International Workshop on Neutrino Masses and Mixings
December 18, 2006, Shizuoka, Japan

Charged Lepton Mass Formula -- Development and Prospect --

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1 Beginning of the charged lepton mass formula

It is generally considered that masses and mixings of the quarks and leptons will obey a simple law of nature, so that we expect that we will find a beautiful relation among those values. It is also considered that the mass matrices of the fundamental particles will be governed by a symmetry.

My dream is to understand masses and mixings from a symmetry, i.e.

not from adjustable parameters, but from Clebsch-Gordan-like coefficients.

For example, **in 1971**, **I tried to understand the Cabibbo mixing from a U(3) symmetry**

The Cabibbo Angle and the Structure of Urbaryons

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By introducing the group $U(4)$ into the world of leptons, we discuss the group theoretical structure of the Hiroshima model, in which urbaryons are assumed to be compound systems of a massive Dirac particle B and lepton-antilepton clounds. Under some assumptions for the properties of the B-particle, we can obtain $\sin\theta = (\sqrt{3}-1)/2\sqrt{2} = 0259$ and $\cos\theta = (\sqrt{3}+1)/2\sqrt{2} = 0966$ as the value of Cabibbo angle θ from kinematical discussions.

Hiroshima model

T.Hayashi, YK, S.Ogawa, PTP (1968)

$$
\begin{aligned}
u &= \langle(\nu_e\overline{\nu}_e \cos\theta + \nu_\mu\overline{\nu}_\mu \sin\theta\rangle)B\rangle \\
c &= \langle(-\nu_e\overline{\nu}_e \sin\theta + \nu_\mu\overline{\nu}_\mu \cos\theta\rangle)B\rangle \\
d &= \langle(e^-\overline{\nu}_e)B\rangle \\
s &= \langle(\mu^-\overline{\nu}_\mu)B\rangle
\end{aligned}
$$

- Semileptonic decays $d \to u$ and $s \to u$ are understood from $e^- \to \nu_e$ with the factor $\cos\theta$ and from $\mu^- \to \nu_\mu$ with the factor $\sin\theta$, respectively.
- Nonleptonic decay $s \to d$ can be understood from $\mu^-\overline{\nu}_\mu \to e^-\overline{\nu}_e$ with the factor 1 without assuming the so-called penguin diagram.

The basis idea was analogy

to the hadronic π^0-η^0-σ^0 mixing

where we assume $(\nu_e, \nu_\mu, \ell) \sim$ 3 of U(3),

and 3 quarks are members of 8+1 of U(3);

Eigenstates of U-spin

$$\pi = \frac{1}{\sqrt{2}}(\nu_e\overline{\nu}_e - \nu_\mu\overline{\nu}_\mu)$$
$$\eta = \frac{1}{\sqrt{6}}(\nu_e\overline{\nu}_e + \nu_\mu\overline{\nu}_\mu - 2\ell\overline{\ell})$$
$$\sigma = \frac{1}{\sqrt{3}}(\nu_e\overline{\nu}_e + \nu_\mu\overline{\nu}_\mu + \ell\overline{\ell})$$

$$u_0 = (\pi B) \qquad c_0 = (\eta B)$$
$$d_0 = (\ell\overline{\nu_e}B) \quad s_0 = (\ell\overline{\nu_\mu}B)$$

$\Rightarrow$

Eigenstates of I-spin

$$\pi = \frac{1}{\sqrt{2}}(\nu_e\overline{\nu}_e - \nu_\mu\overline{\nu}_\mu)$$
$$\omega = \frac{1}{\sqrt{2}}(\nu_e\overline{\nu}_e + \nu_\mu\overline{\nu}_\mu)$$
$$\phi = \ell\overline{\ell}$$

$$u = (\pi' B) \qquad c = (\omega' B)$$
$$d = (e\overline{\nu_e}B) \quad s = (\mu\overline{\nu_\mu}B)$$

$$\pi' = \frac{\sqrt{3}}{2}\pi + \frac{1}{2}\omega = \frac{\sqrt{3}+1}{2\sqrt{2}}\nu_e\overline{\nu}_e - \frac{\sqrt{3}-1}{2\sqrt{2}}\nu_\mu\overline{\nu}_\mu$$

The model was strained and tricky.

Therefore, the paper was rejected to be published by PTP.

I think that the rejection was reasonable.

In **1978**, Harari et al. have proposed a model based on a permutation symmetry S_3:

H. Harari, H. Haut and J. Weyers, PL 78B (1978) 459

They have derived the Cabibbo angle

$$\sin\theta_c = \frac{\sqrt{3}-1}{2\sqrt{2}}, \qquad \cos\theta_c = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Their work was very beautiful, so that I was a little discouraged.

In 1982, I have proposed

a charged lepton mass relation

$$m_e + m_\mu + m_\tau = \frac{2}{3}\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2 \qquad (1.1)$$

It is well-known that the observed charged lepton mass spectrum satisfies the relation with remarkable precision.

[YK, LNC (1982); PLB (1983)]

The mass formula (1.1) is invariant under any exchange $\sqrt{m_i} \leftrightarrow \sqrt{m_j}$.

This suggests that a description by S_3 may be useful for the mass matrix model.

[S_3: Pakvasa and Sugawara (1978); Harari, Haut and Weyers (1978)]

However, this formula was derived from an extension of the π-η-σ mixing model (composite model), and I had never considered an S_3 symmetry until 1999.

In the present talk, I would like to demonstrate

how the S_3 symmetry is promising to understand the masses and mixings.

And, I will comment that

an A_4 symmetry is also promising.

Although we think that there are beautiful relations among quark and lepton masses and mixings, it is hard to see such a symmetry in the quark sector, because the original symmetry will be spoiled by the gluon cloud.

In the present study, I will confine myself to the investigation of the lepton masses and mixings.

2 Leptons under S_3

I would like to show that the S_3 symmetry is promising not only for understanding of the charged lepton mass relation (1.1), but also for that of the observed neutrino mixing, i.e. the so-called tribimaximal mixing

$$U_{TB} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad (2.1)$$

[Harrison, Perkins, Scott, PLB (1999); Xing, PLB (2002); Ma, PRL (2003);]

2.1 Tribimaximal mixing

We define the doublet (ψ_π, ψ_η) and singlet ψ_σ of S_3

$$\begin{pmatrix} \psi_\pi \\ \psi_\eta \\ \psi_\sigma \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \quad (2.2)$$

If the neutrino mass eigenstates are

$(\nu_\pi, \nu_\eta, \nu_\sigma)$ defined by the relation (2.2)

with the mass hierarchy $m^2_{\nu\eta} < m^2_{\nu\sigma} < m^2_{\nu\pi}$

in contrast to the weak eigenstates

$(\nu_1, \nu_2, \nu_3) = (\nu_e, \nu_\mu, \nu_\tau)$,

then we can obtain the tribimaximal mixing

(2.1) because of the relation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_\eta \\ \nu_\sigma \\ \nu_\pi \end{pmatrix} \tag{2.3}$$

2.2 Charged lepton mass formula and Higgs potential

The mass formula (1.1) can be understood from a universal seesaw model with 3-flavor scalars ϕ_i [YK (1990)]

$$M_f = m_L^f M_F^{-1} m_R^f \tag{2.4}$$

Universal seesaw: Gerezhiani (1983); Chang, Mohapatra (1987); Davidson, Wali (1987), Rajpoot (1987), Babu, Mohapatra (1989)

For the charged lepton sector, we take

$$m_L^e = \frac{1}{\kappa} m_R^e = y_e \mathrm{diag}(v_1, v_2, v_3) \tag{2.5}$$

where the VEV $v_i = \langle \phi_i \rangle$ satisfy the relation

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2 \tag{2.6}$$

The VEV relation (2.6) means

$$v_\pi^2 + v_\eta^2 = v_\sigma^2 \tag{2.7}$$

because

$$v_1^2 + v_2^2 + v_3^2 = v_\pi^2 + v_\eta^2 + v_\sigma^2$$

$$= 2v_\sigma^2 = 2\left(\frac{v_1 + v_2 + v_3}{\sqrt{3}}\right)^2 \tag{2.8}$$

The Higgs potential which gives the relation (2.7) is, for example, given by

$$V=\mu^2\sum_i(\overline{\phi}_i\phi_i)+\frac{1}{2}\lambda_1\left[\sum_i(\overline{\phi}_i\phi_i)\right]^2 \tag{2.9}$$

$$+\lambda_2(\overline{\phi}_\sigma\phi_\sigma)(\overline{\phi}_\pi\phi_\pi+\overline{\phi}_\eta\phi_\eta)+V_{soft\ sym.br}$$

Here, the existence of the S_3 invariant λ_2-term is essential. Note that the VEV relation (2.7) can be obtained independently of the explicit values of the parameter λ_2.

YK, PRD73 (2006),

2.3 Yukawa interaction form under S_3

The general form of the S_3 invariant Yukawa interaction is given by

$$H = \left(y_0 \frac{\bar{\psi}_\pi \psi_\pi + \bar{\psi}_\eta \psi_\eta + \bar{\psi}_\sigma \psi_\sigma}{\sqrt{3}} + y_1 \frac{\bar{\psi}_\pi \psi_\pi + \bar{\psi}_\eta \psi_\eta - 2\bar{\psi}_\sigma \psi_\sigma}{\sqrt{6}} \right) \phi_\sigma$$

$$+ y_2 \left(\frac{\bar{\psi}_\pi \psi_\eta + \bar{\psi}_\eta \psi_\pi}{\sqrt{2}} \phi_\pi + \frac{\bar{\psi}_\pi \psi_\pi - \bar{\psi}_\eta \psi_\eta}{\sqrt{2}} \phi_\eta \right)$$

$$+ y_3 \frac{\bar{\psi}_\pi \phi_\pi + \bar{\psi}_\eta \phi_\eta}{\sqrt{2}} \psi_\sigma + y_4 \bar{\psi}_\sigma \frac{\phi_\pi \psi_\pi + \phi_\eta \psi_\eta}{\sqrt{2}} \qquad (2.10)$$

For the charged lepton sector, we have already assumed the form

$$H_e = y_e \left(\bar{\ell}_{L1} \phi^d_{L1} E_{R1} + \bar{\ell}_{L2} \phi^d_{L2} E_{R2} + \bar{\ell}_{L3} \phi^d_{L3} E_{R3} \right) \quad \textbf{(2.11)}$$

The form (2.11) corresponds to the case

$$y_0 = y_e, \quad y_1 = 0, \quad y_2 = \frac{1}{\sqrt{3}} y_e, \quad y_3 = y_4 = \sqrt{\frac{2}{3}} y_e \quad \textbf{(2.12)}$$

in the general form (2.10).

The general study under the S_3 symmetry will be found in YK, a preprint hep-ph/0612058.

Here, I will skip the details and

let us review only a simple example.

2.4 A simple example of the S_3-invariant neutrino interaction

Now, I would like to propose an S_3-invariant neutrino interaction with a concise form

$$H_\nu = y_\nu \left(\frac{\bar{\ell}_\pi N_\pi + \bar{\ell}_\eta N_\eta + \bar{\ell}_\sigma N_\sigma}{\sqrt{3}} \phi^u_\sigma + \frac{\bar{\ell}_\pi N_\eta + \bar{\ell}_\eta N_\pi}{\sqrt{2}} \phi^u_\pi + \frac{\bar{\ell}_\pi N_\pi - \bar{\ell}_\eta N_\eta}{\sqrt{2}} \phi^u_\eta \right) \quad (2.13)$$

Here, we have assumed

the universality of the coupling constants

to the ϕ_σ, ϕ_π and ϕ_η terms.

YK, preprint hep-ph/0605074

Then, we obtain a simple mass spectrum

$$m_{\nu 1} = \left(\frac{1}{\sqrt{6}} - \frac{1}{2}\right)^2 m_0^\nu$$
$$m_{\nu 2} = \frac{1}{6} m_0^\nu$$
$$m_{\nu 3} = \left(\frac{1}{\sqrt{6}} + \frac{1}{2}\right)^2 m_0^\nu$$

$$m_{\nu 1} = (5.3^{+0.4}_{-0.3}) \times 10^{-4}\ \mathrm{eV},$$
$$m_{\nu 2} = (1.05^{+0.07}_{-0.05}) \times 10^{-2}\ \mathrm{eV},$$
$$m_{\nu 3} = (5.22^{+0.35}_{-0.25}) \times 10^{-2}\ \mathrm{eV}.$$

(2.14)

Here, we have used the relation

$$v_\pi^2 + v_\eta^2 = v_\sigma^2 \qquad (2.15)$$

from the S_3 Higgs potential model.

The present case predicts

$$R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{4\sqrt{6}-9}{4\sqrt{6}+9} = 0.0423 \qquad (2.16)$$

The predicted value is somewhat larger than

$$R_{obs} \equiv \frac{\Delta m_{solar}^2}{\Delta m_{atm}^2} = \frac{(7.9^{+0.6}_{-0.5}) \times 10^{-5}\mathrm{eV}^2}{(2.72^{+0.38}_{-0.25}) \times 10^{-3}\mathrm{eV}^2} = (2.9 \pm 0.5) \times 10^{-2} \qquad (2.17)$$

However, the case cannot, at present, be ruled out within three sigma.

If $v_\pi/v_\eta \neq 0$, m_L^ν cannot be diagonal on the basis $(\nu_\pi, \nu_\eta, \nu_\sigma)$. The mixing angle $\theta_{\pi\eta}$ of the further rotation between ν_π-ν_η is given by

$$\tan\theta_{\pi\eta} = v_\pi/v_\eta \qquad (2.18)$$

It is well known that the $2 \leftrightarrow 3$ symmetry is promising for neutrino mass matrix description.

[Fukuyama-Nishiura (1997), Ma-Raidal (2001), Lam (2001), Balaji-Grimus-Schwetz (2001), Grimus-Laboura (2001)]

We also assume the $2 \leftrightarrow 3$ symmetry for $\langle\phi_i^u\rangle$, i.e. $v_2^u = v_3^u$, which leads to

$$\langle\phi_\pi^u\rangle = 0 \qquad \textbf{(2.19)}$$

Therefore, the present model gives the exact tribimaximal mixing:

$$\sin^2 2\theta_{23} = 1, \quad \tan^2\theta_{12} = 1/2, \quad \theta_{13} = 0 \tag{2.20}$$

Note that if we require the $2 \leftrightarrow 3$ symmetry for the fields $\ell_{Li} = (\nu_{Li}, e_{Li})$, the symmetry will affect the charged lepton sector, too.
Here, we have assumed the $2 \leftrightarrow 3$ symmetry only for $\langle\phi_i^u\rangle$, not for $\langle\phi_i^d\rangle$, so that the symmetry does not affect the charged lepton mass matrix.

Summery of the S_3 model

- We have assumed a universal seesaw model.
- The S_3 symmetry in the Yukawa interactions is strictly unbroken.
- S_3 is broken only though the VEV of Higgs scalars ϕ_i .
- We have required the universality of the coupling constants on the basis (e_1, e_2, e_3) for the charged lepton sector, while those on the basis $(\nu_\pi, \nu_\eta, \nu_\sigma)$ for the neutrino sector.

3 Prospect of the Charged Lepton Mass Formula

3.1 Brannen's speculations

[Brannen (2006)]

Recently, Brannen has speculated a neutrino mass relation similar to the charged lepton mass relation (1.1):

$$m_{\nu 1}+m_{\nu 2}+m_{\nu 3}=\frac{2}{3}\left(-\sqrt{m_{\nu 1}}+\sqrt{m_{\nu 2}}+\sqrt{m_{\nu 3}}\right)^2$$

(3.1)

Of course, we cannot extract the values of the neutrino mass ratios $m_{\nu 1}/m_{\nu 2}$ and $m_{\nu 2}/m_{\nu 3}$ from the neutrino oscillation data Δm^2_{solar} and Δm^2_{atm} unless we have more information on the neutrino masses, so that we cannot judge whether the observed neutrino masses satisfy the relation (3.1) or not.

Generally, the masses which satisfy the relations (1.1) and (3.1) can be expressed as

$$m_{fi} = (z_{fi})^2 m_{f0} \qquad (3.2)$$

where

$$\begin{aligned} z_{f1} &= \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \sin \xi_f \\ z_{f2} &= \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \sin(\xi_f + \tfrac{2}{3}\pi) \\ z_{f3} &= \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \sin(\xi_f + \tfrac{4}{3}\pi) \end{aligned} \qquad (4.3)$$

$$(z_{f1})^2 + (z_{f2})^2 + (z_{f3})^2 = 1 \qquad (3.4)$$

Then, Brannen has also speculated the relation

$$\xi_\nu = \xi_e + \frac{\pi}{12} \qquad (3.5)$$

From the observed charged lepton mass values, we obtain

$$\xi_e = \frac{\pi}{4} - \varepsilon = 42.7324^\circ \quad (\varepsilon = 2.2676^\circ) \tag{3.6}$$

Then, the Brannen relation (3.5) gives

$$\xi_\nu = 57.7324^\circ \tag{3.7}$$

which predicts

$$R = \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = 0.0318 \tag{3.8}$$

The value (3.8) is in good agreement with the observed value 0.029 +/- 0.005.

Therefore, the speculations by Brannen are favorable to the observed neutrino data.

We can understand Brannen's first relation (3.1) by assuming two types of ϕ^f i.e. $\phi^u \neq \phi^d$. However, Brannen's second relation (3.5) is hard to be derived from the conventional symmetries. it is an open question that the relation (3.5) is accidental or not .

[YK, hep-ph/0612058]

Besides, Brannen (2006) and G.Rosen (2006) have speculated that the observed value $\xi_e = 42.7324^\circ$ is given by the relation

$$\xi_e = \frac{\pi}{6} + \frac{2}{9} \qquad (3.9)$$

In deed, the value 2/9 is

2/9 rad = 12.732395° (3.10)

It is an amazing coincidence!

However, my opinion is as follows:

Too adhering to this coincidence will again push the formula (1.1) into a mysterious world, so that I do not recommend that you take the relation (3.9) seriously at present.

3.2 From Seesaw to Frogatt-Nielsen

The seesaw model with three $SU(2)_L$ doublet scalars causes the FCNC problem.

Therefore, the seesaw model may, for example, be translated into **a Frogatt-Nielsen-type model**:

$$H_{eff} = y_e \bar{l}_L H_L^d \frac{\phi^d}{\Lambda_d} \frac{\phi^d}{\Lambda_d} e_R + y_\nu \bar{l}_L H_L^u \frac{\phi^u}{\Lambda_u} \nu_R + y_R \overline{\nu}_R \Phi \nu_R^* \tag{3.11}$$

The argument about flavor structure is essentially unchanged under the present model-changing.

However, the scenario for the symmetry breaking energy scale will be considerably changed:

Energy scale of $v_i = \langle \phi_i \rangle$	Seesaw	Frogatt-Nielsen
	10^2 GeV	10^{19} GeV

In the Frogatt-Nielsen-type model, the S_3-broken structure of the effective Yukawa coupling constants is given at the Planck mass scale. However, this is no problem, because the formula (1.1) is not so sensitive to the RGE effects as far as the lepton sector is concerned.

[Xing-Zhang PLB (2006), Li-Ma PRD (2006)]

3.3 From S_3 to A_4

Recently, Ma (UC, Riverside) has explained the observed tribimaximal mixing on the basis of A_4.
This suggests that the A_4 symmetry is also promising as a symmetry of leptons.

Ma, Phys.Rev. D73, 057304 (2006)

- **Irreducible representations of A_4**

When we denote 3 of A_4 as

$$\overline{\psi} = \begin{pmatrix} \overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3 \end{pmatrix} \sim \mathbf{3}, \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \sim \mathbf{3} \qquad (3.12)$$

we can make 1, 1', 1" from 3x3 as follows:

$$(\overline{\psi}\psi)_1 = \frac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2 + \overline{\psi}_3\psi_3)$$
$$(\overline{\psi}\psi)_{1'} = \frac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2\omega + \overline{\psi}_3\psi_3\omega^2)$$
$$(\overline{\psi}\psi)_{1''} = \frac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2\omega^2 + \overline{\psi}_3\psi_3\omega) \qquad (3.13)$$

where

$$\omega = e^{i\frac{2}{3}\pi} = \frac{-1+i\sqrt{3}}{2} \qquad (3.14)$$

When we define

$$(\overline{\psi}\psi)_\sigma = \frac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2 + \overline{\psi}_3\psi_3)$$
$$(\overline{\psi}\psi)_\eta = \frac{1}{\sqrt{6}}(2\overline{\psi}_1\psi_1 - \overline{\psi}_2\psi_2 - \overline{\psi}_3\psi_3) \quad (3.15)$$
$$(\overline{\psi}\psi)_\pi = \frac{1}{\sqrt{2}}(\overline{\psi}_3\psi_3 - \overline{\psi}_2\psi_2)$$

we can express (3.13) as

$$(\overline{\psi}\psi)_1 = (\overline{\psi}\psi)_\sigma$$
$$(\overline{\psi}\psi)_{1'} = \frac{1}{\sqrt{2}}\left[(\overline{\psi}\psi)_\eta - i(\overline{\psi}\psi)_\pi\right] \quad (3.16)$$
$$(\overline{\psi}\psi)_{1''} = \frac{1}{\sqrt{2}}\left[(\overline{\psi}\psi)_\eta + i(\overline{\psi}\psi)_\pi\right]$$

Therefore, if we define $(\phi_\pi, \phi_\eta, \phi_\sigma)$ as

$$\begin{aligned}\phi_1 &= \phi_\sigma \\ \phi_{1'} &= \frac{1}{\sqrt{2}}(\phi_\eta - i\phi_\pi) \\ \phi_{1"} &= \frac{1}{\sqrt{2}}(\phi_\eta + i\phi_\pi)\end{aligned} \tag{3.17}$$

we can write an **A_4 invariant Yukawa interaction** as follows:

$$\begin{aligned}&(\overline{\psi}\psi)_1\phi_1 + (\overline{\psi}\psi)_{1'}\phi_{1"} + (\overline{\psi}\psi)_{1"}\phi_{1'} \\ &= (\overline{\psi}\psi)_\sigma\phi_\sigma + (\overline{\psi}\psi)_\eta\phi_\eta + (\overline{\psi}\psi)_\pi\phi_\pi\end{aligned} \tag{3.18}$$

Thus, we can translate the relations in S_3 into those in A_4. The A_4 symmetry will be also promising in the symmetry of the leptons.

- **Higgs potential which gives** $v_\pi^2 + v_\eta^2 = v_\sigma^2$

The existence of the λ_2-term, $\phi_\sigma^2(\phi_\pi^2 + \phi_\eta^2)$, was essential in the S_3 Higgs potential model. In an A_4 model, the λ_2 -term can be composed as

$$\phi_1\phi_1\left(\phi_{1'}\phi_{1''} + \phi_{1''}\phi_{1'}\right) = \phi_\sigma^2\left(\phi_\pi^2 + \phi_\eta^2\right) \tag{3.19}$$

Thus, the charged lepton mass relation (1.1) can also be derived from the A_4 model.

3.4 From non-SUSY to SUSY

Since the formula (1.1) is so exact, we need to find a condition which is protected against large corrections.

Ma has recently proposed a **supersymmetric S_3-invariant Higgs potential**, because supersymmetry is unbroken even after the superfields acquire VEVs:

$$W = \frac{1}{2} m \left(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2 \right) + \frac{1}{3} \lambda \phi_\sigma^3 + \lambda \phi_\sigma \left(\phi_\pi^2 + \phi_\eta^2 \right) \tag{3.20}$$

which also leads to the relation $v_\pi^2 + v_\eta^2 = v_\sigma^2$.

By extending this idea, and by applying a symmetry $\Sigma(81)$ to the model, Ma has proposed a new model which leads to the charged lepton mass relation (1.1) :

$$W = \frac{1}{2}m_0\chi_0^2 + \frac{1}{2}m_1\chi_1^2 + \frac{1}{2}m_2\chi_2^2 + m_3\chi_1\chi_2 + \frac{1}{3}\lambda(\chi_0^3 + \chi_1^3 + \chi_2^3 + 6\chi_0\chi_1\chi_2) \tag{3.21}$$

Ma, hep-ph/0612022

This will throw new light on the formula (1.1).

4 Summary

The charged lepton mass formula is not supernatural beings, so that the investigation should be done along the conventional mass matrix approach based on the field theory.

A promising possibility has come up:

The charged lepton mass formula (1.1) and the tribimaximal mixing (2.1) are related each other and both can be understood from S_3 or A_4 .

Our next step is to attack the quark masses and mixing by applying the present approach.

I am happy that the understanding of the charged lepton mass formula (1.1) is, step by step, proceeding from the mysterious world to the actual world.