega^2 = -1$$

- Multiplication rule -

Four irreducible representations: $\underline{\mathbf{1}}$, $\underline{\mathbf{1}}$’, $\underline{\mathbf{1}}$” and $\underline{\mathbf{3}}$

$$\underline{\mathbf{1}}' \times \underline{\mathbf{1}}' = \underline{\mathbf{1}}'', \quad \underline{\mathbf{1}}'' \times \underline{\mathbf{1}}'' = \underline{\mathbf{1}}', \quad \underline{\mathbf{1}}' \times \underline{\mathbf{1}}'' = \underline{\mathbf{1}},$$

$$\underline{\mathbf{3}}_{\mathbf{1}} \times \underline{\mathbf{3}}_{\mathbf{2}} = \underline{\mathbf{1}} + \underline{\mathbf{1}}' + \underline{\mathbf{1}}'' + \underline{\mathbf{3}}_{\mathbf{A}} + \underline{\mathbf{3}}_{\mathbf{B}},$$

where, denoting $\underline{\mathbf{3}}_{\boldsymbol{i}}$ as $(\boldsymbol{a_i}, \boldsymbol{b_i}, \boldsymbol{c_i})$, we have

$$\begin{aligned}
\underline{1} &= a_1 a_2 + b_1 b_2 + c_1 c_2, \\
\underline{1}' &= a_1 a_2 + \omega b_1 b_2 + \omega^2 c_1 c_2, \\
\underline{1}'' &= a_1 a_2 + \omega^2 b_1 b_2 + \omega c_1 c_2, \\
\underline{3}_{\mathrm{A}} &\sim (b_1 c_2, c_1 a_2, a_1 b_2), \\
\underline{3}_{\mathrm{B}} &\sim (c_1 b_2, a_1 c_2, b_1 a_2).
\end{aligned}$$

Note that $\underline{\mathbf{3}} \times \underline{\mathbf{3}} \times \underline{\mathbf{3}} = \underline{\mathbf{1}}$ is possible in A_4 and

$$\underline{\mathbf{1}}' \times \underline{\mathbf{1}}' \times \underline{\mathbf{1}}', \ \underline{\mathbf{1}}'' \times \underline{\mathbf{1}}'' \times \underline{\mathbf{1}}'', \ \underline{\mathbf{1}} \times \underline{\mathbf{1}}' \times \underline{\mathbf{1}}'' = \underline{\mathbf{1}}.$$

● A4 model for lepton sector

- A simple example - **[Ma, Rajasekaran ('01)]**

A_4 assignments for leptons, $L_i = (\nu_i, l_i)$, l_i^c :

	$(\nu_i, l_i)_L$	$l_1^c = e_R$	$l_2^c = \mu_R$	$l_3^c = \tau_R$	$(\phi_i^0, \phi_i^-)_l$
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}''$	$\underline{3}$

$$M_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+\omega b+\omega^2 c & d \\ e & d & a+\omega^2 b+\omega c \end{pmatrix},$$

parameters (a, b, c) come from $(\underline{1}, \underline{1}', \underline{1}'')$ and (d, e, f) from $\underline{3}$.

$$M_l = \begin{pmatrix} g_1 v_{l1} & g_2 v_{l1} & g_3 v_{l1} \\ g_1 v_{l2} & g_2 \omega v_{l2} & g_3 \omega^2 v_{l2} \\ g_1 v_{l3} & g_2 \omega^2 v_{l3} & g_3 \omega v_{l3} \end{pmatrix}$$

$$= \begin{pmatrix} v_{l1} & 0 & 0 \\ 0 & v_{l2} & 0 \\ 0 & 0 & v_{l3} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_3 \end{pmatrix}.$$

g_i: Yukawa couplings, $v_{li} = \langle \phi_i^0 \rangle_l$

How HPS mixing matrix is derived ...

$$U_{lL}^{\dagger} M_\nu U_{lL}^{*} = U_{lL}^{\dagger} U_{\nu L} M_\nu^{\rm diag} U_{\nu L}^{T} U_{lL}^{*} = U_{l\nu} M_\nu^{\rm diag} U_{l\nu}^{T}$$

Taking $v_l \equiv v_{l1} = v_{l2} = v_{l3}$ $(v_{li} = \langle \phi_i^0 \rangle_l)$,

$$M_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_3 \end{pmatrix} \sqrt{3} v_l = U_{lL} M_l^{\rm diag}.$$

In the case $b = c$ and $e = f = 0$, then one has

$$U_{l\nu} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$: HPS matrix

with $(m_e, m_\mu, m_\tau) = (\sqrt{3} v_l g_1, \sqrt{3} v_l g_2, \sqrt{3} v_l g_3)$ and

$$M_\nu^{\rm diag} = diag(a - b + d, a + 2b, -a + b + d).$$

● A4 model for quark sector

Following the successful lepton assignments, if we assign A4 representation for quarks as

	(ν_i, l_i)	l_i^c	$(\phi_i^0, \phi_i^-)_l$	(u_i, d_i)	$u_i^c,\, d_i^c$	$(\phi_i^0, \phi_i^-)_q$
A_4	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}},\, \underline{\mathbf{1}}',\, \underline{\mathbf{1}}''$	$\underline{\mathbf{3}}$	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}},\, \underline{\mathbf{1}}',\, \underline{\mathbf{1}}''$	$\underline{\mathbf{3}}$

Taking $v_q \equiv v_{q1} = v_{q2} = v_{q3}$ $(v_{qi} = \langle \phi_i^0 \rangle_q)$,

$$M^{U(D)} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} g_1^{U(D)} & 0 & 0 \\ 0 & g_2^{U(D)} & 0 \\ 0 & 0 & g_3^{U(D)} \end{pmatrix} \sqrt{3} v_q,$$

$$U^U = U^D = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad \Rightarrow V_{\rm CKM} = (U^U)^\dagger U^D = 1$$

CKM small mixings come from higher operators!?

[Babu, Ma, Valle ('03)]

SU(5) motivated Quark-Lepton assignment

[Ma, Sawanaka, Tanimoto ('06)]

Remember the successful lepton assignments:

	$L=(\nu,l)$	l^c	$\Phi_l=(\phi^0,\phi^-)_l$
A_4	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}},\ \underline{\mathbf{1}}',\ \underline{\mathbf{1}}''$	$\underline{\mathbf{3}}$

$$\underline{\mathbf{5}}^*_i : (L=(\nu,l), d^c)_i \sim \underline{\mathbf{3}}$$

$$\underline{\mathbf{10}}_i : (l^c, u^c, Q=(u,d))_i \sim \underline{\mathbf{1}}, \underline{\mathbf{1}}', \underline{\mathbf{1}}''$$

$$\underline{\mathbf{5}}_i : (\phi^0_1, \phi^-_1)_U \sim \underline{\mathbf{1}}', \quad (\phi^0_2, \phi^-_2)_U \sim \underline{\mathbf{1}}''$$

$$\underline{\mathbf{5}}^*_i + \underline{\mathbf{45}}_i : (\phi^0_i, \phi^-_i)_l, \quad (\phi^0_i, \phi^-_i)_D \sim \underline{\mathbf{3}}$$

- Down sector -

$$M_D = \begin{pmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} v_1 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_3 \end{pmatrix}$$

$v_i \equiv \langle \phi_i^0 \rangle_D$ and the Yukawa couplings h_i can be real.

To get quark mass hierarchy, $v_1 \ll v_2 \ll v_3$.

(Cf. $v_{li} \equiv \langle \phi_i^0 \rangle_l = v_l$)

- Up sector -

$$M_U = \begin{pmatrix} 0 & \mu_2 & \mu_3 \\ \mu_2 & m_2 & 0 \\ \mu_3 & 0 & m_3 \end{pmatrix}$$

$$\underline{1}' \times \underline{1}' \times \underline{1}' = \underline{1}$$
$$\underline{1}'' \times \underline{1}'' \times \underline{1}'' = \underline{1}$$
$$\underline{1} \times \underline{1}' \times \underline{1}'' = \underline{1}$$

m_2, μ_3: from $\phi_1^0 \sim \underline{1}'$, m_3, μ_2: from $\phi_1^0 \sim \underline{1}''$.

μ_2 and $m_{2,3}$ can be real, with μ_3 complex.

We have 10 parameters: h_i, v_i, m_2, m_3, μ_2, μ_3,
for 6 masses, 3 mixing angles and 1 CP phase

- Masses -

$$m_b^2 \simeq (h_3 v_3)^2, \quad m_s^2 \simeq 3(h_2 v_2)^2, \quad m_d^2 \simeq 9(h_3 v_2)^2$$

$$m_t \simeq |m_3|, \quad m_c \simeq |m_2|, \quad m_u \simeq \left| \frac{\mu_2^2}{m_2} + \frac{\mu_3^2}{m_3} \right|$$

- Mixing angles -

$$V^{\mathrm{CKM}} = V_U^\dagger V_D = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{us}^{\mathrm{CKM}} \simeq V_{ds} - V_{uc} = \frac{h_1}{h_2} - \frac{\mu_2}{m_2}$$

$$V_{cb}^{\mathrm{CKM}} \simeq V_{sb} = \frac{h_2}{h_3}$$

$$V_{ub}^{\mathrm{CKM}} \simeq V_{db} - V_{uc} V_{sb} - V_{ut} = \frac{h_1}{h_3} - \frac{\mu_2}{m_2}\frac{h_2}{h_3} - \frac{\mu_3}{m_3}$$

$$\frac{h_1}{h_2} \simeq \lambda = 0.22, \quad \frac{h_2}{h_3} \simeq \lambda^2, \quad \frac{v_1}{v_3} \simeq \lambda^2, \quad \frac{v_2}{v_3} \simeq \lambda - \lambda^{1/2}$$

- CP phase -

CP violation is predicted by ω in down sect. and the imaginary part of μ_3

$$J_{CP} \simeq \frac{\sqrt{3}}{2}\frac{h_1^2}{h_3^2}\left(\frac{v_2^2 - v_1^2}{v_2^2 + v_1^2}\right)\left(1 + \frac{\mathrm{Re}(\mu_3) + \frac{1}{\sqrt{3}}\mathrm{Im}(\mu_3)}{m_t}\frac{h_3}{h_1}\right)$$

mainly comes from the A_4 phase ω

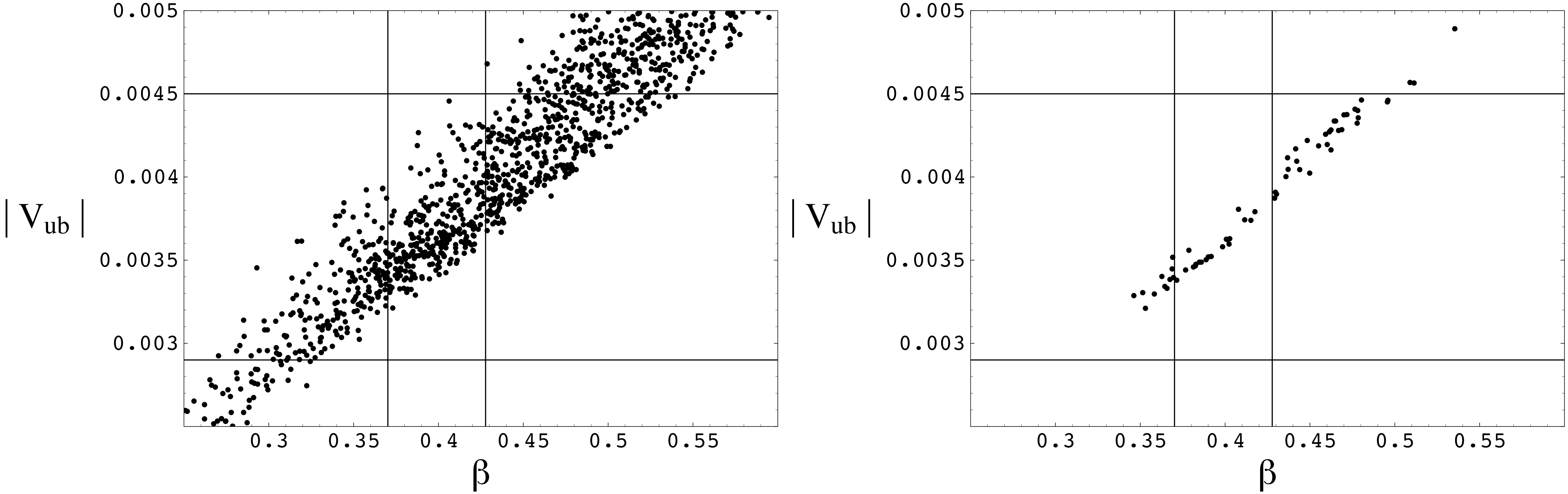
The correlation between V_{ub} and $\sin 2\beta$ is found !

since V_{ub} depends on the phase of μ_3

$$V_{ub} \simeq \frac{h_1}{h_3} - \frac{h_2}{h_3}\frac{\mu_2}{m_2} - \frac{\mu_3}{m_3}$$

Our model is testable !

$\beta - |V_{ub}|$, ($\beta = 0.370 \sim 0.427$, $|V_{ub}| = 0.0029 \sim 0.0045$ at 90% C.L.)



Left) with the following nine experimental inputs:

$m_u = 0.9 \sim 2.9$ (MeV), $m_c = 530 \sim 680$ (MeV), $m_t = 168 \sim 180$ (GeV),
$m_d = 1.8 \sim 5.3$ (MeV), $m_s = 35 \sim 100$ (MeV), $m_b = 2.8 \sim 3$ (GeV),
$|V_{us}| = 0.221 \sim 0.227$, $|V_{cb}| = 0.039 \sim 0.044$, $J_{CP} = (2.75 \sim 3.35) \times 10^{-5}$

Right) restrict with narrow parameter ranges as

$m_u = 1.4 \sim 1.5$ (MeV), $m_c = 600 \sim 610$ (MeV), $m_t = 172 \sim 176$ (GeV),
$m_d = 3.4 \sim 3.6$ (MeV), $m_s = 60 \sim 70$ (MeV), $m_b = 2.85 \sim 2.95$ (GeV),
$|V_{us}| = 0.221 \sim 0.227$, $|V_{cb}| = 0.041 \sim 0.042$, $J_{CP} = (3.0 \sim 3.1) \times 10^{-5}$

● Summary

- A4 family symmetry is successfully applied to quarks motivated by the quark-lepton assignments of SU(5).

- The realistic quark masses and mixings angles are obtained, ebtirely within the A4 context.

- A decisive test of our model can be allowed, since $|V_{ub}|$ strongly correlates to the CP phase.

Future: to examine models in which
- quarks and leptons are unified in SU(5), SO(10), ...
- the seesaw mechanism can be applied.