Can Four-Zero-Texture mass matrix model reproduce the quark and lepton mixing angles and CP violating phases ?

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Our Strategy for the Four-Zero-Texture (FZT) mass matrix model

- Many people have discussed the FZT mass matrix model;
  H. Nishiura, K. M, and T. Fukuyama, PRD **60**,013006 (1999).

- And they applied the FZT mass matrix model to SO(10)GUT;
  K. M, T. Fukuyama, and H. Nishiura, PRD **61**, 053001 (2000).

Recently, some people say "the FZT in quark sector is dying";

modify

We check this opinion.
K. M and H. Nishiura, PRD **74,** 033014 (2006)

the solutions is remained

Recently, some people say "the some FZT in the SO(10) GUT is dying";

Apply the results

Perhaps, if there are solutions, people will say " it must be fine-tuning."

We will check the our model.
In this talk

Is there solutions?

change the parameters

We check this opinion.
In this talk (if there is a time.)

Is the bi-large mixing stable?

Four-zero-texture (FZT) mass matrix model

- We would like to discuss the Four-Zero-Texture mass matrix model, quickly .

$$M_f = \begin{pmatrix} 0 & a_f e^{+i\tau_f} & 0 \\ a_f e^{-i\tau_f} & b_f & c_f e^{+i\sigma_f} \\ 0 & c_f e^{-i\sigma_f} & d_f \end{pmatrix}$$

for $f = u, d, e, D, L, R$

- If we assume the Hermite matrix, we get

$$a_f = \sqrt{-\frac{m_{f1}\, m_{f2}\, m_{f3}}{d_f}}\,, \quad c_f = \sqrt{-\frac{(d_f - m_{f1})(d_f - m_{f2})(d_f - m_{f3})}{d_f}}$$

$$b_f = (m_{f1} + m_{f2} + m_{f3}) - d_f$$

Here $0 < m_{f1} < -m_{f2} < m_{f3}$ for $|m_{f1}| < d_f < |m_{f2}|$

$0 < -m_{f1} < m_{f2} < m_{f3}$ for $|m_{f2}| < d_f < |m_{f3}|$

The number of parameters in the quark sector

- The number of parameters in the quark sector.

$$
\begin{array}{lccr}
 & M_u & \Rightarrow & 6 \\
+) & M_d & \Rightarrow & 6 \\
\hline
 & N(pmt) & & =12
\end{array}
$$

- The number of constraints from experiments.

$$
\begin{array}{lccr}
 & \text{masses} & \Rightarrow & 3\times 2 = 6 \\
+) & \text{CKM} & \Rightarrow & 3+1=4 \\
\hline
 & N(exp) & & =10
\end{array}
$$

- The following two phase parameters can not be determined.

$$N(free) = N(pmt) - N(exp) = 12 - 10 = 2$$

$$\Rightarrow (\tau_u + \tau_d), (\sigma_u + \sigma_d)$$

Diagonalization

- The FZT matrix is diagonalized as follows

$$U_f^\dagger M_f U_f = \mathrm{diag}(m_1, m_2, m_3)$$

Here,

$$U_f \equiv P_f^\dagger O_f, \quad P_f \equiv (1, \tau_f, \sigma_f + \tau_f) \equiv (1, \alpha_{f2}, \alpha_{f3})$$

$$O_f \equiv \begin{pmatrix} \sqrt{\frac{(d_f - m_{f1}) m_{f2} m_{f3}}{R_{f1} d_f}} & \sqrt{\frac{(d_f - m_{f2}) m_{f3} m_{f1}}{R_{f2} d_f}} & \sqrt{\frac{(d_f - m_{f3}) m_{f1} m_{f2}}{R_{f3} d_f}} \\ -\sqrt{-\frac{(d_f - m_{f1}) m_{f1}}{R_{f1}}} & \sqrt{-\frac{(d_f - m_{f2}) m_{f2}}{R_f}} & \sqrt{-\frac{(d_f - m_{f3}) m_{f3}}{R_{f3}}} \\ \sqrt{\frac{m_{f1}(d_f - m_{f2})(d_f - m_{f3})}{R_{f1} d_f}} & -\sqrt{\frac{m_{f2}(d_f - m_{f3})(d_f - m_{f1})}{R_{f2} d_f}} & \sqrt{\frac{m_{f3}(d_f - m_{f1})(d_f - m_{f2})}{R_{f3} d_f}} \end{pmatrix}$$

$$R_{f1} = (m_{f1} - m_{f2})(m_{f1} - m_{f3}), \quad R_{f2} = (m_{f2} - m_{f3})(m_{f2} - m_{f1})$$

$$R_{f3} = (m_{f3} - m_{f1})(m_{f3} - m_{f2})$$

The CKM matrix

- The CKM quark mixing matrix $U_{\rm CKM} \equiv U_u^\dagger U_d$ is given by

$$(U_{\rm CKM})_{12} \simeq \sqrt{\frac{|m_d|}{m_s}} - e^{i\alpha_2}\sqrt{\frac{|m_u|}{m_c}}\, x_u x_d - e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_c}}\,(1-x_u)(1-x_d)$$

$$(U_{\rm CKM})_{23} \simeq \sqrt{\frac{|m_d|\, m_s}{m_b^2}} - e^{i\alpha_2}\sqrt{\frac{|m_u|}{m_c}}\, x_u(1-x_d) + e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_c}}\,(1-x_u)x_d$$

$$(U_{\rm CKM})_{23} \simeq \sqrt{\frac{|m_u|}{m_c}\frac{|m_d|\, m_s}{m_b^2}\frac{(1-x_d)}{x_d}} + e^{i\alpha_2}\sqrt{x_u(1-x_d)} - e^{i\alpha_3}\sqrt{(1-x_u)x_d}$$

$$\delta_f \simeq \arg \frac{\left(e^{i\alpha_3}\sqrt{(1-x_u)(1-x_d)} + e^{i\alpha_2}\sqrt{x_u x_d}\right)^*}{\left(e^{i\alpha_3}\sqrt{(1-x_u)x_d} - e^{i\alpha_2}\sqrt{x_u(1-x_d)}\right)\left(e^{i\alpha_2}\sqrt{(1-x_u)x_d} - e^{i\alpha_3}\sqrt{x_u(1-x_d)}\right)^*}$$

where $x_f \equiv d_f/m_{f3}$,

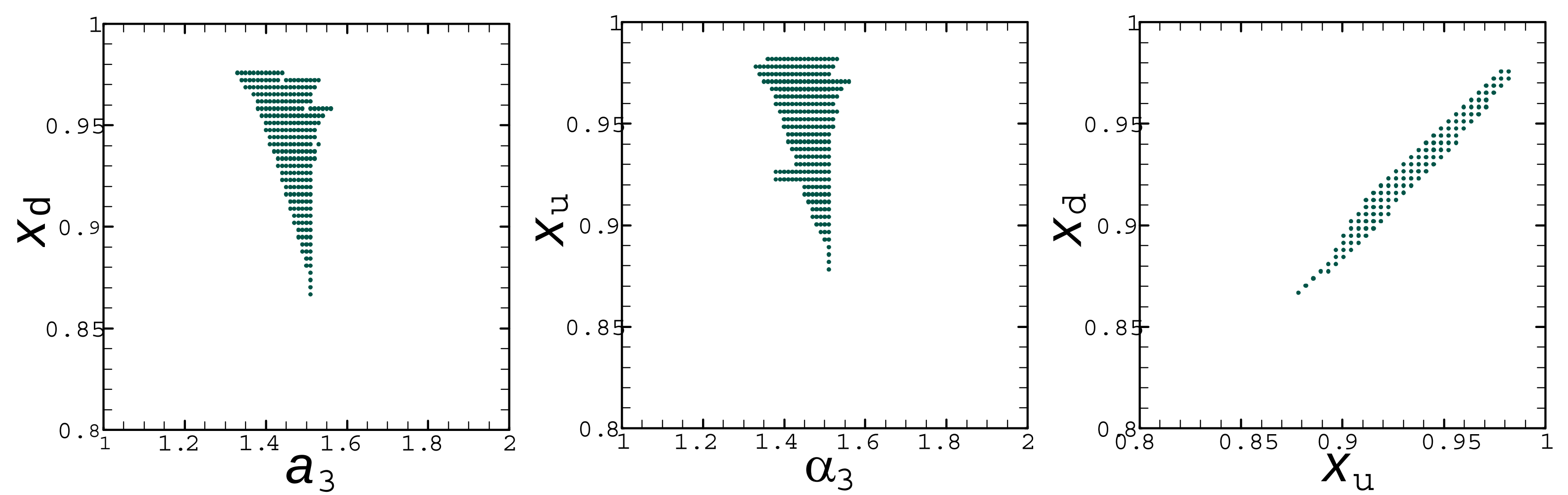
$$\alpha_2 \equiv \alpha_{u2} - \alpha_{d2} = \tau_u - \tau_d \equiv \Delta\tau ,$$

$$\alpha_3 \equiv \alpha_{u3} - \alpha_{d3} = (\tau_u - \tau_d) + (\sigma_u - \sigma_d) \equiv \Delta\tau + \Delta\sigma .$$

Note that m_u/m_t and m_c/m_t are not sensitive to CKM matrix.

Numerical estimation

- We fix the quark masses by the observed masses.
- Two component parameters x_u and x_d and two phase parameters α_2 and α_3 are left as free parameters.
- In our former paper, we find that if α_2 takes a value as $\alpha_2 \simeq \pi/2$, there are the allowed region in the dotted regions.



The numerical results

- We show the best fit values as a example

$$\Delta\tau = \pi/2,\ \Delta\sigma = -0.121,\ x_u = 0.9560,\ x_d = 0.9477$$

		Our results	The values estimated from exp data in MSSM ($\tan\beta=10$)
$\lvert m_u(M_x)\rvert$	=	1.04 [MeV]	$1.04^{+0.19}_{-0.20}$ [MeV]
$m_c(M_x)$	=	302 [MeV]	302^{+25}_{-27} [MeV]
$m_t(M_x)$	=	129 [GeV]	129^{+196}_{-40} [GeV]
$\lvert m_d(M_x)\rvert$	=	1.33 [MeV]	$1.33^{+0.17}_{-0.19}$ [MeV]
$m_s(M_x)$	=	26.5 [MeV]	$26.5^{+3.3}_{-3.7}$ [MeV]
$m_b(M_x)$	=	1.00 [GeV]	1.00 ± 0.04 [GeV]
$\lvert(U_{CKM})_{12}\rvert$	=	0.2251	0.2226 – 0.2259
$\lvert(U_{CKM})_{12}\rvert$	=	0.0340	0.0295 – 0.0387
$\lvert(U_{CKM})_{12}\rvert$	=	0.0032	0.0024 – 0.0038
δ_ε	=	58.86	46° – 74°

General SO(10)

- Each SM family + a right-handed neutrino in a single 16-dim rep.

$$W^{Y}_{SO(10)} = Y^{10}_{ij}\, 16_i 16_j 10_H + Y^{120}_{ij}\, 16_i 16_j 120_H + Y^{126}_{ij}\, 16_i 16_j 126_H$$

Here the matrices $\boldsymbol{Y}^{10}$, $\boldsymbol{Y}^{126}$ are symmetric, and $\boldsymbol{Y}^{120}$ is anti-symmetric.

- Each terms include the following mass terms, respectively.

$$16\,16\,10 \supset 5(uu^c + \nu\nu^c) + \bar{5}(dd^c + ee^c)$$
$$16\,16\,120 \supset 5\,\nu\nu^c + 45\,uu^c + \bar{5}(dd^c + ee^c) + \overline{45}(dd^c - 3ee^c)$$
$$16\,16\,126 \supset 1\,\nu\nu^c + 15\,\nu\nu + 5(uu^c - 3\nu\nu^c) + \overline{45}(dd^c - 3ee^c)$$

The relations from the SO (10) GUT

- The resulting tree level mass matrices as follows

$$M_u = S + \delta' A + \varepsilon S'$$

$$M_d = \alpha S + \delta A + S'$$

$$M_e = \alpha S + A - 3 S'$$

$$M_D = S + \delta'' A - 3\varepsilon S'$$

$$M_L = \beta S'$$

$$M_R = \gamma S'$$

Higgs : 10 (sym), 120 (anti-sym), 126 (sym)

from SO(10) GUT

The relations of quark and charged leptons in the SO (10) GUT

- First, we only discuss the mass matrices of up, down and charged lepton.

$$M_u = S + \delta' A + \varepsilon S'$$

$$M_d = \alpha S + \delta A + S'$$

$$M_e = \alpha S + A - 3 S'$$

Here, we define $r \equiv \delta'/\delta$ for the sake of later discussion.

The number of parameters in SO(10) GUT

- The number of parameters in our model.

$$S_f, A, S'_f \Rightarrow 4+2+4=10$$

$$M_f = (S_f + A_f) \Rightarrow 6\times 3 = 18$$

$$+)\ \alpha, \delta, \delta', \varepsilon \Rightarrow \quad = 4$$

$$N(pmt) \quad = 32$$

- The number of constraints from equations.

$$N(eqs) = 6\times 3 \quad = 18$$

- The number of constraints from experiments.

$$\text{masses} \Rightarrow 3\times 3 = 9$$

$$+)\ \text{CKM} \Rightarrow 3+1 = 4$$

$$N(exp) \quad = 13$$

- The number of free parameters in our model.

$$N(free) = N(pmt) - N(eqs) - N(exp) = 1$$

- After summarizing these eqs., two parameters d_e and r remain as free parameters in one equation.

$$F(r)^2\left[4\alpha a_u\cos(\Delta\tau+\tau_d)-(3+K)a_d\cos\tau_d\right]^2-\left[4\alpha c_u\cos(\Delta\sigma+\sigma_d)-(3+K)c_d\cos\sigma_d\right]^2$$
$$=(1-K)^2\left[a_e^2F(r)^2-c_e^2\right]$$

where $\Delta\tau\equiv\tau_u-\tau_d$, $\Delta\sigma\equiv\sigma_u-\sigma_d$

The parameters $d_u, d_d, \Delta\tau, \Delta\sigma$ are fixed by the CKM angles and phase in former discussion.

$$\alpha(d_u,d_d,d_e)=\frac{(3d_d+d_e)(\Sigma_d-\Sigma_e)-(d_d-d_e)(3\Sigma_d+\Sigma_e)}{4\{d_u(\Sigma_d-\Sigma_e)-(d_d-d_e)\Sigma_u\}}$$

$$K(d_u,d_d,d_e)=-\frac{(3d_d+d_e)\Sigma_u-d_u(3\Sigma_d+\Sigma_e)}{(d_d-d_e)\Sigma_u-d_u(\Sigma_d-\Sigma_e)}$$

where $\Sigma_u\equiv m_u+m_c+m_t$, $\Sigma_d\equiv m_d+m_s+m_b$, $\Sigma_e\equiv m_e+m_\mu+m_\tau$

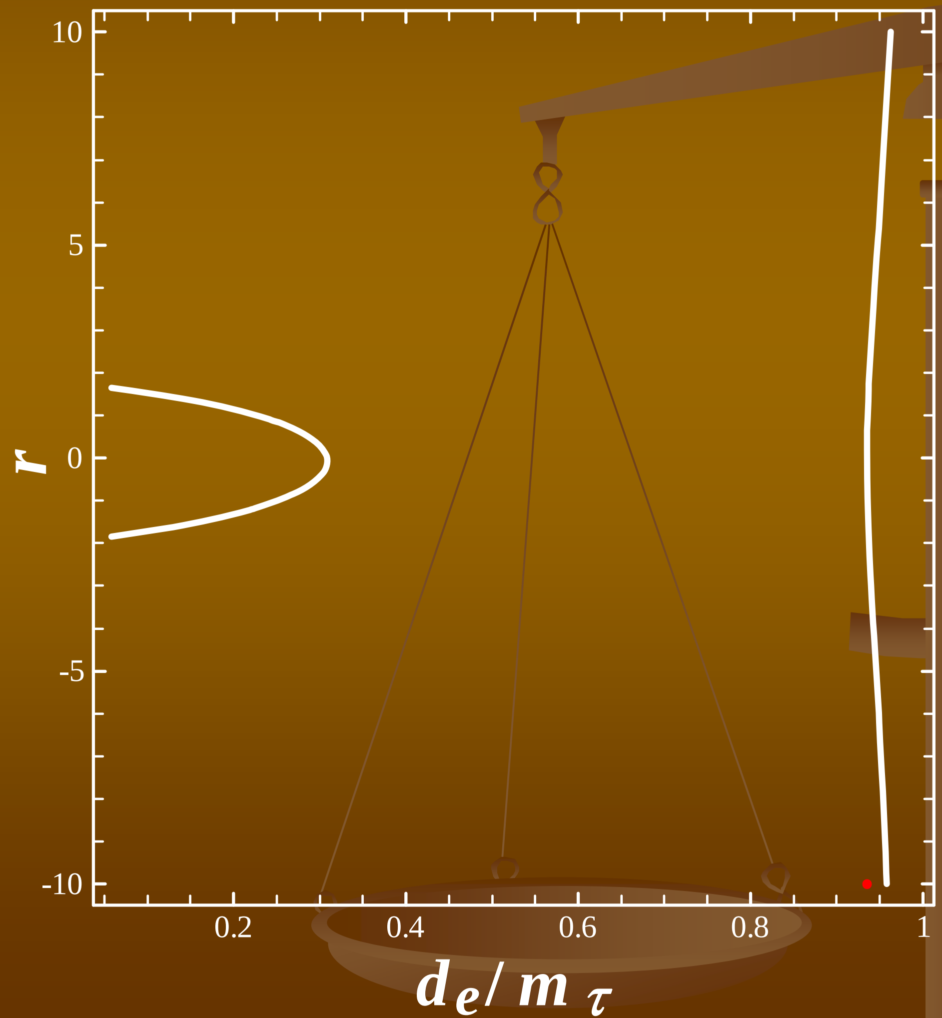
$$a_f(d_f)=\sqrt{-\frac{m_{f1}m_{f2}m_{f3}}{d_f}},\quad c_f(d_f)=\sqrt{-\frac{(d_f-m_{f1})(d_f-m_{f2})(d_f-m_{f3})}{d_f}}\quad \text{for } f=u,d,e$$

$$\tan\tau_d(r,d_u,d_d,\Delta\tau)=\frac{a_u\sin\Delta\tau}{r\,a_d-a_u\cos\Delta\tau},\quad \tan\sigma_d(r,d_u,d_d,\Delta\sigma)=\frac{c_u\sin\Delta\sigma}{r\,c_d-c_u\cos\Delta\sigma}$$

$$F(r,d_d)=\frac{c_d\sin\sigma_d}{a_d\sin\tau_d}$$

- The contour lines on which the following equation is satisfied.

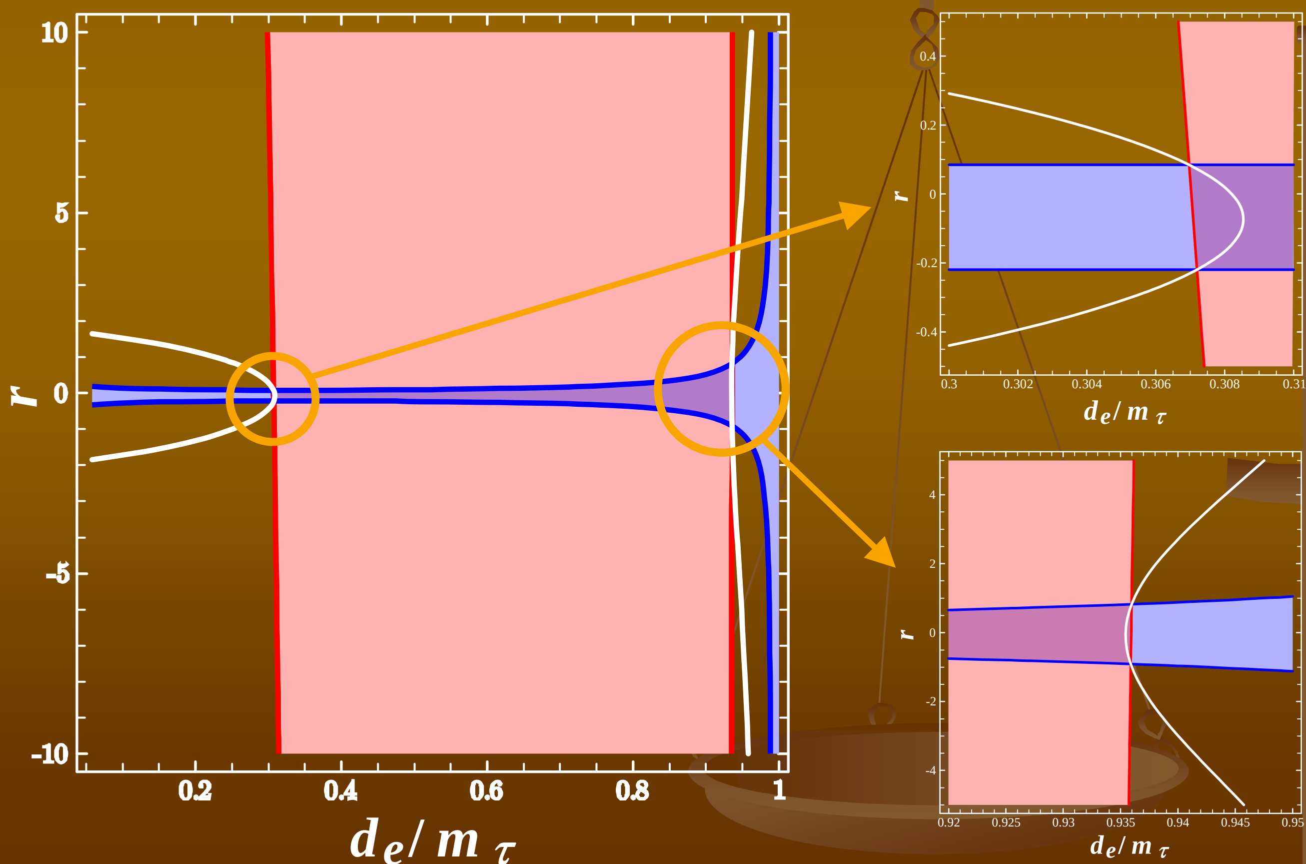
$$F(r)^2\left[4\alpha a_u\cos(\Delta\tau+\tau_d)-(3+K)a_d\cos\tau_d\right]^2-\left[4\alpha c_u\cos(\Delta\sigma+\sigma_d)-(3+K)c_d\cos\sigma_d\right]^2$$
$$=(1-K)^2\left[a_e^2F(r)^2-c_e^2\right]$$



- The additional conditions which come from the phases in the charged lepton mass matrix.

$$-1 \le \cos\tau_e(d_e, r) = \frac{4\alpha\, a_u \cos(\Delta\tau + \tau_d) - (3+K)\, a_d \cos\tau_d}{(1-K)\, a_e} \le +1 \rightarrow$$

$$-1 \le \cos\sigma_e(d_e, r) = \frac{4\alpha\, C_u \cos(\Delta\sigma + \sigma_d) - (3+K)\, C_d \cos\sigma_d}{(1-K)\, C_e} \le +1 \rightarrow$$



Sol. (a)

Zoomed view

Sol. (b)

Zoomed view

Sol. (a)

$$\underset{\sim}{S} = \begin{pmatrix} 0 & 3.1\times10^{2} & 0 \\ 3.1\times10^{2} & 3.5\times10^{4} & 3.6\times10^{4} \\ 0 & 3.6\times10^{4} & 1.0\times10^{5} \end{pmatrix}$$

$$\underset{\sim}{S}' = \begin{pmatrix} 0 & -2.0 & 0 \\ -2.0 & -2.0\times10^{2} & -6.7\times10 \\ 0 & -6.7\times10 & 1.5\times10^{2} \end{pmatrix}, \quad A = i\begin{pmatrix} 0 & -2.5\times10^{-1} & 0 \\ 2.5\times10^{-1} & 0 & 1.1 \\ 0 & -1.1 & 0 \end{pmatrix}$$

$$\alpha = 7.9\times10^{-3}, \quad \delta = 2.3\times10, \quad \delta' = -5.2, \quad \varepsilon = 1.5\times10^{2}$$

Sol. (b)

$$\underset{\sim}{S} = \begin{pmatrix} 0 & 3.6\times10^{2} & 0 \\ 3.6\times10^{2} & 1.3\times10^{4} & 3.3\times10^{4} \\ 0 & 3.3\times10^{4} & 1.4\times10^{4} \end{pmatrix}$$

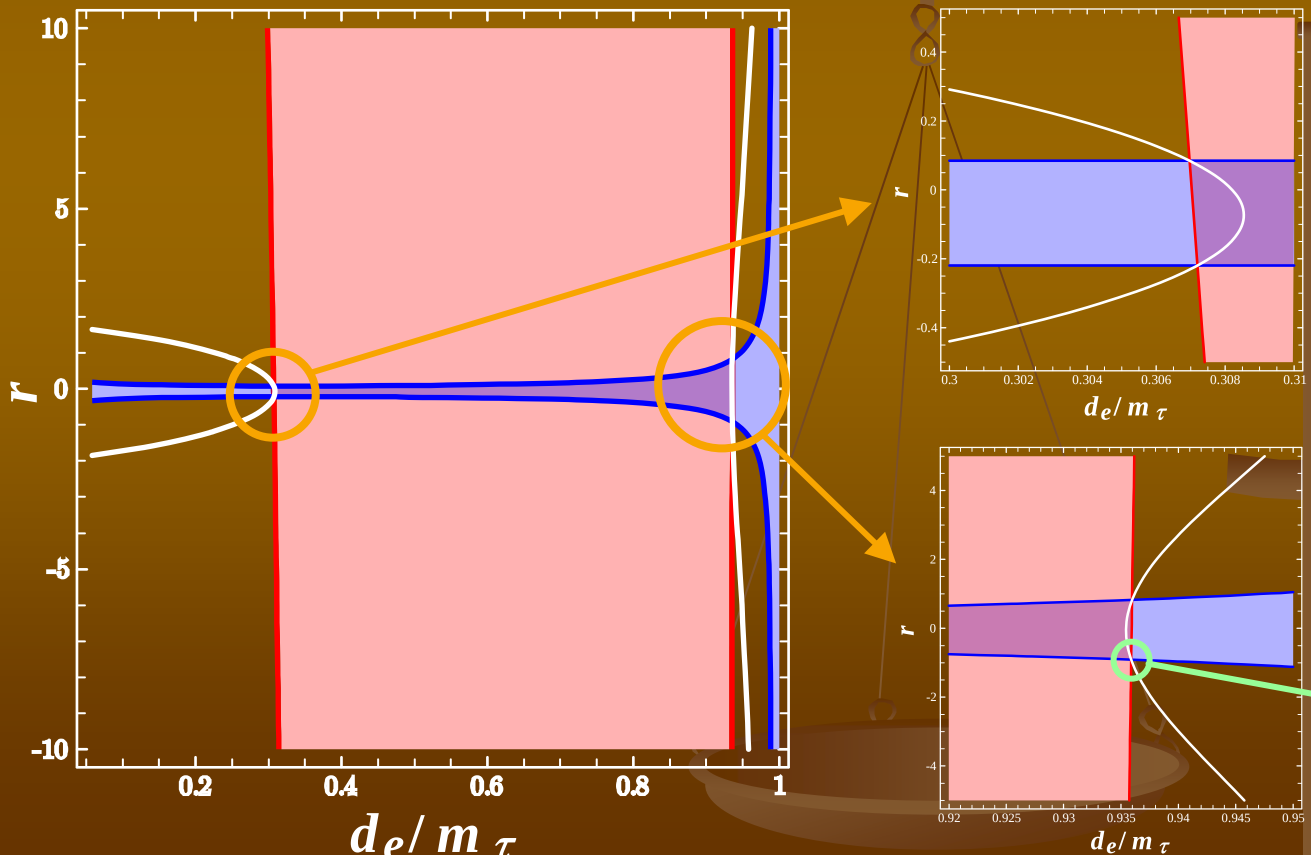
$$\underset{\sim}{S}' = \begin{pmatrix} 0 & -7.7\times10^{-1} & 0 \\ -7.7\times10^{-1} & -1.6\times10 & -1.4\times10 \\ 0 & -1.5\times10 & -3.7\times10 \end{pmatrix}, \quad A = i\begin{pmatrix} 0 & 5.3 & 0 \\ -5.3 & 0 & -2.4\times10 \\ 0 & 2.4\times10 & 0 \end{pmatrix}$$

$$\alpha = 7.0\times10^{-3}, \quad \delta = 1.0\times10^{2}, \quad \delta' = -9.3\times10, \quad \varepsilon = 4.5\times10^{2}$$

- The additional conditions which come from the phases in the charged lepton mass matrix.

$$-1 \le \cos\tau_e(d_e, r) = \frac{4\alpha a_u \cos(\Delta\tau + \tau_d) - (3+K) a_d \cos\tau_d}{(1-K) a_e} \le +1 \rightarrow$$

$$-1 \le \cos\sigma_e(d_e, r) = \frac{4\alpha c_u \cos(\Delta\sigma + \sigma_d) - (3+K) c_d \cos\sigma_d}{(1-K) c_e} \le +1 \rightarrow$$



In this talk, I will concentrate at this point. Namely, the contribution of 120 is small.

The neutrino mass matrix predicted from FZT in the SO (10) GUT

- As we have shown, the quark and charged lepton parts are OK.
- Next, Let's discuss the neutrino mass matrix.

$$M_u = S + \delta' A + \varepsilon S'$$
$$M_d = \alpha S + \delta A + S'$$
$$M_e = \alpha S + A - 3 S'$$

OK! & All parameters are determined by the former discussion

$$M_D = S + \delta'' A - 3\varepsilon S'$$
$$M_L = \beta S'$$
$$M_R = \gamma S'$$
$$m_\nu = M_L - M_D M_R^{-1} M_D^T$$

There are three free pmt in the ν part.

The neutrino mass matrix predicted from FZT in the SO (10) GUT

- As we have shown, the quark and charged lepton parts are OK.
- Next, Let's discuss the neutrino mass matrix.
- We use the following global analysis of neutrino experiments.

$$0.25 < \sin^2\theta_{12} < 0.38$$

$$0.35 < \sin^2\theta_{23} < 0.65$$

$$\sin^2\theta_{13} < 0.03$$

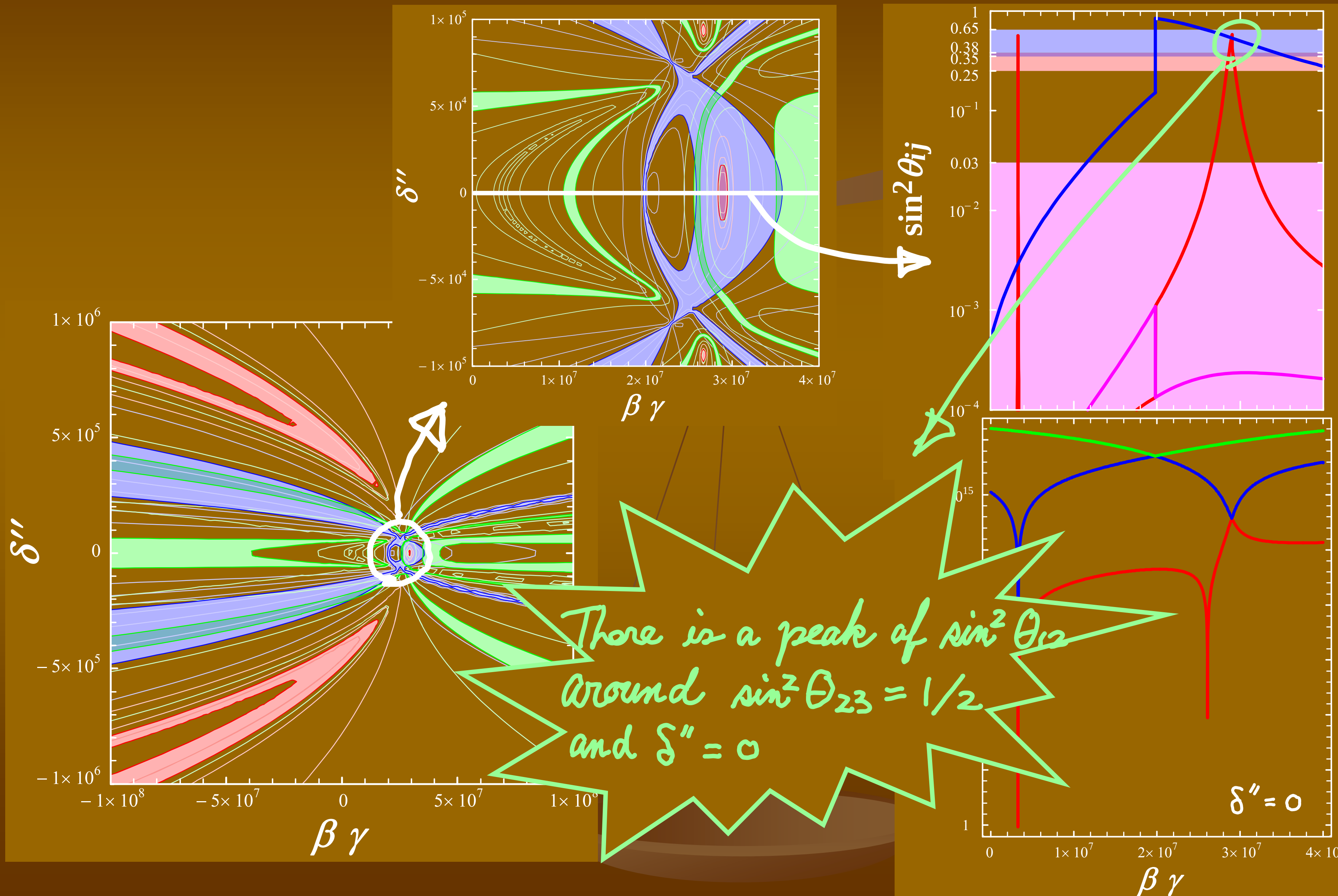
$$\begin{cases} \Delta m_{21}^2 = (7.2 - 8.9) \times 10^{-5}\ \mathrm{eV}^2 \\ |\Delta m_{32}^2| = (2.1 - 3.1) \times 10^{-3}\ \mathrm{eV}^2 \end{cases}$$

$$\rightarrow \frac{\Delta m_{21}^2}{|\Delta m_{32}^2|} = (2.3 - 4.2) \times 10^{-2}$$

at 99% CL

A. Strumia, F. Vissani, hep-ph/0606054

- The allowed regions of neutrino masses and mixing angles in the case of the normal hierarchy at Sol. (b)



Sol. (b)

$$M_e = \alpha S + A - 3S' = \begin{pmatrix} 0 & 4.8 - 5.7\times10^{-2} i & 0 \\ 4.8 + 5.7\times10^{-2} i & 1.4\times10^{2} & 2.8\times10^{2} + 2.6\times10^{-1} i \\ 0 & 2.8\times10^{2} - 2.6\times10^{-1} i & 1.1\times10^{3} \end{pmatrix}$$

$$M_D = S + \delta'' A - 3\varepsilon S' = \begin{pmatrix} 0 & 1.4\times10^{3} & 0 \\ 1.4\times10^{3} & 3.5\times10^{4} & 5.3\times10^{4} \\ 0 & 5.3\times10^{4} & 1.9\times10^{5} \end{pmatrix}$$

$\delta'' = 0$

$$M_L = \beta S', \quad M_R = \gamma S' = \gamma \begin{pmatrix} 0 & -7.7\times10^{-1} & 0 \\ -7.7\times10^{-1} & -1.6\times10 & -1.4\times10 \\ 0 & -1.5\times10 & -3.7\times10 \end{pmatrix}$$

$$m_\nu = M_L - M_D M_R^{-1} M_D = \frac{1}{\gamma}\left[\beta\gamma\, S' - M_D (S')^{-1} M_D^T\right]$$

$\beta\gamma = 2.85\times10^{7}$

$$= \frac{1}{\gamma}\begin{pmatrix} 0 & -2.0\times10^{7} & 0 \\ -2.0\times10^{7} & -3.9\times10^{8} & -2.1\times10^{8} \\ 0 & -2.1\times10^{8} & -1.2\times10^{8} \end{pmatrix}$$

$\sin^2\theta_{23} = 0.53, \quad \sin^2\theta_{13} = 2.3\times10^{-4}$

$\sin^2\theta_{12} = 0.29, \quad \Delta m^2_{12}/\Delta m^2_{13} = 4.4\times10^{-4}$

It is too small.

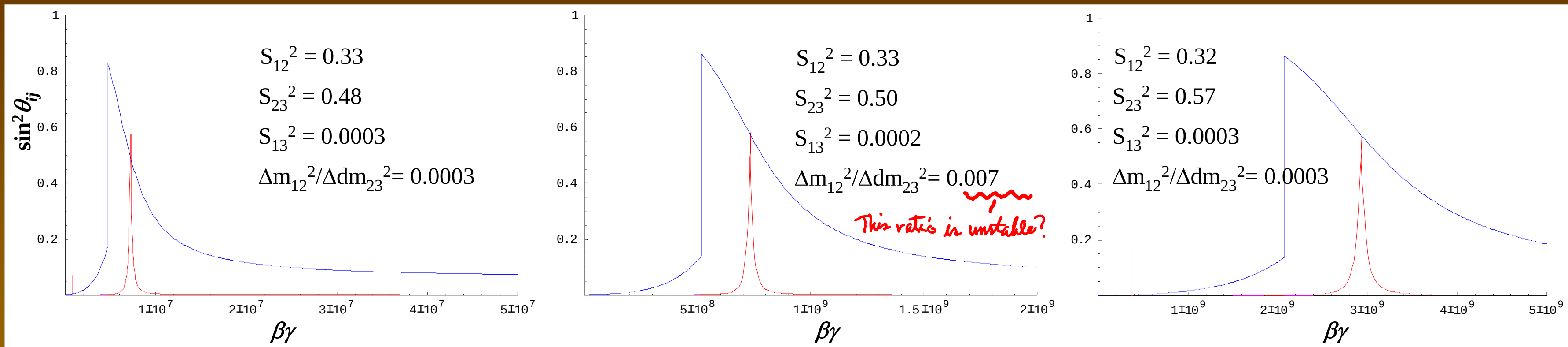
The uncertainty from the RG effects

- We don't know how much the true masses of fermions are in the GUT scale because nobody know what is really happened above TeV scale.
- However, in the most case, the ratio m_{f1}/m_{f2} is stable.
- Therefore, by changing only m_{f3} in the very wide range even where it is unreasonable value theoretically, we will check the stability of the bi-large mixing.

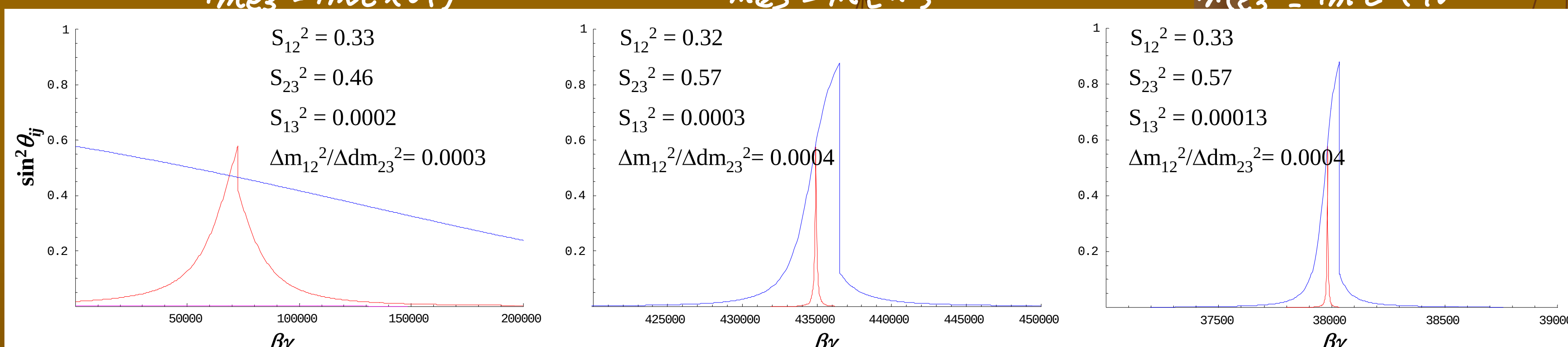
- I regret that the discussions about the uncertainty from the CKM matrix and so on are not finished.

- There is always the peak of $\sin^2\theta_{12}$ around $\sin^2\theta_{12}=1/2$, even if m_{u3} and m_{e3} are changed.

$m_{u3} = m_t \times 0.5$ $\quad$ $m_{u3} = m_t \times 5$ $\quad$ $m_{u3} = m_t \times 10$



$m_{e3} = m_\tau \times 0.5$ $\quad$ $m_{e3} = m_\tau \times 5$ $\quad$ $m_{e3} = m_\tau \times 10$



- The rates of change of each elements with respect to m_{e3} are incoherent as follows.

$$\frac{S_{ij}(m_{e3}=0.5\,m_\tau)}{S_{ij}(m_{e3}=10\,m_\tau)} = \begin{pmatrix} \text{Indeterminate} & -10.7516 & \text{Indeterminate} \\ -10.7516 & -5.03675 & -3.10112 \\ \text{Indeterminate} & -3.10112 & -2.81708 \end{pmatrix}$$

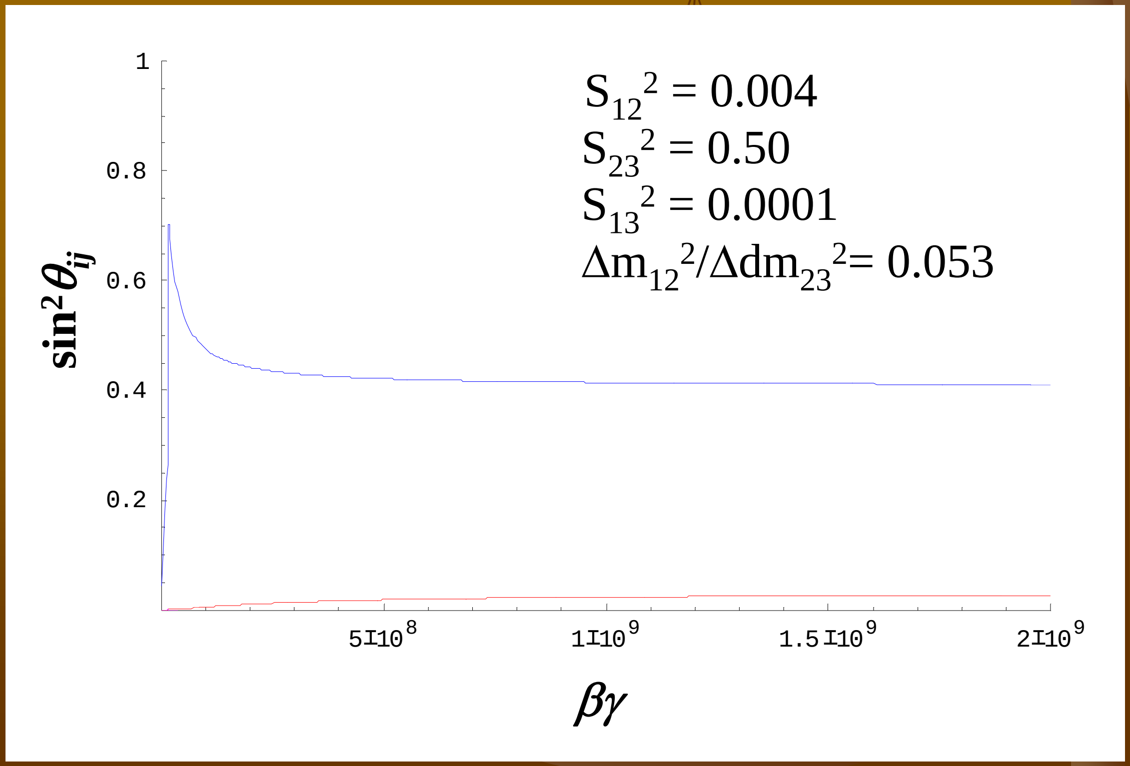
$$\frac{S'_{ij}(m_{e3}=0.5\,m_\tau)}{S'_{ij}(m_{e3}=10\,m_\tau)} = \begin{pmatrix} \text{Indeterminate} & 1.17063 & \text{Indeterminate} \\ 1.17063 & 0.0894184 & -0.0262295 \\ \text{Indeterminate} & -0.0262295 & -0.0407902 \end{pmatrix}$$

$$\frac{A_{ij}(m_{e3}=0.5\,m_\tau)}{A_{ij}(m_{e3}=10\,m_\tau)} = \begin{pmatrix} \text{Indeterminate} & 1.22501 & \text{Indeterminate} \\ 1.22501 & \text{Indeterminate} & 1.20363 \\ \text{Indeterminate} & 1.20363 & \text{Indeterminate} \end{pmatrix}$$

Because the CKM matrix is sensitive to m_{d3}, we must check more carefully when we change m_{d3}. However I have no time to check….

- Note that we can not take the bi-large mixing for granted in the general FZT.
- If you change other masses, the bi-large mixing is sometimes forbidden. For example, if muon was more heavy, the large $\sin^2\theta_{12}$ can not be derived.

$m_{ez} = m_\mu \times 3$



Summary of our FZT mass matrix model

- Many people have discussed the FZT mass matrix model;
  H. Nishiura, K. M, and T. Fukuyama, PRD **60**,013006 (1999).
- And they applied the FZT mass matrix model to SO(10)GUT;
  K. M, T. Fukuyama, and H. Nishiura, PRD **61**, 053001 (2000).

Recently, some people say "the FZT in quark sector is dying";

modified

We check this opinion.
K. M and H. Nishiura, PRD **74,** 033014 (2006)

the solutions is remained

Apply the results

Recently, some people say "the some FZT in the SO(10) GUT is dying";

Perhaps people will say "if there are solutions, it must be fine-tuning."

We will check the our model.
In this talk

We find the solutions in the SO(10) GUT.

change the parameters

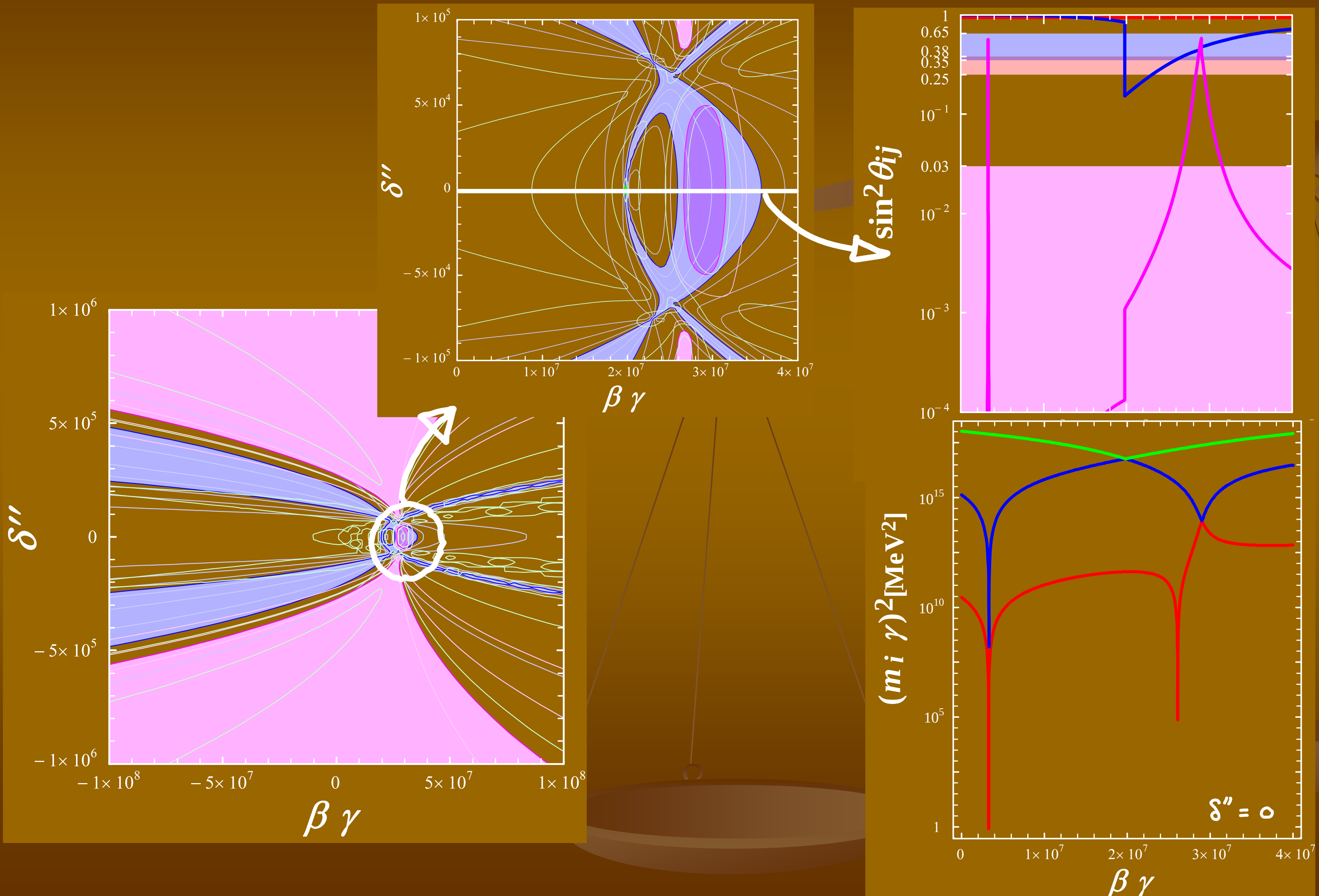
We'll also check this opinion.
In this talk (if there is time.)

There is the hidden property in FZT which always makes the peak of $\sin^2\theta_{12}$ around $\sin^2\theta_{23}$=1/2.

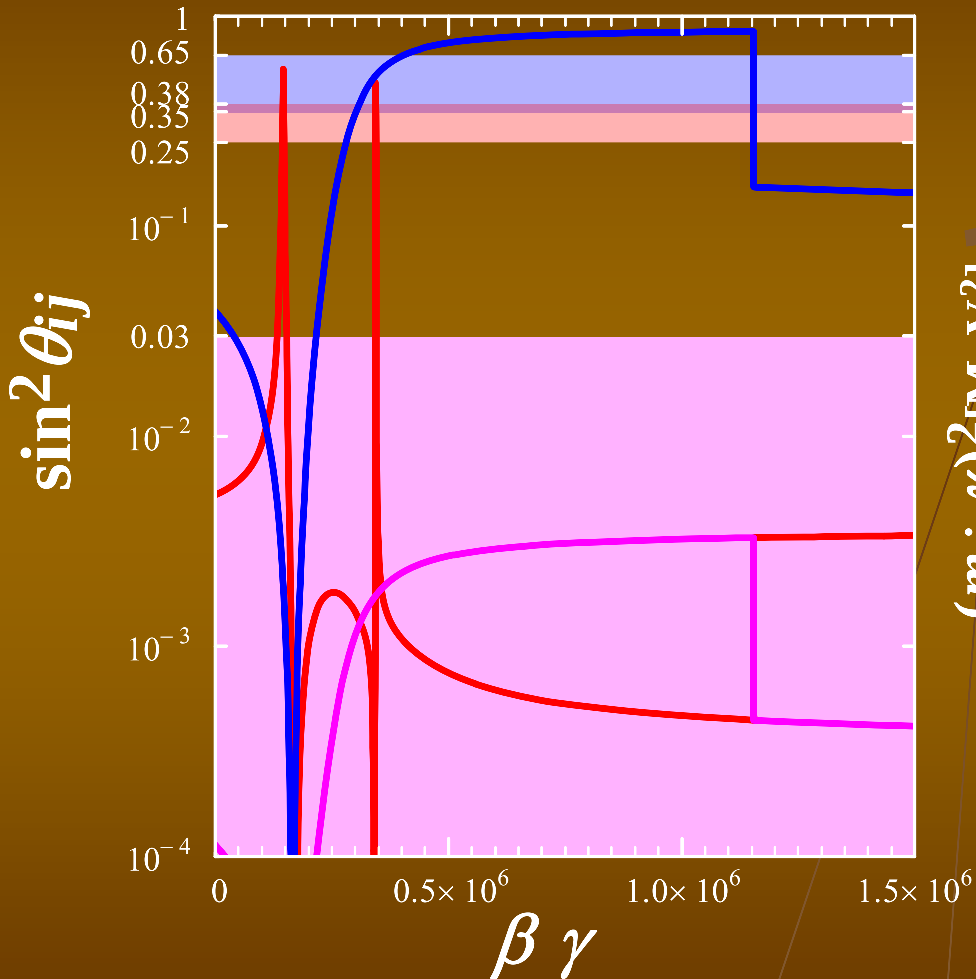
The remaining problems in the FZT in SO(10) GUT

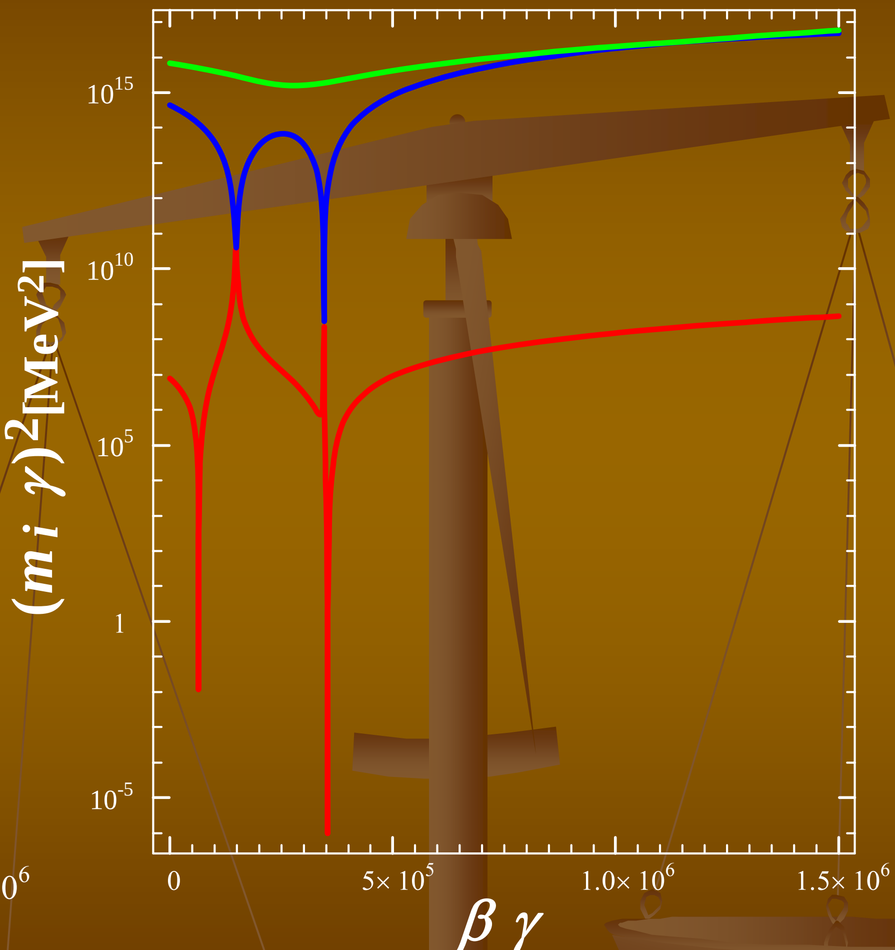
- This is the model under consideration.
- $\Delta m_{12}{}^2$ is too small.
- We don't know the reasons or the mechanisms which always make the peak of $\sin^2\theta_{12}$ around $\sin^2\theta_{23} = 1/2$ under some conditions.
- The stability of the bi-large mixing is not sufficiently discussed yet.
- We don't know where the FZT come from.

- The allowed regions of neutrino masses and mixing angles in the case of the inverse hierarchy at Sol. (b)

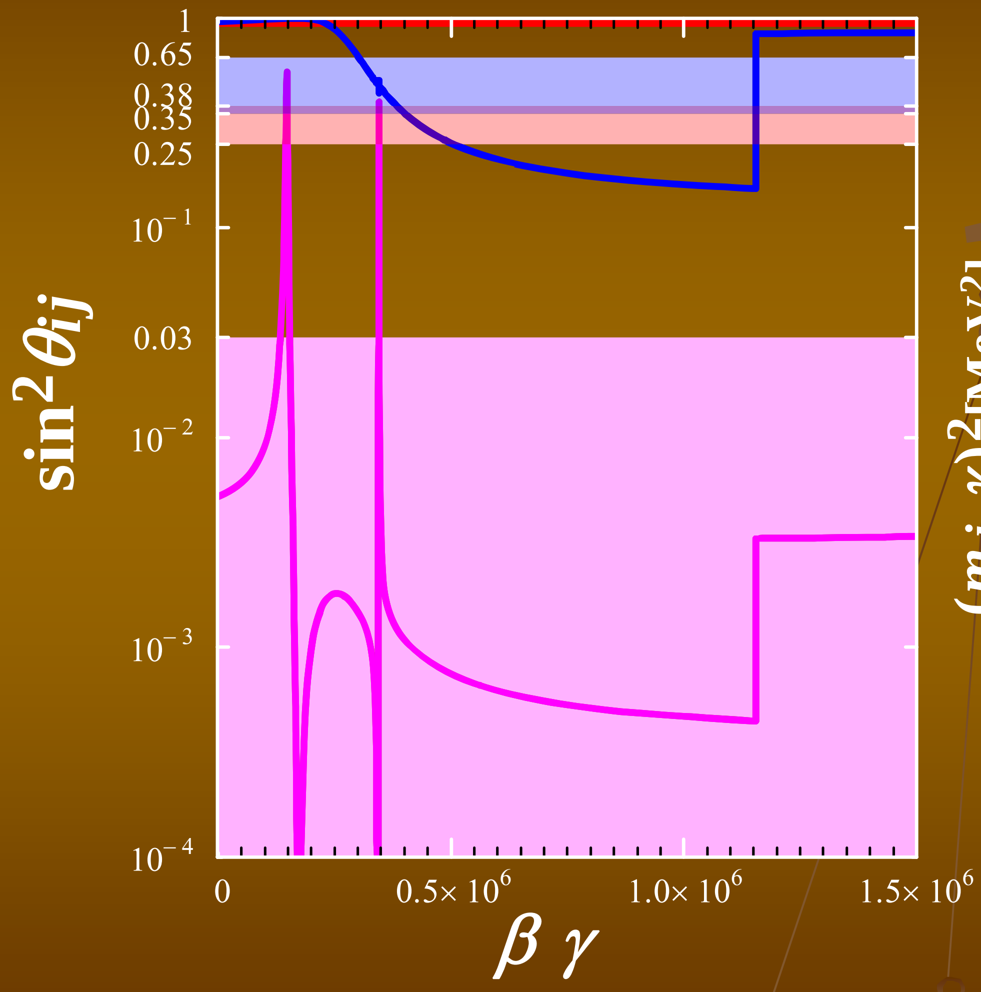


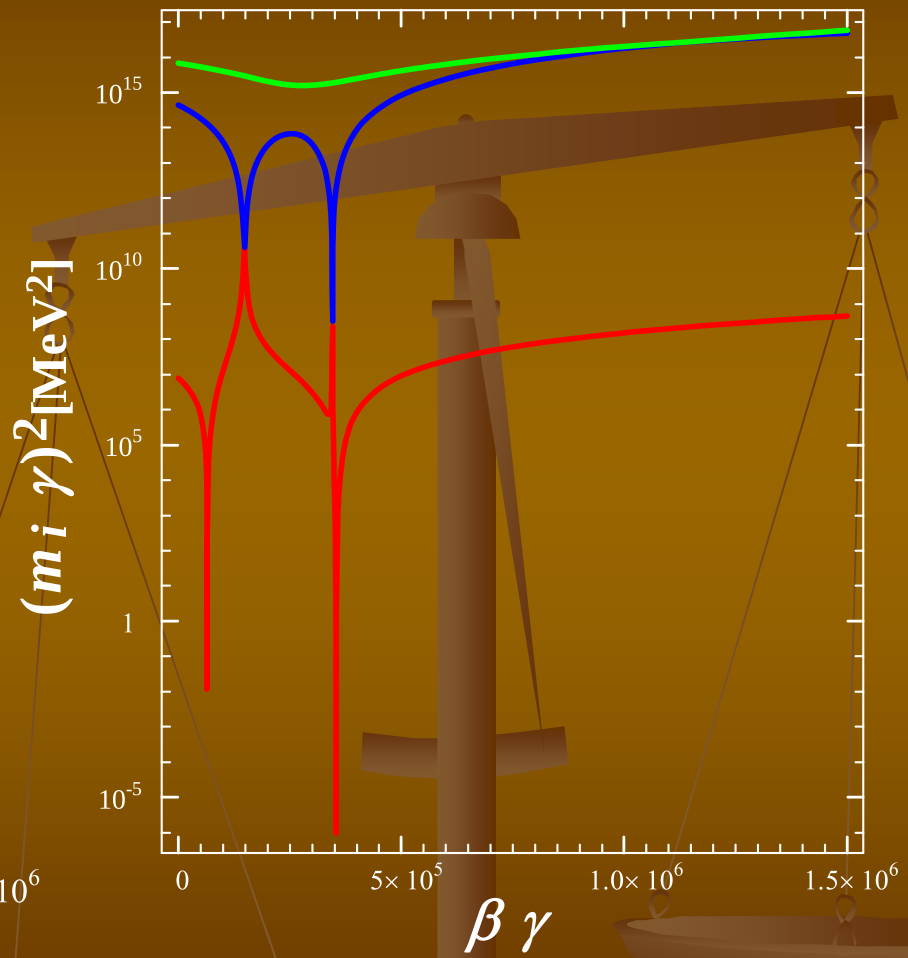
- The allowed regions of neutrino masses and mixing angles in the case of the normal hierarchy at Sol. (a)





- The allowed regions of neutrino masses and mixing angles in the case of the inverse hierarchy at Sol. (a)





By substituting " $S = \frac{3S_d + S_e}{4\alpha}$ and $S' = \frac{S_d - S_e}{4}$ "
in the GUT relations, these relation is given

$$4\alpha S_u = (3S_d + S_e) + K(S_d - S_e)$$

where $K \equiv \alpha \varepsilon$.

$$4\alpha d_u = [3d_d + d_e] + K[d_d - d_e]$$

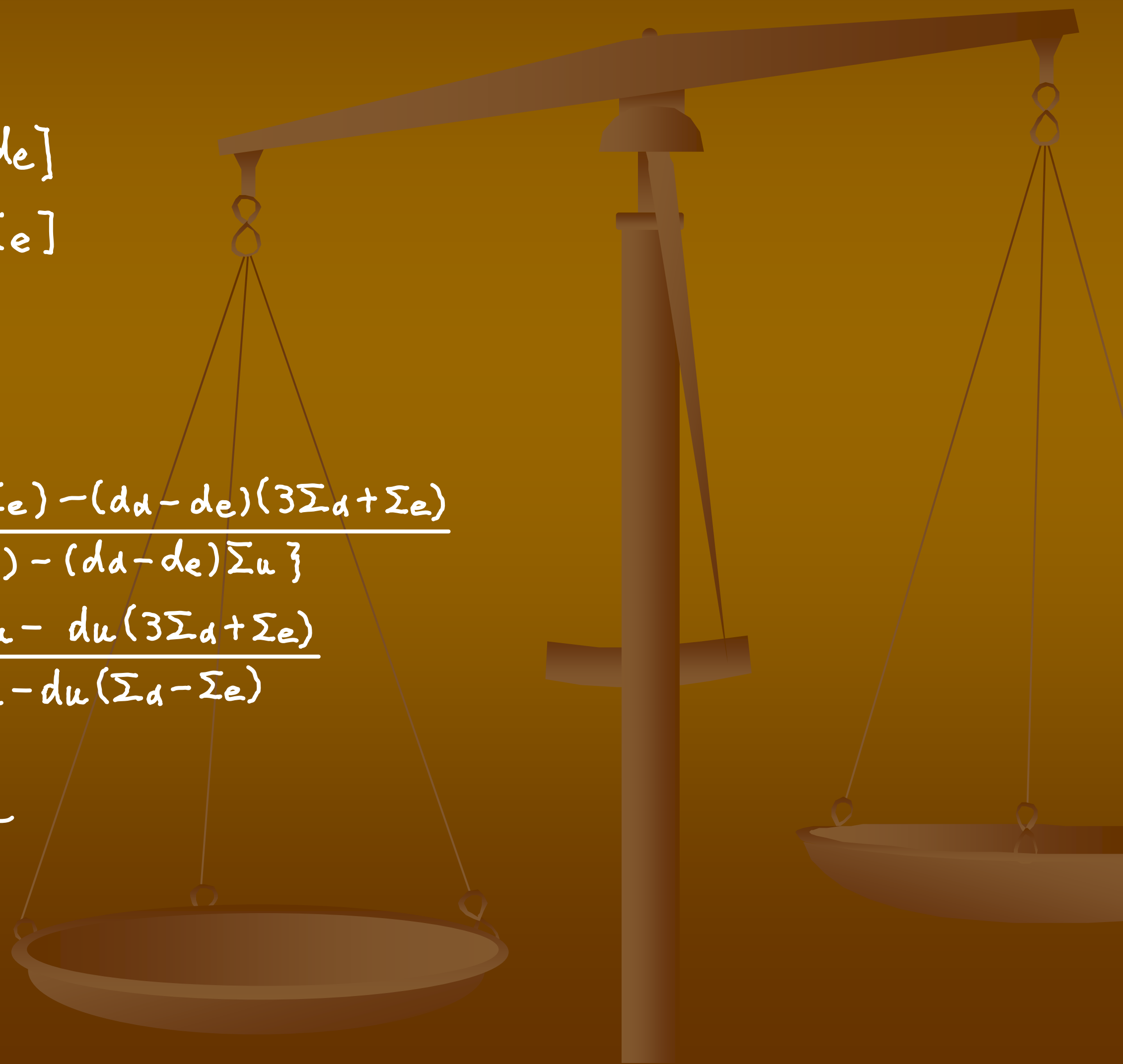
$$4\alpha \Sigma_u = [3\Sigma_d + \Sigma_e] + K[\Sigma_d - \Sigma_e]$$

where
$$\begin{cases} \Sigma_u \equiv m_u + m_c + m_t \\ \Sigma_d \equiv m_d + m_s + m_b \\ \Sigma_e \equiv m_e + m_\mu + m_\tau \end{cases}$$

$$\alpha(d_u, d_d, d_e) = \frac{(3d_d + d_e)(\Sigma_d - \Sigma_e) - (d_d - d_e)(3\Sigma_d + \Sigma_e)}{4\{d_u(\Sigma_d - \Sigma_e) - (d_d - d_e)\Sigma_u\}}$$

$$K(d_u, d_d, d_e) = -\frac{(3d_d + d_e)\Sigma_u - d_u(3\Sigma_d + \Sigma_e)}{(d_d - d_e)\Sigma_u - d_u(\Sigma_d - \Sigma_e)}$$

α, Kは d_u, d_d, d_e の関数



From $\frac{1}{\delta} A_d = \frac{1}{\delta'} A_u$, we get

$$\frac{1}{\delta} a_d \sin \tau_d = \frac{1}{\delta'} a_u \sin(\tau_d + \Delta\tau)$$

$$= \frac{1}{\delta'} a_u \left[\sin \tau_d \cos \Delta\tau + \cos \tau_d \sin \Delta\tau\right]$$

$$\frac{1}{\delta} c_d \sin \sigma_d = \frac{1}{\delta'} c_u \sin(\sigma_d + \Delta\sigma)$$

$$= \frac{1}{\delta'} c_u \left[\sin \sigma_d \cos \Delta\sigma + \cos \sigma_d \sin \Delta\sigma\right]$$

Here, $a_f = \sqrt{-\frac{m_{f1} m_{f2} m_{f3}}{d_f}}$, $c_f = \sqrt{-\frac{(d_f - m_{f1})(d_f - m_{f2})(d_f - m_{f3})}{d_f}}$

$$\begin{cases} \tan \tau_d(r, d_u, d_d) = \dfrac{\frac{1}{\delta'} a_u \sin \Delta\tau}{\frac{1}{\delta} a_d - \frac{1}{\delta'} a_u \cos \Delta\tau} = \dfrac{a_u \sin \Delta\tau}{r a_d - a_u \cos \Delta\tau} \\[2ex] \tan \sigma_d(r, d_u, d_d) = \dfrac{\frac{1}{\delta'} c_u \sin \Delta\sigma}{\frac{1}{\delta} c_d - \frac{1}{\delta'} c_u \cos \Delta\sigma} = \dfrac{c_u \sin \Delta\sigma}{r c_d - c_u \cos \Delta\sigma} \end{cases}$$

where, $r \equiv \delta'/\delta$

残る方程式は

$$a_e \sin\tau_e = \frac{1}{\delta} a_d \sin\tau_d$$

$$c_e \sin\sigma_e = \frac{1}{\delta} c_d \sin\sigma_d$$

$$4\alpha a_u \cos(\Delta\tau+\tau_d) = (3+K) a_d \cos\tau_d + (1-K) a_e \cos\tau_e$$

$$4\alpha c_u \cos(\Delta\sigma+\sigma_d) = (3+K) c_d \cos\sigma_d + (1-K) c_e \cos\sigma_e$$

} # of Eqs = 4

$d_u, d_d, \Delta\tau \equiv \tau_u-\tau_d, \Delta\sigma \equiv \sigma_u-\sigma_d$ はCKM決定の自由度に使われる

上の方程式を使って残るのは $\underbrace{d_e, \tau_e, \sigma_e, \delta, r}_{5コ}$ のどれか1つ

恐らく d_e をfree pmtにするのが良い

以下 d_e をinputして議論 → a_f, b_f, c_f, d_f は全て決まる。定数として取り扱う

$$\delta = \frac{a_d \sin \tau_d}{a_e \sin \tau_e} = \frac{c_d \sin \sigma_d}{c_e \sin \sigma_e} \Leftrightarrow \frac{c_e \sin \sigma_e}{a_d \sin \tau_d}\delta = \frac{c_e \sin \sigma_e}{a_e \sin \tau_e} = \frac{c_d \sin \sigma_d}{a_d \sin \tau_d} \equiv F(r)$$

$$\begin{cases} \sin \tau_e(r, \sigma_e) = \dfrac{c_e \sin \sigma_e}{a_e F(r)} \Rightarrow \left[a_e \cos \tau_e\right]^2 = a_e^2 - \left(\dfrac{c_e \sin \sigma_e}{F(r)}\right)^2 \\ \delta(r, \sigma_e) = \dfrac{1}{c_e \sin \sigma_e} a_d \sin \tau_d F(r) \equiv \dfrac{1}{c_e \sin \sigma_e} G(r) \end{cases}$$

残るpmtは r と $\sin \sigma_e$ のみ！

$$
\begin{cases}
4\alpha a_u \cos(\Delta\tau+\tau_d) = (3+K) a_d \cos\tau_d + (1-K) a_e \cos\tau_e \\
4\alpha C_u \cos(\Delta\sigma+\sigma_d) = (3+K) C_d \cos\sigma_d + (1-K) C_e \cos\sigma_e
\end{cases}
$$

$$
\begin{cases}
\left[4\alpha a_u \cos(\Delta\tau+\tau_d) - (3+K) a_d \cos\tau_d\right]^2 = (1-K)^2 \left[a_e^2 - \left(\dfrac{C_e \sin\sigma_e}{F(r)}\right)^2\right] \\
\left[4\alpha C_u \cos(\Delta\sigma+\sigma_d) - (3+K) C_d \cos\sigma_d\right]^2 = (1-K)^2 \left[C_e^2 - (C_e \sin\sigma_e)^2\right]
\end{cases}
$$

$$
F(r)^2\left[4\alpha a_u \cos(\Delta\tau+\tau_d) - (3+K) a_d \cos\tau_d\right]^2 - \left[4\alpha C_u \cos(\Delta\sigma+\sigma_d) - (3+K) C_d \cos\sigma_d\right]^2
$$

$$
= (1-K)^2\left[a_e^2 F(r)^2 - C_e^2\right]
$$

… d_e と r のみの方程式!

d_eを動かしつつ
この方程式を満たす
rを求める

→ rが見つかる! ☺ → σ_eの決定

↓

全て決定 → ν-mass, MNS へ

to be continued

↓

rが見つからない ☹ → The End

Sol. (a)

$$\underset{\sim}{S} = \begin{pmatrix} 0 & 3{,}1\times10^{2} & 0 \\ 3{,}1\times10^{2} & 3.5\times10^{4} & 3{,}6\times10^{4} \\ 0 & 3{,}6\times10^{4} & 1{,}0\times10^{5} \end{pmatrix}$$

$$\underset{\sim}{S}' = \begin{pmatrix} 0 & -2{,}0 & 0 \\ -2{,}0 & -2{,}0\times10^{2} & -6.7\times10 \\ 0 & -6.7\times10 & 1.5\times10^{2} \end{pmatrix}, \quad A = i\begin{pmatrix} 0 & -2{,}5\times10^{-1} & 0 \\ 2.5\times10^{-1} & 0 & 1{,}1 \\ 0 & -1{,}1 & 0 \end{pmatrix}$$

$$\alpha = 7.9\times10^{-3}, \quad \delta = 2.3\times10, \quad \delta' = -5{,}2, \quad \varepsilon = 1.5\times10^{2}$$

$$M_u = \begin{pmatrix} 0 & 1.8\times10 + 1{,}3i & 0 \\ 1{,}8\times10 - 1{,}3i & 6.0\times10^{3} & 2{,}6\times10^{4} - 5{,}8i \\ 0 & 2{,}6\times10^{4} + 5{,}8i & 1{,}2\times10^{5} \end{pmatrix}$$

$$M_d = \begin{pmatrix} 0 & 4.4\times10^{-1} - 6{,}1i & 0 \\ 4.4\times10^{-1} + 6{,}1i & 7.7\times10 & 2.2\times10^{2} + 2{,}6\times10i \\ 0 & 2{,}2\times10^{2} - 2{,}6\times10i & 9{,}5\times10^{2} \end{pmatrix}$$

$$M_e = \begin{pmatrix} 0 & -8{,}5 - 2.6\times10^{-1}i & 0 \\ 8{,}5 + 2{,}6\times10^{-1} & 8{,}8\times10^{2} & 4{,}8\times10^{2} + 1{,}1i \\ 0 & 4{,}8\times10^{2} - 1.1i & 3.6\times10^{2} \end{pmatrix}$$

Sol. (b)

$$S = \begin{pmatrix} 0 & 3.6\times10^{2} & 0 \\ 3.6\times10^{2} & 1.3\times10^{4} & 3.3\times10^{4} \\ 0 & 3.3\times10^{4} & 1.4\times10^{4} \end{pmatrix}$$

$$S' = \begin{pmatrix} 0 & -7.7\times10^{-1} & 0 \\ -7.7\times10^{-1} & -1.6\times10 & -1.4\times10 \\ 0 & -1.5\times10 & -3.7\times10 \end{pmatrix}, \quad A = i\begin{pmatrix} 0 & 5.3 & 0 \\ -5.3 & 0 & -2.4\times10 \\ 0 & 2.4\times10 & 0 \end{pmatrix}$$

$$\alpha = 7.0\times10^{-3}, \quad \delta = 1.0\times10^{2}, \quad \delta' = -9.3\times10, \quad \varepsilon = 4.5\times10^{2}$$

$$M_u = \begin{pmatrix} 0 & 1.7\times10+5.3i & 0 \\ 1.7\times10-5.3i & 6.0\times10^{3} & 2.6\times10^{4}-2.4\times10\,i \\ 0 & 2.6\times10^{4}+2.4\times10\,i & 1.2\times10^{5} \end{pmatrix}$$

$$M_d = \begin{pmatrix} 0 & 1.8-5.8i & 0 \\ 1.8+5.8i & 7.7\times10 & 2.2\times10^{2}+2.6\times10\,i \\ 0 & 2.2\times10^{2}-2.6\times10\,i & 9.5\times10^{2} \end{pmatrix}$$

$$M_e = \begin{pmatrix} 0 & 4.8-5.7\times10^{-2}i & 0 \\ 4.8+5.7\times10^{-2}i & 1.4\times10^{2} & 2.8\times10^{2}+2.6\times10^{-1}i \\ 0 & 2.8\times10^{2}-2.6\times10^{-1}i & 1.1\times10^{3} \end{pmatrix}$$

The numerical results

$$|m_u(M_X)| = 1.04^{+0.19}_{-0.20}\ [\mathrm{MeV}]$$

$$m_c(M_X) = 302^{+25}_{-27}\ [\mathrm{MeV}]$$

$$m_t(M_X) = 129^{+196}_{-40}\ [\mathrm{GeV}]$$

$$|m_d(M_X)| = 1.33^{+0.17}_{-0.19}\ [\mathrm{MeV}]$$

$$m_s(M_X) = 26.5^{+3.3}_{-3.7}\ [\mathrm{MeV}]$$

$$m_b(M_X) = 1.00 \pm 0.04\ [\mathrm{GeV}]$$

$$|m_e(M_X)| = 0.3250\cdots\ [\mathrm{MeV}]$$

$$m_\mu(M_X) = 68.598\cdots\ [\mathrm{MeV}]$$

$$m_\tau(M_X) = 1171.4 \pm 0.2\ [\mathrm{MeV}]$$

$$|(U_{CKM})_{12}| = 0.2226 - 0.2259$$

$$|(U_{CKM})_{12}| = 0.0295 - 0.0387$$

$$|(U_{CKM})_{12}| = 0.0024 - 0.0038$$

$$\delta_\varepsilon = 46^\circ - 74^\circ$$

- The allowed regions of neutrino masses and mixing angles in the case of the normal hierarchy at Sol. (b)