On the Fukugita-Tanimoto-Yanagida Ansatz with Partially Non-degenerate Right-handed Majorana Neutrinos

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M. O. and Z.Z. Xing, hep-ph/0608280, to be published in PLB

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§ 1. Introduction

Recent neutrino experimental results (at 99% C.L.)

Mixing angles

$$0.35 < \sin^2\theta_{23} < 0.65$$
$$0.25 < \sin^2\theta_{12} < 0.38$$
$$\sin^2\theta_{13} < 0.030$$

Mass-squared differences

$$2.1 \times 10^{-3} < \Delta m_{32}^2 < 3.1 \times 10^{-3}\ \text{eV}^2$$
$$7.2 \times 10^{-5} < \Delta m_{21}^2 < 8.9 \times 10^{-5}\ \text{eV}^2$$

Neutrinos have two large mixing angles and one small mixing angle.

Cosmological bound on neutrino masses

$$\sum_i m_{\nu_i} \lesssim 0.6\ \text{eV}$$

Neutrinos have very small masses.

Why are neutrino masses so small?

Seesaw Mechanism $M_\nu = Y_{\nu_D}^T M_R^{-1} Y_{\nu_D} \cdot v^2, \quad M_R \gg v$

Various phenomenological ansatz for lepton mass matrices have actively been studied.

"Texture zeros"

An empirical relation

$$\tan\theta_C = \sqrt{\frac{m_d}{m_s}}$$

R. Gatto, G. Sartori and M. Tonin (1968),
N. Cabibbo and Maiani (1968),
R.J. Oakes (1969)

$$M_u = \begin{pmatrix} 0 & A_u \\ A_u & B_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & A_d e^{i\gamma} \\ A_d e^{i\gamma} & B_d \end{pmatrix}$$

S. Weinberg (1977), H. Fritzsch (1977),
F. Wilczek and A. Zee (1977)

Fritzsch texture for lepton mass matrices

$$M_l = \begin{pmatrix} 0 & C_l & 0 \\ C_l & 0 & B_l \\ 0 & B_l & A_l \end{pmatrix} \qquad M_\nu = \begin{pmatrix} 0 & C_\nu & 0 \\ C_\nu & 0 & B_\nu \\ 0 & B_\nu & A_\nu \end{pmatrix}$$

Z.Z. Xing (2002)

Within the seesaw framework ...

M. Fukugita. M. Tanimoto and T. Yanagida (2003)

Fukugita-Tanimoto-Yanagida (FTY) ansatz

$$M_l = \begin{pmatrix} 0 & C_l & 0 \\ C_l & 0 & B_l \\ 0 & B_l & A_l \end{pmatrix} \qquad M_D = \begin{pmatrix} 0 & C_D & 0 \\ C_D & 0 & B_D \\ 0 & B_D & A_D \end{pmatrix}$$

$M_R = \mathrm{diag}\,(M_1, M_2, M_3)$ **with** $M_1 = M_2 = M_3 \equiv M_0$

$$\Rightarrow \quad M_\nu = M_D M_R^{-1} M_D^T = \frac{M_D^2}{M_0}$$

No CP violation!
No leptogenesis!

G.C. Branco. T. Morozumi, B.M. Nobre and M.N. Rebelo (2001)

The FTY model is consistent with the current experimental data.

In this work

M. O. and Z.Z. Xing, hep-ph/0608280

We generalize the FTY ansatz by allowing the masses of M_R to be partially non-degenerate and examine how the deviation from the mass degeneracy can affect the neutrino observables.

(A) $M_3 = M_2 \neq M_1$ (B) $M_2 = M_1 \neq M_3$ (C) $M_3 = M_1 \neq M_2$

$$M_{\rm R} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix} \qquad M_{\rm R} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_2 \end{pmatrix} \qquad M_{\rm R} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_1 \end{pmatrix}$$

§ 2. Model

We take the Fritzsch texture for M_l and M_D.

$$M_l = \begin{pmatrix} 0 & C_l & 0 \\ C_l & 0 & B_l \\ 0 & B_l & A_l \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & C_D & 0 \\ C_D & 0 & B_D \\ 0 & B_D & A_D \end{pmatrix}$$

M_l can be decomposed by a diagonal phase matrix as

$$M_l = P_l \overline{M}_l P_l^T$$

$$P_l = \mathrm{diag}\left(e^{i(\varphi_l - \phi_l)}, e^{i\phi_l}, 1\right)$$

$$O_l^\dagger \overline{M}_l O_l^* = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

$$\varphi_l = \arg[B_l]$$
$$\phi_l = \arg[C_l]$$

➡ $V_l^\dagger M_l V_l^* = M_l^{\mathrm{diag}}$

where $V_l = P_l O_l$

Similarly,

$$M_D = P_D \overline{M}_D P_D^T$$

$$P_D = \mathrm{diag}\left(e^{i(\varphi_D - \phi_D)}, e^{i\phi_D}, 1\right)$$

$$O_D^\dagger \overline{M}_D O_D^* = \mathrm{diag}(d_1, d_2, d_3)$$

$$\varphi_D = \arg[B_D]$$
$$\phi_D = \arg[C_D]$$

➡ $V_D^\dagger M_D V_D^* = M_D^{\mathrm{diag}}$

where $V_D = P_D O_D$

$O_{l(D)}$ is given as follows:

$$O_{11} = +\left[\frac{1-y}{(1+x)(1-xy)(1-y+xy)}\right]^{1/2} \qquad O_{21} = +\left[\frac{x(1-y)}{(1+x)(1-xy)}\right]^{1/2}$$

$$O_{12} = -i\left[\frac{x(1+xy)}{(1+x)(1+y)(1-y+xy)}\right]^{1/2} \qquad O_{22} = +i\left[\frac{1+xy}{(1+x)(1+y)}\right]^{1/2}$$

$$O_{13} = +\left[\frac{xy^3(1-x)}{(1-xy)(1+y)(1-y+xy)}\right]^{1/2} \qquad O_{23} = +\left[\frac{y(1-x)}{(1-xy)(1+y)}\right]^{1/2}$$

$$O_{31} = -\left[\frac{xy(1-x)(1+xy)}{(1+x)(1-xy)(1-y+xy)}\right]^{1/2}$$

$$O_{32} = -i\left[\frac{y(1-x)(1-y)}{(1+x)(1+y)(1-y+xy)}\right]^{1/2}$$

$$O_{33} = +\left[\frac{(1-y)(1+xy)}{(1-xy)(1+y)(1-y+xy)}\right]^{1/2}$$

$x_l \equiv m_e/m_\mu \approx 0.00484$
$y_l \equiv m_\mu/m_\tau \approx 0.0595$

for O_l

$x \equiv d_1/d_2$
$y \equiv d_2/d_3$

for O_D

Here we assume that M_D is **real**, i.e. $\phi_D = \varphi_D = 0$

$$\begin{aligned} M_\nu &= M_{\mathrm{D}} M_R^{-1} M_{\mathrm{D}}^T \\ &= O_D M_D^{\mathrm{diag}} O_D^T M_R^{-1} O_D M_D^{\mathrm{diag}} O_D^T \\ &= O_D Q M'_\nu Q^T O_D^T \end{aligned}$$

$Q = \mathrm{diag}\,(1, i, 1)$

where

Mass splitting parameter → $\delta_{12} \equiv (M_2 - M_1)/M_2$

$$M'_\nu = \frac{d_3^2}{M_1} \times \mathrm{diag}\,(x^2 y^2, y^2, 1) - \delta_{12} M''_\nu$$

We can express the deviation from the FTY case in M_ν to leading order of δ_{12}.

We define

$$\omega \equiv d_3^2/M_1$$

$$D_\nu \equiv (1+x)(1-xy)(1+y)(1-y+xy)$$

$$D'_\nu \equiv (1+x)(1-xy)(1+y)$$

$$F_{x,y} \equiv \sqrt{xy(1-x^2)(1-y^2)}$$

$$F_{x,xy} \equiv \sqrt{y(1-x^2)(1-x^2y^2)}$$

$$F_{y,xy} \equiv \sqrt{x(1-y^2)(1-x^2y^2)}$$

M''_ν is given as follows:

(A) $M_3 = M_2 \neq M_1$ **(B)** $M_2 = M_1 \neq M_3$ **(C)** $M_3 = M_1 \neq M_2$

Case	M''_ν	Function
A	$\frac{\omega}{D_\nu}\begin{pmatrix} x^3y^2F^A_{11} & xy^2F_{y,xy} & -xy^2F_{x,y} \\ xy^2F_{y,xy} & y^2F^A_{22} & xy^2F_{x,xy} \\ -xy^2F_{x,y} & xy^2F_{x,xy} & F^A_{33} \end{pmatrix}$	$F^A_{11} \equiv (1+y)(1-y-x^2y^2+y^2)$ $F^A_{22} \equiv (1-xy)(1+xy+x^2y^2-y^2)$ $F^A_{33} \equiv (1+x)(1-y^2+xy^2-x^2y^2)$
B	$\frac{\omega}{D_\nu}\begin{pmatrix} x^3y^3F^B_{11} & xy^3(1-x)F_{y,xy} & -xy(1+xy)F_{x,y} \\ xy^3(1-x)F_{y,xy} & y^3F^B_{22} & y(1-y)F_{x,xy} \\ -xy(1+xy)F_{x,y} & y(1-y)F_{x,xy} & F^B_{33} \end{pmatrix}$	$F^B_{11} \equiv (1-x)(1+xy)(1+y)$ $F^B_{22} \equiv (1-x)(1-xy)(1-y)$ $F^B_{33} \equiv (1+x)(1+xy)(1-y)$
C	$\frac{\omega}{D'_\nu}\begin{pmatrix} x^3y^2F^C_{11} & xy^2F_{y,xy} & xyF_{x,y} \\ xy^2F_{y,xy} & y^2F^C_{22} & yF_{x,xy} \\ xyF_{x,y} & yF_{x,xy} & F^C_{33} \end{pmatrix}$	$F^C_{11} \equiv (1-y^2)$ $F^C_{22} \equiv (1-x^2y^2)$ $F^C_{33} \equiv y(1-x^2)$

$$O_\nu'^T M_\nu' O_\nu' = \text{diag}\,(m_1, m_2, m_3)$$

$O_\nu' = R_{23}(\theta'_{23}) R_{12}(\theta'_{12}) R_{13}(\theta'_{13})$

$\Rightarrow$ $V_\nu^\dagger M_\nu V_\nu^* = M_\nu^{\text{diag}}$ **where** $V_\nu = O_D Q O_\nu'$

Case	Mass eigenvalues for M_ν'	Mixing angles for M_ν'
A	$m_1 \simeq \omega x^2 y^2 \cdot \{1 - \frac{x(1-y+y^2)\,\delta_{12}}{(1+x)(1-xy)(1-y+xy)}\}$ $m_2 \simeq \omega y^2 \cdot \{1 - \frac{(1+xy-y^2)\,\delta_{12}}{(1+x)(1+y)(1-y+xy)}\}$ $m_3 \simeq \omega \cdot \{1 - \frac{(1-y^2)\,\delta_{12}}{(1-xy)(1+y)(1-y+xy)}\}$	$\tan 2\theta'_{23} \simeq \frac{-2xy^2 F_{x,xy}\,\delta_{12}}{(1+x-2y^2)-(1+x-2y^2)\,\delta_{12}}$ $\tan 2\theta'_{12} \simeq \frac{-2x F_{y,xy}\,\delta_{12}}{(1+x-x^2-y^2)+(1-y^2)\,\delta_{12}}$ $\tan 2\theta'_{13} \simeq \frac{2xy^2 F_{x,y}\,\delta_{12}}{(1+x-y^2)-(1+x-y^2)\,\delta_{12}}$
B	$m_1 \simeq \omega x^2 y^2 \cdot \{1 - \frac{xy(1-x+xy)\,\delta_{12}}{(1+x)(1-xy)(1-y+xy)}\}$ $m_2 \simeq \omega y^2 \cdot \{1 - \frac{y(1-x)(1-y)\,\delta_{12}}{(1+x)(1+y)(1-y+xy)}\}$ $m_3 \simeq \omega \cdot \{1 - \frac{(1-y+xy)\,\delta_{12}}{(1-xy)(1+y)(1-y+xy)}\}$	$\tan 2\theta'_{23} \simeq \frac{-2y(1-y) F_{x,xy}\,\delta_{12}}{(1+x-2y^2)-(1+x-y)\,\delta_{12}}$ $\tan 2\theta'_{12} \simeq \frac{-2xy(1-x) F_{y,xy}\,\delta_{12}}{(1+x-x^2-y^2)-y(1-x-y)\,\delta_{12}}$ $\tan 2\theta'_{13} \simeq \frac{2xy(1+xy) F_{x,y}\,\delta_{12}}{(1+x-y^2)-(1+x-y)\,\delta_{12}}$
C	$m_1 \simeq \omega x^2 y^2 \cdot \{1 - \frac{x(1-y)\,\delta_{12}}{(1+x)(1-xy)}\}$ $m_2 \simeq \omega y^2 \cdot \{1 - \frac{(1+xy)\,\delta_{12}}{(1+x)(1+y)}\}$ $m_3 \simeq \omega \cdot \{1 - \frac{y(1-x)\,\delta_{12}}{(1+y)(1-xy)}\}$	$\tan 2\theta'_{23} \simeq \frac{-2y F_{x,xy}\,\delta_{12}}{(1+x+y-y^2)-y(1-y-x^2)\,\delta_{12}}$ $\tan 2\theta'_{12} \simeq \frac{-2x F_{y,xy}\,\delta_{12}}{(1+x+y-x^2)-\delta_{12}}$ $\tan 2\theta'_{13} \simeq \frac{-2xy F_{x,y}\,\delta_{12}}{(1+x+y)-y(1-x^2)\,\delta_{12}}$

where we have neglected the terms of $\mathcal{O}(x^3)$, $\mathcal{O}(y^3)$, $\mathcal{O}(x^2y)$, $\mathcal{O}(xy^2)$ **and** $\mathcal{O}(\delta_{12}^2)$ **by assuming** $0 < x < 1, 0 < y < 1$ **and** $|\delta_{12}| \ll 1$.

$$V_{\rm MNS} = V_l^\dagger V_\nu = O_l^\dagger P_l^\dagger O_D Q O'_\nu$$

where $P_l = \mathrm{diag}\,(e^{i\alpha}, e^{i\beta}, 1)$

with $\alpha \equiv -\varphi_l - \beta$ **and** $\beta \equiv -\phi_l$

Parametrization

$$V_{\rm MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{|V_{e2}|^2}{1-|V_{e3}|^2}$$

$$\sin^2\theta_{23} = \frac{|V_{\mu 3}|^2}{1-|V_{e3}|^2}$$

$$\sin^2\theta_{13} = |V_{e3}|^2$$

$$R_\nu \equiv \left|\frac{\Delta m^2_{21}}{\Delta m^2_{32}}\right| = y_\nu^2 \frac{1-x_\nu^2}{|1-y_\nu^2|}$$

$x_\nu \equiv m_1/m_2$ $y_\nu \equiv m_2/m_3$

$$\mathcal{J} = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta$$

$$\langle m\rangle_e^2 = \sum_{i=1}^{3}(m_i^2|V_{ei}|^2) = m_3^2(x_\nu^2 y_\nu^2 c_{12}^2 c_{13}^2 + y_\nu^2 s_{12}^2 c_{12}^2 + s_{13}^2)$$

$$\langle m\rangle_{ee} = \left|\sum_{i=1}^{3}(m_i V_{ei}^2)\right| = m_3\left|x_\nu y_\nu c_{12}^2 c_{13}^2 e^{2i\rho} + y_\nu s_{12}^2 c_{12}^2 e^{2i\sigma} + s_{13}^2\right|$$

§ 3. Numerical Analysis

We have **seven** parameters in our model: $d_3, x, y, \alpha, \beta, M_1, \delta_{12}$

These parameters are constrained by the neutrino experimental results.

Results

In the case of $\delta_{12} = 0$ (the FTY case)

$\omega \equiv d_3^2/M_1 (= m_3) \approx 47 \sim 56$ meV

$x\,(=\sqrt{x_\nu}) \approx 0.22 \sim 0.56$

$y\,(=\sqrt{y_\nu}) \approx 0.39 \sim 0.45$

$\alpha \approx 0 \sim 2.0\pi$

$\beta \approx 0.61\pi \sim 1.4\pi$

Predicted values

$\sin^2\theta_{23} \approx 0.35 \sim 0.49$, $\sin^2\theta_{12} \approx 0.25 \sim 0.38$, $\sin^2\theta_{13} \approx 0.0050 \sim 0.030$,

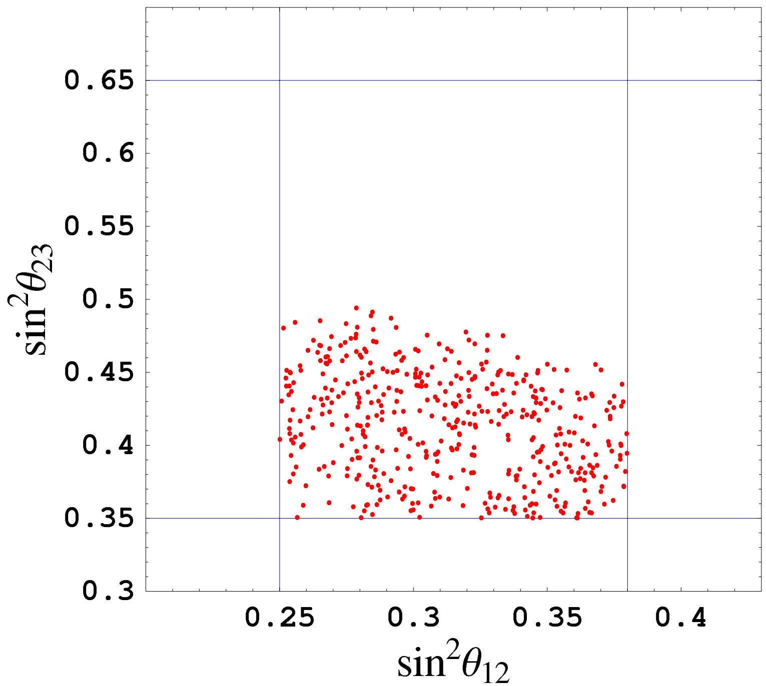
$R_\nu \approx 0.023 \sim 0.041$,

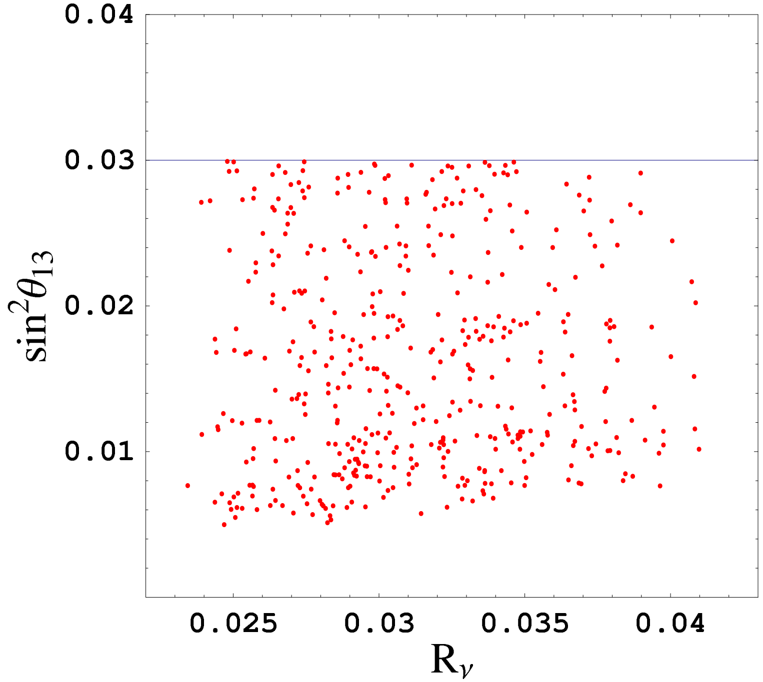
$x_\nu \approx 0.049 \sim 0.32$, $y_\nu \approx 0.15 \sim 0.20$,

$\delta \approx -0.26\pi \sim 0.26\pi$, $\rho \approx -0.12\pi \sim 0.12\pi$, $\sigma \approx -0.16\pi \sim 0.16\pi$,

$\langle m \rangle_e \approx 5.8 \sim 11$ meV , $\langle m \rangle_{ee} \approx 2.8 \sim 6.6$ meV , $\mathcal{J} \approx (-2.3 \sim 2.3) \times 10^{-2}$,

In the case of $\delta_{12} = 0$





$\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ have **maximum** and **minimum** values, respectively.

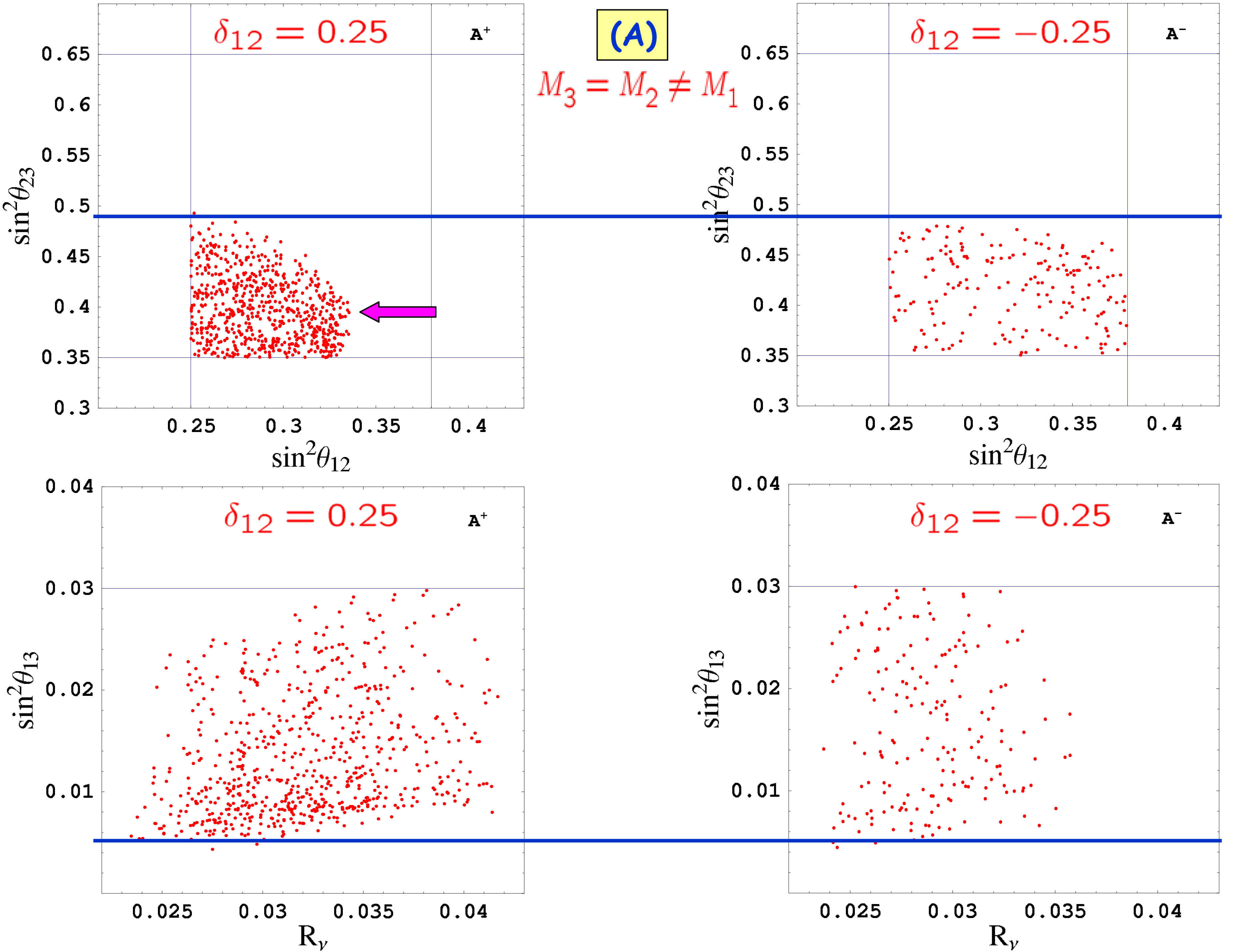
In the case of $\delta_{12} \neq 0$

The differences of parameter regions between $\delta_{12} = 0$ and $\delta_{12} \neq 0$ cases can be **distinguishable** in $\mathcal{O}(\delta_{12}) \sim 10^{-1}$.

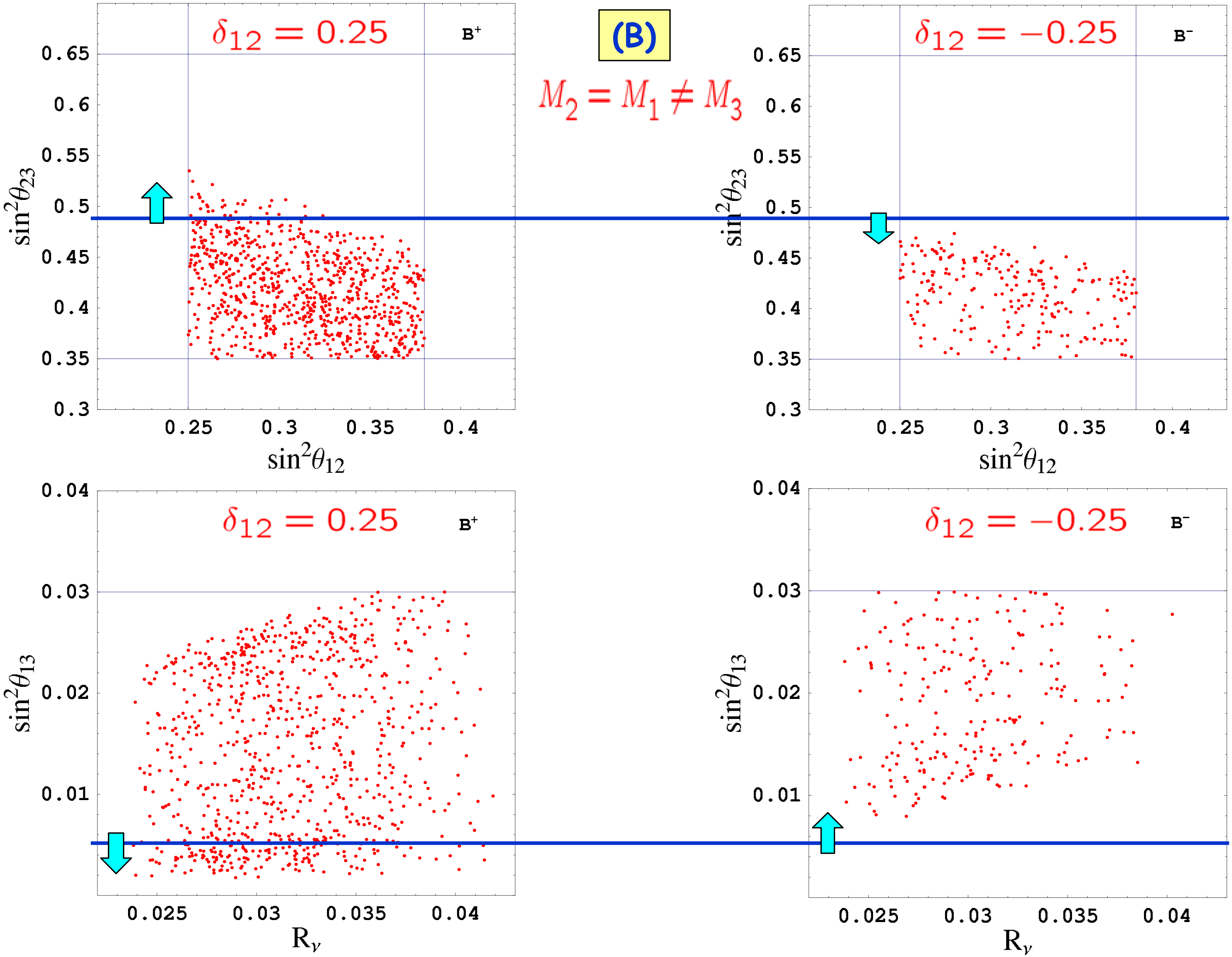
We show the allowed range for the parameters at the typical value of $|\delta_{12}| = 0.25$ in the three cases.

(A) $M_3 = M_2 \neq M_1$	(B) $M_2 = M_1 \neq M_3$	(C) $M_3 = M_1 \neq M_2$

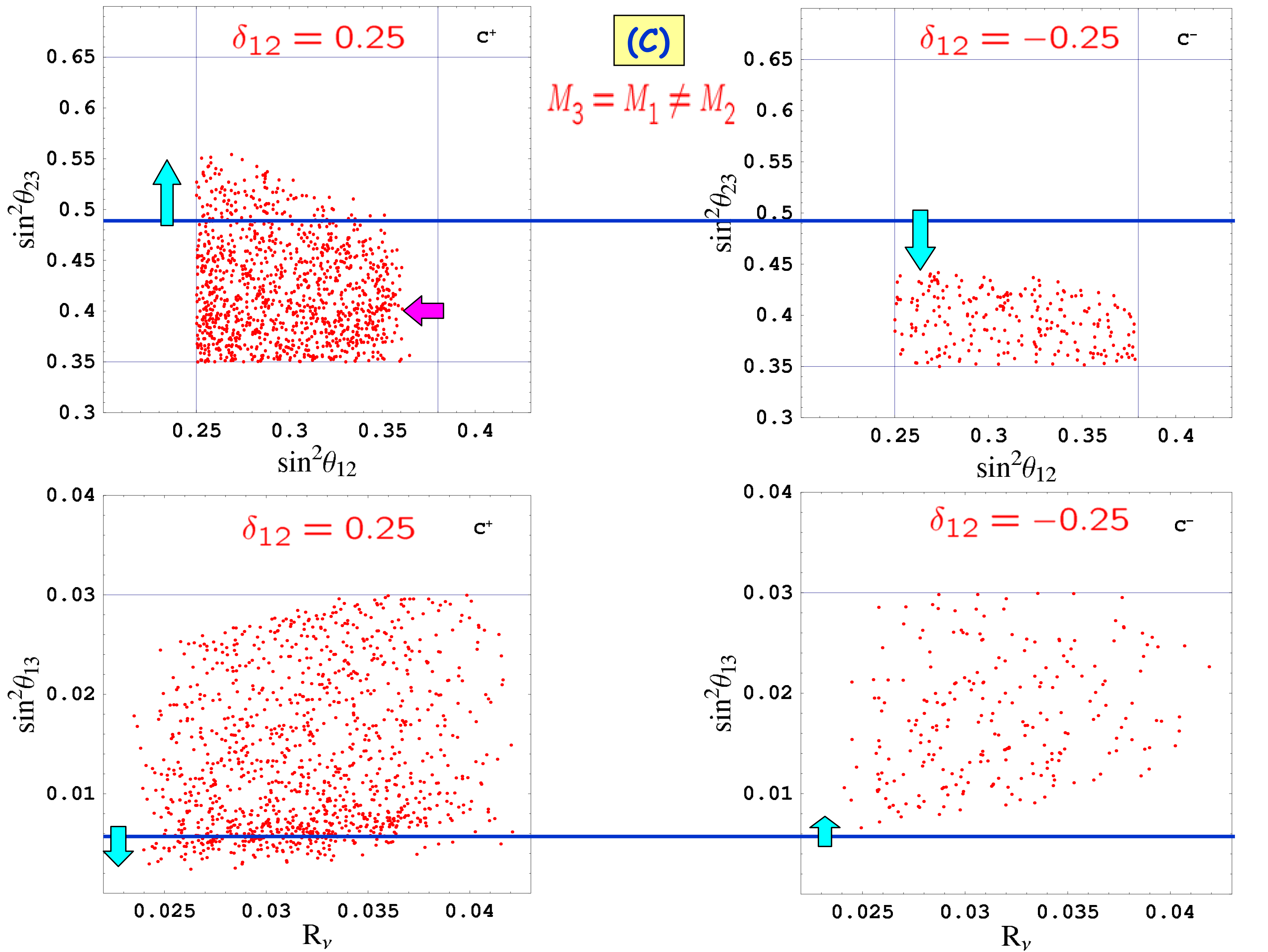
$$M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_2 \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_2 \end{pmatrix} \qquad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_1 \end{pmatrix}$$



(A)
$M_3 = M_2 \neq M_1$
$\delta_{12} = 0.25$
A+
$\delta_{12} = -0.25$
A−
$\sin^2\theta_{23}$
$\sin^2\theta_{12}$
$\sin^2\theta_{13}$
R_ν



(B)
$M_2 = M_1 \neq M_3$
$\delta_{12} = 0.25$
B+
$\delta_{12} = -0.25$
B−
$\sin^2\theta_{23}$
$\sin^2\theta_{12}$
$\sin^2\theta_{13}$
R_ν
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.04
0.03
0.02
0.01
0.025
0.035


(C)
$M_3 = M_1 \neq M_2$
$\delta_{12} = 0.25$
c+
$\delta_{12} = -0.25$
c−
$\sin^2\theta_{23}$
$\sin^2\theta_{12}$
$\sin^2\theta_{13}$
R_ν
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.3
0.35
0.4
0.04
0.03
0.02
0.01
0.025
0.03
0.035
0.04

Typical results at $|\delta_{12}| = 0.25$

	A^+	A^-	B^+	B^-	C^+	C^-
δ_{12}	0.25	-0.25	0.25	-0.25	0.25	-0.25
ω (meV)	$62 \sim 75$	$40 \sim 45$	$57 \sim 72$	$40 \sim 48$	$49 \sim 60$	$44 \sim 54$
x	$0.26 \sim 0.68$	$0.20 \sim 0.47$	$0.23 \sim 0.67$	$0.21 \sim 0.51$	$0.22 \sim 0.75$	$0.21 \sim 0.45$
y	$0.37 \sim 0.43$	$0.40 \sim 0.44$	$0.36 \sim 0.41$	$0.42 \sim 0.48$	$0.42 \sim 0.51$	$0.38 \sim 0.43$
α (π)	$0 \sim 0.9$ $1.1 \sim 2.0$	$0 \sim 0.9$ $1.1 \sim 2.0$	$0 \sim 2.0$	$0.013 \sim 0.70$ $1.3 \sim 2.0$	$0 \sim 2.0$	$0.019 \sim 0.85$ $1.3 \sim 2.0$
β (π)	$0.64 \sim 1.4$	$0.63 \sim 1.4$	$0.61 \sim 1.4$	$0.67 \sim 1.4$	$0.51 \sim 1.5$	$0.69 \sim 1.3$
$\sin^2\theta_{23}$	$0.35 \sim 0.49$	$0.35 \sim 0.48$	$0.35 \sim 0.54$	$0.35 \sim 0.47$	$0.35 \sim 0.55$	$0.35 \sim 0.44$
$\sin^2\theta_{12}$	$0.25 \sim 0.34$	$0.25 \sim 0.38$	$0.25 \sim 0.38$	$0.25 \sim 0.38$	$0.25 \sim 0.36$	$0.25 \sim 0.38$
$\sin^2\theta_{13}$	$0.0043 \sim 0.030$	$0.0045 \sim 0.030$	$0.0018 \sim 0.030$	$0.0080 \sim 0.030$	$0.0024 \sim 0.030$	$0.0066 \sim 0.030$
R_ν	$0.023 \sim 0.042$	$0.024 \sim 0.036$	$0.024 \sim 0.042$	$0.024 \sim 0.040$	$0.024 \sim 0.042$	$0.024 \sim 0.042$
m_3 (meV)	$47 \sim 57$	$50 \sim 56$	$47 \sim 56$	$47 \sim 56$	$47 \sim 56$	$47 \sim 56$
x_ν	$0.076 \sim 0.45$	$0.033 \sim 0.20$	$0.052 \sim 0.43$	$0.044 \sim 0.26$	$0.058 \sim 0.56$	$0.039 \sim 0.18$
y_ν	$0.15 \sim 0.22$	$0.15 \sim 0.19$	$0.15 \sim 0.21$	$0.15 \sim 0.20$	$0.15 \sim 0.24$	$0.15 \sim 0.20$
δ (π)	$-0.26 \sim 0.25$	$-0.26 \sim 0.25$	$-0.30 \sim 0.30$	$-0.23 \sim 0.23$	$-0.29 \sim 0.29$	$-0.24 \sim 0.23$
ρ (π)	$-0.13 \sim 0.13$	$-0.12 \sim 0.12$	$-0.18 \sim 0.18$	$-0.10 \sim 0.10$	$-0.16 \sim 0.16$	$-0.09 \sim 0.09$
σ (π)	$-0.16 \sim 0.16$	$-0.15 \sim 0.15$	$-0.20 \sim 0.20$	$-0.13 \sim 0.13$	$-0.19 \sim 0.18$	$-0.13 \sim 0.13$
$\langle m \rangle_e$ (meV)	$5.5 \sim 10$	$5.7 \sim 11$	$4.9 \sim 11$	$6.5 \sim 11$	$5.2 \sim 11$	$6.3 \sim 11$
$\langle m \rangle_{ee}$ (meV)	$2.9 \sim 6.7$	$2.7 \sim 5.8$	$2.7 \sim 7.6$	$2.9 \sim 6.1$	$2.7 \sim 8.0$	$2.9 \sim 5.5$
$\mathcal{J}$ ($\cdot 10^{-2}$)	$-2.1 \sim 2.2$	$-2.1 \sim 2.3$	$-2.1 \sim 2.1$	$-2.3 \sim 2.2$	$-2.2 \sim 2.1$	$-2.1 \sim 2.1$

Remarks

- The **maximal atmospheric** neutrino mixing angle $\sin^2\theta_{23} = 0.5$ can only be achieved in the cases B^+ and C^+.
- In all cases, $\sin^2\theta_{12}$ is **not** well restricted.
- The smallest mixing angle $\sin^2\theta_{13}$ has an **lower bound** in each case.

$\theta_{13} > 3.76^\circ$ (A^+),	$\theta_{13} > 3.85^\circ$ (A^-),
$\theta_{13} > 2.43^\circ$ (B^+),	$\theta_{13} > 5.13^\circ$ (B^-),
$\theta_{13} > 2.81^\circ$ (C^+),	$\theta_{13} > 4.66^\circ$ (C^-)

⟺ In future experiments $\sin^2 2\theta_{13} = 0.01$ $(\theta_{13} = 2.87^\circ)$

- In **all** cases, the allowed range for $\mathcal{J}$ is **roughly the same.**

⇒ $|\mathcal{J}| \sim \mathcal{O}(10^{-2})$ could be measured in the future long-baseline neutrino experiments.

Current experimental upper bounds on $\langle m \rangle_e$ and $\langle m \rangle_{ee}$:

$$\langle m \rangle_e < 2.1 \text{ eV}$$

$$\langle m \rangle_{ee} < 0.39 \text{ eV}$$

Future →

$$\langle m \rangle_e \simeq 0.2 \text{ eV}$$

$$\langle m \rangle_{ee} \sim \mathcal{O}(10^{-2}) \text{ eV}$$

- **Both the predicted values of $\langle m \rangle_e$ and $\langle m \rangle_{ee}$ in our model are too small to be experimentally accessible in the near future.**

§ 4. Summary

We have generalized the FTY ansatz by allowing the masses of M_R to be partially non-degenerate and examined how the deviation from the mass degeneracy can affect the neutrino observables.

(A) $M_3 = M_2 \neq M_1$ (B) $M_2 = M_1 \neq M_3$ (C) $M_3 = M_1 \neq M_2$

The dependence of mixing angles on δ_{12}

	$\delta_{12}(+)$	$\delta_{12}(-)$
θ_{23}	increase	decrease
θ_{12}	decrease	—
θ_{13}	decrease	increase

The case C is the most sensitive to the effect of the deviation from the mass degeneracy.

Hierarchical structure

For example, taking the case B^+, we obtain

$$M_{\rm D} \sim d_3 \begin{pmatrix} 0 & 0.21\cdots 0.36 & 0 \\ 0.21\cdots 0.36 & 0 & 0.34\cdots 0.55 \\ 0 & 0.34\cdots 0.55 & 0.68\cdots 0.87 \end{pmatrix}$$

Comparing M_D with the Fritzsch-type up-quark mass matrix

$$M_{\rm up} \sim m_t \begin{pmatrix} 0 & 2.0\times 10^{-4} & 0 \\ 2.0\times 10^{-4} & 0 & 6.5\times 10^{-2} \\ 0 & 6.5\times 10^{-2} & 1 \end{pmatrix}$$

we can see that the hierarchy of M_D is much weaker than that of $M_{\rm up}$.

The weak hierarchy of M_D is the main source of large flavor mixing in the lepton sector.

Future work

- Including the complex phases into M_D and/or M_R.

⇨ Leptogenesis